Integrated inventory and pricing policies for supply chain networks

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Prepared for presentation at the Annual AIChE Meeting (paper 421c)
November 7-12, 2004, Austin TX

Keywords: *Supply chain management, Model-based predictive control, Inventory control, Pricing policy, Demand elasticity*

UNPUBLISHED

Abstract

An optimization-based control framework that simultaneously determines the optimal inventory and product pricing policies is developed for multi-product, multi-echelon supply chain networks. The optimization problem aims at adjusting the available manufacturing resources, product transportation, inventories and prices for the entire supply chain network to satisfy demand while maximizing network's revenues and service level, through the minimization of unsatisfied demand, over a specified rolling time horizon. The control scheme employs model predictive control principles with local feedback inventory controllers for the satisfaction of the overall objectives. Customer demand responses to product prices are taken directly into consideration through the explicit utilization of demand elasticity. The optimal manipulation of the product prices acts as an additional instrument for the efficient operation of the supply chain through the direction of product demand in less congested parts of the network

1. Introduction

The purpose of supply chain management is to minimize the cost of transporting and storing products within a supply chain network, while satisfying end-point customers[1-2]. Operating network cost, average inventory level, and customer service level (fill rate of customer orders) are commonly employed performance measures[3]. The underlying modeling of supply chain dynamics has recently been thoroughly reviewed[2]. Decentralized inventory control strategies suffer from demand amplification between echelons[4]. Model-predictive control principles have been applied to individual echelons of the network[5]. However, the achieved control performance is suboptimal because it is restricted by the calculated control actions in a downstream echelon. In a recent paper[6] a model predictive control strategy was employed for the optimization of production/distribution systems, including a simplified scheduling model for the manufacturing function. The suggested control strategy considers only deterministic type of demand, which reduces the need for an inventory control mechanism.

The present paper attempts to associate this standard problem with the profit maximizing pricing policy of the network, and to tackle both simultaneously in an integrated fashion. In a nutshell, product price manipulation can be used to alleviate congested transportation routes, or to relieve heavily utilized inventory nodes, by altering appropriately the demand profile at the end-point nodes of the supply chain network. Essentially, a flexible node-level pricing policy can be understood as a substitute instrument to supply chain management, as it succeeds in altering the flow of orders customers place either by increasing or decreasing aggregate network demand for specific products, or by redirecting orders from one end-point node to others. As a result, the supply chain network does not have to reroute as strenuously inventories from one node to another to accommodate demand fluctuations, avoiding thus excessive costs due to clogged transportation routes and delays in deliveries. Instead, it finds preferable to persuade its customers, via the appropriate pricing policy, to redirect their orders to the desired end-point nodes.

For the purposes of our study and the time scales of interest, a discrete time difference model is developed, capable of analyzing networks of arbitrary structure. To treat product demand uncertainty within a deterministic supply chain network model, a rolling horizon model predictive control approach is suggested. A centralized optimization-based control strategy determines simultaneously the optimal product inventory, distribution and pricing policies for the maximization of network profits (gross of manufacturing costs) and satisfaction of service quality specifications. The algorithm uses a rolling horizon, to allow the incorporation of past and present control actions to future predictions. Optimal forecasting models are employed for the prediction of future product demand variation. Through illustrative simulations it is demonstrated that the model can accommodate supply chain networks of realistic size under a variety of stochastic and deterministic disturbances.

2. Supply Chain Model

Following the formulation in Seferlis and Giannelos[7], let *DP* denote the set of desired products in the supply chain. These can be manufactured at plants, *P*, utilizing various resources, *RS*. In the present study, product manufacturing considers independent production lines for the distributed products. The products are subsequently transported to and stored at warehouses, *W*. Products from warehouses are transported upon customer demand, either to distribution centers, *D*, or directly to retailers, *R*. Retailers set the prices for the various products and receive time-varying orders from different customers for each product. The level of the orders for one product depends on the price of the product, the price of the same product at neighboring retailer nodes, and the prices of other similar product, whether complimentary or substitute, in the same or neighboring nodes. Satisfaction of customer demand and maximization of network profits are the primary targets in the supply chain management mechanism. Unsatisfied demand is recorded as back-orders for the next time period. A discrete time difference model is used to describe the supply chain network dynamics. It is assumed that decisions are taken within equally spaced time periods (e.g., hours, days, or weeks). The duration of the base time period depends on the dynamic characteristics of the network. As a result, dynamics of higher frequency than that of the selected time-scale are considered negligible and completely attenuated by the network.

Plants *P*, warehouses *W*, distribution centers *D*, and retailers *R* constitute the nodes of the system. For each node, *k*, there is a set of upstream (predecessor) nodes, indexed by k', which can supply node *k*. There is also a set of downstream (successor) nodes, indexed by k'', which can be supplied by *k*. All valid (k',k) and/or (k,k'') pairs constitute permissible routes within the network. All variables in the supply chain network (e.g., inventory, transportation loads) are assumed to be continuous variables. This is definitely valid for bulk commodities and products. For unit products, continuous variables can still be utilized, with the addition of a post-processing rounding step to identify neighboring integer solutions. This approach, though clearly not formally optimal, may be necessary to retain computational tractability in systems of industrial relevance.

