Design and Scheduling of Multi-Period and Multipurpose Batch Plants

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Abstract

This paper presents a new method for simultaneous cyclic scheduling and design based on a Mixed Integer Nonlinear Programming (MINLP) formulation. It takes task completion time, cycle time, batch size and relative equipment size as determinant variables to consider design and scheduling simultaneously. In particular, previous formulation did not take into account the important issue of how to divide the production ratio within a cycle; they simply split it by intuition. The method proposed here tackles this problem by including a determinant variable specifying how to divide the production ratio in a cycle. If the system to be solved is sufficiently large that no feasible solution can be found within a reasonable computation time, we decompose the original MINLP model into MILP and NLP subproblems and find the solutions using an iterative procedure.

1. Introduction

Multipurpose batch plants have been extensively studied due to their advantageous characteristics, particularly their process flexibility and suitability for producing high-value-added small products. Because companies receive large benefits from high-value-added products with small amounts instead, the technology itself is expensive. Therefore, research aimed at improving optimization techniques for use in process design holds an important place in industry. Most previous attempts to optimize the multipurpose batch process have used the Mixed Integer Linear Programming (MILP) model. The MILP model can be solved using a branch and bound algorithm; however, this approach suffers from the critical shortcoming that branch and bound nodes increase exponentially with the number of integer or binary variables. To overcome this shortcoming, researchers have attempted to mathematically decompose the model into smaller models while leaving the process characteristics unchanged. The elimination of meaningless binary variables from the model also helps to overcome this problem.

Fuchino et al. determined the optimal scheduling of a multiproduct and multipurpose plants[1] and subsequently introduced an evolutionary design method and solved the design and cyclic scheduling problems[2]. In addition they later extended their method to multi-period production plans[3]. Heo et al.[4] proposed a modified model that contained both relative equipment size and scheduling variables, and showed that this model gave much better solutions than the formulation of Fuchino et al.[2] They pointed the locality due to their intuitive selection of relative equipment in Fuchino et al.[2]

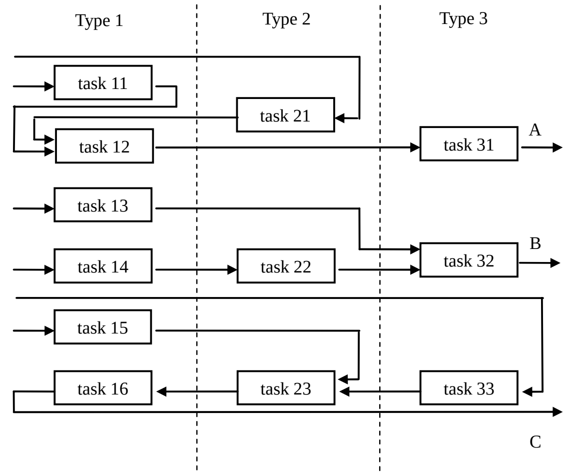


Figure 1. Recipe data of example 1

This paper points out the locality of the solution in that Fuchino et al.[3] determined a production split ratio in convenience. Our MINLP model includes the production split ratio as a determinant variable, which divides the total requirement into several splits. In addition, we determine the relative equipment, batch size and completion time simultaneously. The solution could be found using the Generalized Benders Decomposition method. Although a heuristic decomposition steps to MILP and NLP subproblems are accompanied because hundreds of binary variables belong to a bigger MINLP problem, much better solutions are found in this work.

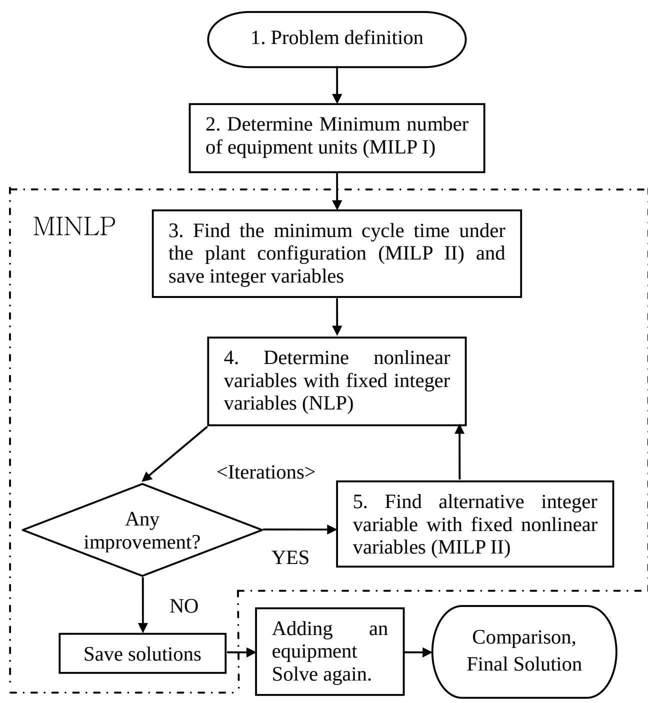


Figure 2. Schematic diagram for design and scheduling

1. Problem Definition

This work tackles the problem of determining the optimal design and scheduling for complicated manufacturing processes producing high-value-added products in small quantities. An example of such a process is shown in **Figure 1**. Given data involved in these processes is the number of products, *N*; the number of periods, *T*, which have a constant time horizon; the equipment units and their types required for manufacturing the *N* products; the task sequence for each product, which includes divergence and merge of task flow, processing time and size factor of each task, transportation time from one unit to another, and sequence dependent setup time and cost data. Determinant variables include the number of equipment units of each type, volume of equipment, relative volume of equipment, relative batch size, cycle time, makespan per cycle, production split ratio and start and completion times of all the tasks.

The objective function is to minimize the investment cost for the plant configuration while satisfying the production requirements of whole periods and other constraints under the No Intermediate Storage policy.

Several assumptions are made based on the characteristics of high-value-added products and the mathematical model: satisfaction of the minimum operating ratio, which means that the batch size must be over the defined portion of the equipment size; allowance of common usage within the same type of equipment and in-phase operation of a task in the same type of equipment; possibility that requirements are satisfied after repeated cycles or sum of product split with small amount; same processing times of split tasks with original amount of tasks and so on. Even though the batch size is small all, the tasks must abide by their size factors.

Table1. Data for example 1

(a) Requirement of three products within three time periods

	A	B	C
t_1	3000	2000	1000
t_2	2000	1000	3000
t_3	1000	3000	2000

(b) Processing times and size factors

Type	Task *i*	PT_i	FS_i
1	11	0.20	0.50
	12	0.15	1.00
	13	0.20	0.60
	14	0.15	0.40
	15	0.15	0.30
	16	0.15	1.00
2	21	0.25	0.50
	22	0.25	0.40
	23	0.25	1.00
3	31	0.20	1.00
	32	0.15	1.00
	33	0.15	0.70

(c) Sequence dependent cleanup times

From/To	A	B	C
A	-	0.05	0.10
B	0.10	-	0.15
C	0.05	0.15	-

Table 2. Cost data

	Type 1	Type 2	Type 3
α_j	250.0	350.0	300.0
β_j	0.6	0.6	0.6

3. Solution Strategies

The proposed strategy for optimizing the design and scheduling of batch processes is shown in **Figure 2**. Step 1 is described in the previous section. In step 2 (MILP I), we use MILP to determine an initial plant configuration with the minimum number of equipment units that could feasibly satisfy the requirements.

MILP I

MILP I gives the minimum number of equipment units that can satisfy production requirements.

Objective function: minimize $NOU = \sum_{j}^{JN} WV_j$ (1)

where WV_j is a binary variable that equals 1 when the equipment unit *j* is introduced and 0 otherwise, and *JN* is the maximum number of equipment units. This objective function, which is the sum of introducing equipment units, is to be minimized subject to:

$$W_{pisjt} \le WV_j \quad for \quad \forall p, i(p), s(p,t), j, t \qquad (2)$$

If task *i* of product *p* and its split *s* is processed in unit *j* within period *t*, binary variable W_{pisjt} equals 1. When W_{pisjt} = 1, the unit *j* must be introduced.

$$\sum_{j(i)} W_{pisjt} \ge 1 \quad for \quad \forall p, i(p), s(p,t), t \qquad (3)$$

Each task must be processed at least once and can be performed with in-phase operation.

$$B_{pisjt} + M_1 \cdot (1 - W_{pisjt}) \ge MOR \cdot RV_j \quad for \quad \forall p, i(p), s(p,t), j(i), t \qquad (4)$$

$$B_{pisjt} \le RV_j + M_1 \cdot (1 - W_{pisjt}) \quad for \quad \forall p, i(p), s(p,t), j(i), t \qquad (5)$$

When task *i* of product *p* and its split *s* is processed in unit *j* within period *t*, the relative batch size (B_{pisjt}) must be greater than the minimum operation ratio (*MOR*) of the relative volume of equipment (RV_j) and smaller than the relative volume of equipment. A sufficiently large positive number, M_1, is used in Eqs. (4)-(6), and in other equations below.

$$B_{pisjt} \le M_1 \cdot W_{pisjt} \quad for \quad \forall p, i(p), s(p,t), j(i), t \qquad (6)$$

where, *i(p)* : task *i* of product *p*; *j(i)* : unit which is able to process task *i* ; *s(p,t)* : split of product *p* in time period *t*. Tasks can be separately processed with in-phase operation. The summation of the relative batch sizes is equal to the total amount of task *i*, which is calculated by multiplication of requirement of product *p* in period *t* (RPT_{pt}), size factor of task *i* (FS_i) and proportion of split *s* of product *p* in period *t* (PS_{pst}).

$$FS_i \cdot RPT_{pt} \cdot PS_{pst} = \sum_{j(i)} B_{pisjt} \quad for \quad \forall p, i(p), s(p,t), t \qquad (7)$$

The summation of the split ratios must equal unity.

$$\sum_{s(p,t)} PS_{pst} = 1 \quad for \quad \forall p, t \qquad (8)$$

Two points in the present formulation should be noted. First, the variable PS_{pst} is introduced, which specifies how to divide product *p* into split *s* in period *t*. Second, an additional constraint is imposed, namely that the sum of the production split ratio equals unity. In previous work, for example, when 5000 units of a product were required, this total was divided into two splits of 2500 for convenience. However, the introduction in the present work of a variable that allows all possible splits increases the effectiveness with which the formulation finds out objective values. After determining the number of equipment units, we must solve an MINLP model under the obtained plant configuration.

MINLP model

Objective function: minimize $Cost = \sum_j \alpha_j \cdot \left(\frac{RV_j}{H / CTT} \right)^{\beta_j}$ (9)

where α_j and β_j are cost data of equipment unit *j*, RV_j is the relative volume of equipment unit *j*, *CTT* is maximum cycle time among periods, and *H/CTT* is the number of cycles within time horizon *H*. The volume of purchasing equipment (V_j) is calculated by dividing the volume of relative equipment unit by the number of cycles in time horizon (i.e., $V_j = RV_j/(H/CTT)$).

Subject to: Equations (3)-(8) and

$$CTT \geq CT_t \qquad (10)$$

By Eq. 10, the cycle time in the objective function is the maximum one of all the periods. The maximum cycle time must be chosen because of common usage of equipment in multi-period. If a smaller cycle time is used, the number of cycles increases. This causes the volumes of purchasing equipment unit to become smaller, which makes infeasibility at other periods.

$$CT_t = MS_t - SLD_t + Max_{pp'} \{ CLT_{pp'} \} \quad for \quad \forall t \qquad (11)$$

In Eq. 11, the cycle time of each period *t* is derived by subtracting the slack time between cycles (SLD_t) and by addition of a cleanup time to the makespan per cycle (MS_t). The cleanup time (C*LT*) can be fixed at the largest value required because it constitutes less than about 10% of the cycle time and, in most cases, the largest cleanup time falls between cycles. Generally, the term “makespan” refers to the total time required to complete a group of tasks. The cycle time is less than the makespan of a cycle because of heads (*SLF*) and tails (*SLL*):

$$SLF_{jt} \le C_{pist} - HT_{ist} - PT_i - tr_i + M_2 \cdot (1 - W_{pisjt}) \quad for \quad \forall p, i(p), s(p,t), j, t \qquad (12)$$

$$SLL_{jt} \le MS_t - tr_i - C_{pist} + M_2 \cdot (1 - W_{pisjt}) \quad for \quad \forall p, i(p), s(p,t), j, t \qquad (13)$$

$$SLD_t \le SLF_{jt} + SLL_{jt} \quad for \quad \forall j, t \qquad (14)$$

where M_2 is a sufficiently large positive number. In Eqs. 12 and 13, the head of unit *j* in period *t* is less than the start time of all the tasks and the tail of unit *j* in period *t* is greater than the makespan per cycle minus completion time (*C*) of all the tasks. The slack time between cycles is calculated by summing of the two terms in Eq. 14.

$$MS_t \ge C_{pist} + tr_i \quad for \quad \forall p, i, s, t \qquad (15)$$

$$C_{pist} - HT_{ist} - PT_i - tr_i \ge 0 \quad for \quad \forall p, i(p), s(p,t), t \qquad (16)$$

The makespan is greater than the completion times of all the tasks plus the transportation time (*tr*). The completion time of task *i* of split *s* of product *p* in period *t* is greater than the sum of the holding time (*HT*), processing time (*PT*), and the first transportation time for task *i*. In other words, the start time of each task is greater than the first transportation time.

$$C_{pist} + tr_i = C_{pi'st} - HT_{i'st} - PT_{i'} \quad for \quad \forall p, i(p), i'(i,p), s(p,t), t \qquad (17)$$

where, *i'(i,p)* : task *i'* following task *i* of product *p*. The start time of task *i'* of split *s* of product *p* in period *t* is expressed in the right hand side of Eq. 17. It is equal to the completion time of immediately before task *i'* (task *i*) plus transportation time.

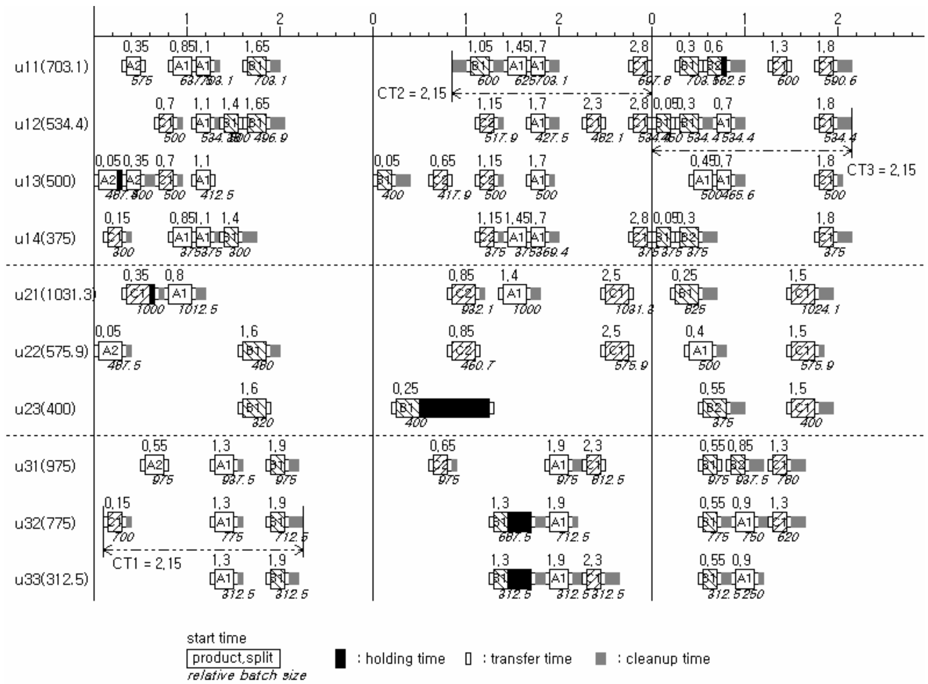
$$\begin{aligned} & C_{pist} + CLT_{pp'} + tr_i - M_2 \cdot (1 - Z_{psp's't}) \le C_{p'i's't} - HT_{i's't} - PT_{i'} - tr_i + M_2 \cdot (2 - W_{pisjt} - W_{p'i's'jt}) \\ & \quad for \quad \forall p, i(p), s(p,t), p', i'(p'), s'(p',t), j, t : p \ne p' \ or \ s \le s' \end{aligned} \qquad (18)$$

$$\begin{aligned} & C_{p'i''s't} + M_2 \cdot (1 - Z_{psp's't}) \ge C_{pist} + CLT_{pp'} + tr_i - M_2 \cdot (2 - W_{pisjt} - W_{p'i's'jt}) \\ & \quad for \quad \forall p, i(p), s(p,t), p', i''(p'), i'(i'', p'), s'(p',t), j, t : p \ne p' \ or \ s \le s' \end{aligned} \qquad (19)$$

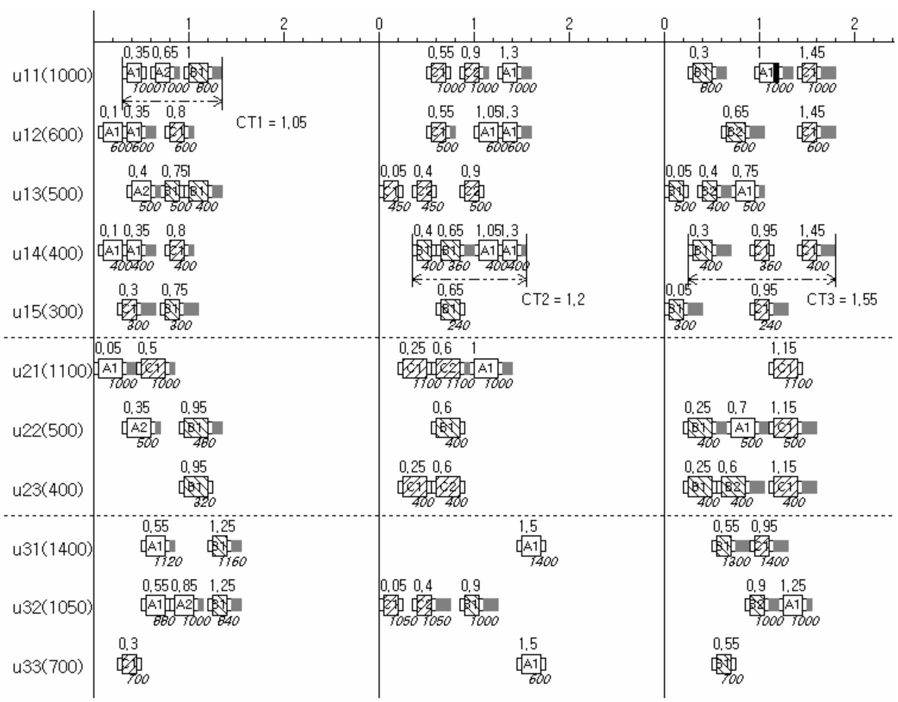
If product *p* and its split *s* is processed before product *p'* and its split *s'*, the value of binary variable $Z_{psp's'}$ equals unity. Equation 18 implies that the processing time, holding time and sequence dependent cleanup time have to be considered in adjacent tasks of the same unit *j*. In Eq. 19, if task *i'* is directly after task of task *i"*of the same product *p'*, the completion of task *i"* must be delayed when task *i* of another product *p* is processed in unit *j*. This MINLP problem can be solved using the Generalized Benders Decomposition method.

Next, we use the evolutionary search method of Fuchino et al.[2] to determine the optimal plant structure. Under this method, neighboring plant configurations are created by adding equipment. The solutions of these created configurations are obtained using

MINLP solver. The optimal neighboring configuration is then found by comparing the solutions for the various neighboring plant configurations. By repeating this process, we finally determine the solution for the overall plant configuration.



(a) In case of 10 units



(b) In case of 11 units

Figure 3. Solution of example 1 with relative volume

4. Illustrative Examples

First we consider a system producing three products, A, B, and C. The parameters for this system, referred to here as example 1, are shown in **Tables 1** and **2** and **Figure 1**. The minimum operating ratio is 80% and the transportation time is 0.05. The time horizon is 300. MILP I gives the minimum number of equipment units as ten, four of type 1 and three each of types 2 and 3. The investment cost is 7,138.39. When five equipment units of type 1 are used, the investment cost is lower (6,982.04). The Gantt charts for example 1 are shown in **Figure 3**. In these charts, the names and sizes of relative equipment units are indicated, with dotted lines separating equipment units of different type. The solutions were obtained using the software GAMS21.3/DICOPT running on a 2.54GHz Pentium IV PC. The final objective value obtained using our approach is 23.9% better than that obtained by Fuchino et al.[3]

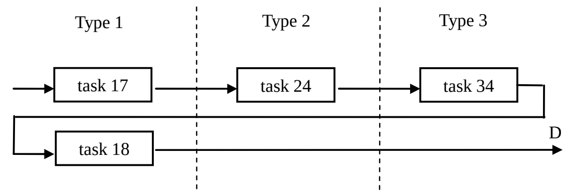


Figure 4. Recipe data of product D

In example 2, we consider a larger system with four products through four periods. The data for this example are shown in **Table 3** and **Figure 4**. As for example 1, the minimum plant configuration determined by MILP I is comprised of ten equipment units, four of type 1 and three each of types 2 and 3, which is less than in previous work[3]. The lower number of equipment units in the present work seems to be due to the introduction of the variable for the production split ratio (PS_{pst}).

Table 3. Data for example 2

(a) Requirement of four products within four time periods

	A	B	C	D
t_1	5000	2500	2000	3000
t_2	2500	1000	1000	6000
t_3	1000	7000	2500	1000
t_4	2000	2000	6000	1000

(b) Processing times and size factors

Type	Task *i*	PT_i	FS_i
1	17	0.15	1.00
	18	0.15	1.00
2	24	0.25	1.00
3	34	0.20	1.00

(c) Sequence dependent cleanup times

From/To	A	B	C	D
A	-	0.05	0.10	0.10
B	0.10	-	0.15	0.10
C	0.05	0.15	-	0.10
D	0.15	0.10	0.05	-

Using a 2.54GHz Pentium IV PC, for example, it may take several days to determine just one feasible solution, and the time required for the feasible solutions to reach the final solution by converging iterations of Generalized Benders Decomposition method may be impractically long.

Neighboring plant structures of example 1 and example 2 are such cases. Below we discuss a heuristic decomposition method of the MINLP model that overcomes this problem.

MINLP box of dotted line in the **Figure 2** shows the heuristic decomposition strategy. In step 3, the minimum cycle time under a given plant configuration is found using MILP II. The cycle time, one of the nonlinear variables of the original MINLP model, is the objective function of MILP II. In step 4, better values of nonlinear variables (RV_j and CT_t) are obtained by using a set of fixed binary variables derived in step 3. Then, in step 5, the relative volumes of equipment from the NLP solution are fixed and another set of binary variables is found. Repeated applications of steps 4 and 5 in an iterative procedure will give a good solution, but that solution is not guaranteed to be the optimal solution. When additional iterations no longer improve the objective function, the solution of the corresponding plant configuration is determined. Next, the Evolutionary Search method is used to determine the final solution.

MILP II

The MILP II model determines the cycle times of the production periods.

Objective: minimize $SCT = \sum_t CT_t$ (20)

Subject to: Equations (3)-(8) and (11)-(19)

The best solution of example 2, obtained through 10 iterations, had cycle times of 3.2, 3.1, 2.6 and 2.7 for the four periods, respectively, and equipment cost of 10,598.78. The Gantt chart for this system is shown in **Figure 5**. This result is 12.8% better than that obtained previously[3]. In example 2, no neighboring plant configuration gave a better solution.

A summary of the mathematical models for examples 1 and 2 is given in **Table 4**. Some problems are solved by Generalized Benders Decomposition, others by a heuristic decomposition method of the proposed MINLP solving strategy. Although the heuristic decomposition method is not guaranteed to give the optimal solution, it gave much better solutions for the large and complicated formulations involved in design and scheduling problems of multipurpose batch plants within a reasonable computation time.

The proposed strategies are easier to apply and give better solutions than those used previously. Heo et al.[4] used the repeated MILP by solving steps with separable programming after obtaining minimum cycle time and solving again with alternatively fixed cycle times (see Figure 1[4]).

5. Conclusions

This paper outlines a new approach to determining the optimal design and scheduling of multipurpose batch plants using the MINLP formulation. Previous works have been limited to the MILP form of this model. Their solution strategies consisted of the use of intuitive decisions or separable programming methods for one variable to treat the power of bilinear terms with directional search method for the other variable. In contrast, the proposed MINLP formulation and heuristic decomposition, which contain a new determinant variable governing the split ratio, gives much better solutions with simpler procedures.

Acknowledgment

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Table 4. Summary of models

example	equipments	binary	linear	Duration	iterations	Objective
1	4-3-3	250	664	3*d*15*h*35*m*11*s*	3*	7138.39
	5-3-3	290	737	413.3*s*	8	<u>6928.04</u>
	4-4-3	262	697	122.9*s*	6	7396.45
	4-3-4	262	697	87.6*s*	6	7065.36
	6-3-3	360	810	407.6*s*	5	7142.33
2	4-3-3	676	1532	9*h*56*m*16*s*	10	<u>10598.78</u>
	5-3-3	718	1616	4*h*4*m*49*s*	3	10795.30
	4-4-3	656	1528	13*h*31*m*50*s*	10	10953.02
	4-3-4	656	1528	2*h*20*m*7*s*	2	-

[Solution by Generalized Benders Decomposition*; *d*, *h*, *m*, *s* imply days, hours, minutes and seconds, respectively; lined objective is the final solution; 4-3-4 case of example 2, the volume of the last unit(u34) is zero]

References

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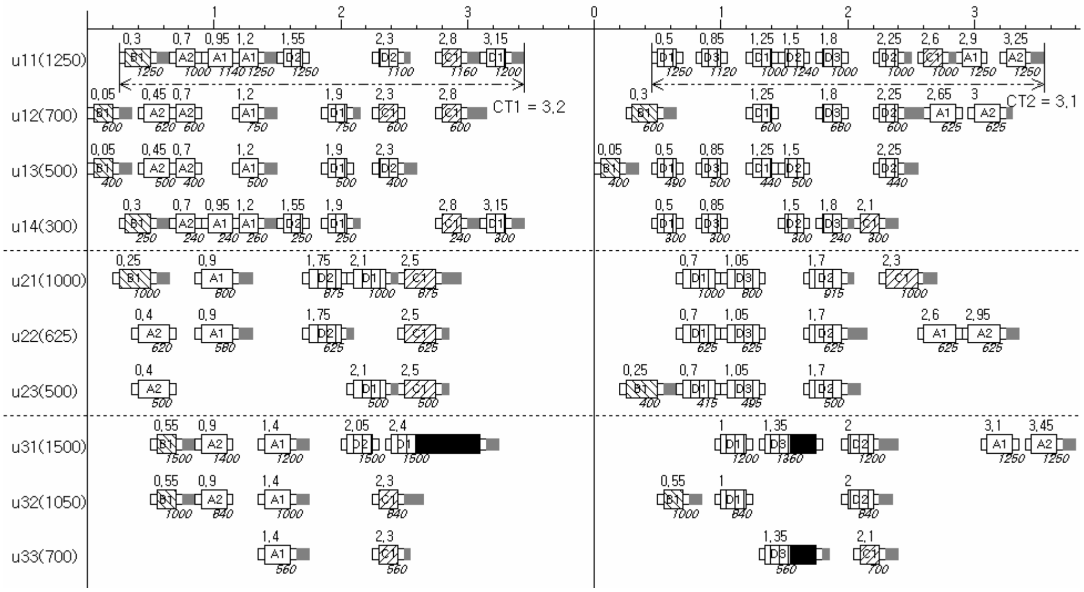
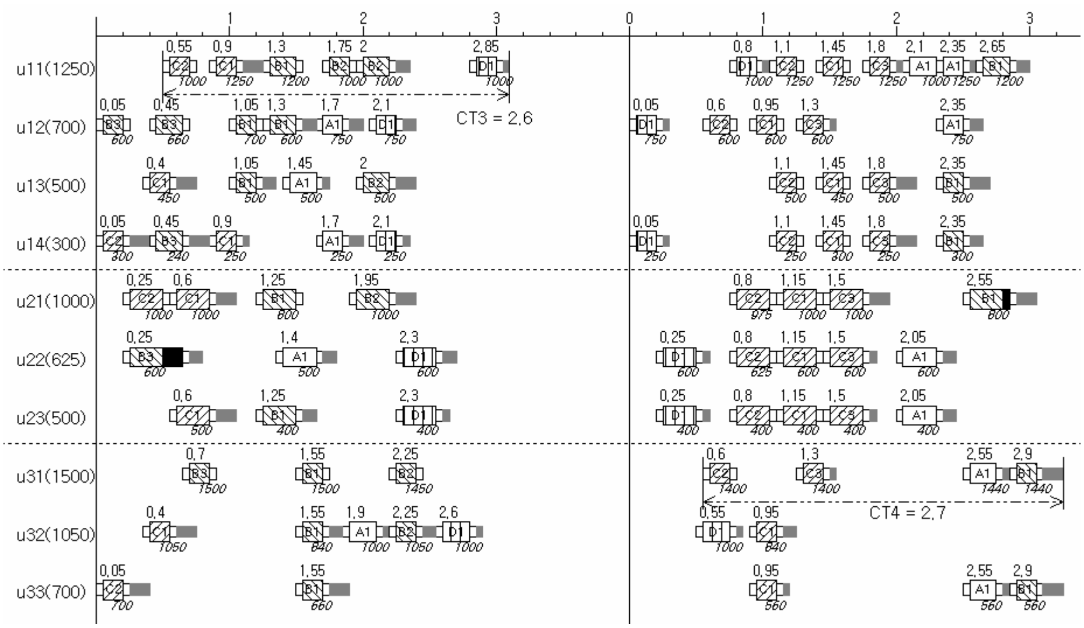


(a) Period 1 and 2



(b) Period 3 and 4

Figure 5. Solution of example 2