A product balance around any network node involves the inventory level in the node at time instances *t* and *t-1*, as well as the total inflow of products from upstream nodes and total outflow to downstream nodes. The following balance equation is valid for nodes that are either warehouses or distribution centers:

$$y_{i,k}(t) = y_{i,k}(t-1) + \sum_{k'} x_{i,k',k}\left(t - L_{k',k}\right) - \sum_{k''} x_{i,k,k''}(t) \qquad \forall\ k \in \{W, D\},\, t \in T,\, i \in DP \tag{1}$$

$y_{i,k}$ is the inventory of product *i* stored in node k. $x_{i,k',k}$ denotes the amount of the *i*-th product transported through route (k',k). $L_{k',k}$ denotes the lead time for the ordered products. Lead time includes the production time, when employed to orders placed at plant nodes, and the transportation lag (delay time) for route (k',k), i.e. the required time periods for the transfer of material from the supplying node to the current node. Lead-time is assumed to be an integer multiple of the base time period and independent of the size of the order as product manufacturing is solely limited by resource constraints.

For retailer nodes, the inventory balance is slightly modified to account for the actual delivery of the i-th product attained, denoted by $d_{i,k}$:

$$y_{i,k}(t) = y_{i,k}(t-1) + \sum_{k'} x_{i,k',k}(t - L_{k',k}) - d_{i,k}(t) \qquad \forall\ k \in R, t \in T\ , i \in DP \tag{2}$$

The amount of unsatisfied demand is recorded as back-orders for each product and time period. Hence, the balance equation for back-orders takes the following form:

$$BO_{i,k}(t) = BO_{i,k}(t-1) + R_{i,k}(t) - d_{i,k}(t) - LO_{i,k}(t) \qquad \forall\ k \in R, t \in T\ , i \in DP \tag{3}$$

where $R_{i,k}(t)$ denotes the demand for the *i*-th product at the *k*-th retailer node and time period *t*. $LO_{i,k}(t)$ denotes the amount of cancelled back-orders (lost orders) because the network failed to satisfy them within a reasonable time limit. Lost orders are usually expressed as a percentage of unsatisfied demand at time *t*. Due to the association of delivered products with the current prices at the retailer nodes, the back orders must also be associated with the prices at the time of order. In such a case a detailed record of the time and price history back orders should be maintained. In order to simplify the model, it is assumed that unsatisfied demand at any given time period is lost for the network, i.e., $BO_{i,k}(t-1) = LO_{i,k}(t)$.

At each node capable of carrying inventory (nodes of type *W*, *D*, and *R*), capacity constraints are in effect that account for a maximum allowable inventory level:

$$\sum_i a_i y_{i,k}(t) \le V_k^{max} \qquad \forall\ k \in \{W, D, R\}, t \in T \tag{4}$$

where $y_{i,k}$ denotes the inventory of the *i*-th product in the *k*-th node, a_i the storage volume factor for the i-th product, and V_k^{max} the total volumetric capacity of the *k*-th node. Similarly, a maximum allowable transportation capacity is defined for each permissible transportation route within the supply chain network:

$$\sum_i b_i x_{i,k,k''}(t) \le T_{k,k''}^{\max} \qquad \forall\ k \in \{P, W, D\}, t \in T \tag{5}$$

where b_i denotes the transportation volume factor for each product, and $T_{k,k''}^{\max}$ the maximum allowable transportation volume for the route.

Each plant is assumed to have installed independent production lines for each product or product family, thus allowing the simultaneous production of any combination of products at any given time period. The individual production lines share common resources. For each manufacturing resource *j*, there is a maximum level of availability in each plant:

$$\sum_i \sum_{k''} k_{i,j} x_{i,k,k''}(t) \le C_{j,k}^{max} \qquad \forall\ k \in P, t \in T\ , j \in RS \tag{6}$$

where $k_{i,j}$ denotes the utilization factor of the *j*-th resource for the *i*-th product and $C_{j,k}^{max}$ is the maximum availability of the i-th resource.

Product demand, $r_{i,k}(t)$, is considered to be a function of the prices of all related products being offered by the supply chain network. Demand is assumed to be isoelastic. It should be noted that demand elasticity represents the sensitivity of product demand to changes in the prices of the related products offered by the supply chain network. In addition, product demands are assumed to be subject to uncorrelated idiosyncratic stochastic shocks every period. In a reduced form, the expression for the changes in product demand due to pricing decisions and realizations of stochastic shocks is provided by:

$$\frac{r_{i,k}(t)-r_{ref,i,k}}{r_{ref,i,k}} = \sum_{m}^{R}\sum_{j}^{DP}\frac{\partial \ln r_{i,k}}{\partial \ln p_{j,m}}\left(\frac{p_{j,m}(t)-p_{ref,j,m}}{p_{ref,j,m}}\right)+\frac{r^{s}_{i,k}(t)-r_{ref,i,k}}{r_{ref,i,k}} \tag{7}$$

where $r_{ref,i,k}$ and $p_{ref,i,k}$ denote the reference values for the demand and product price levels and $r^{s}_{i,k}$ accounts for the stochastic variation of the product demand. The logarithmic sensitivity of product demand with respect to price is called demand elasticity and provides a measure of the change in demand for a unit change in product price. Own elasticity ($\frac{\partial \ln r_{i,k}}{\partial \ln p_{i,k}} \forall i,k$) refers to the change in demand for a product when its price changes. Own elasticity is negative implying that an increase in the price of the product would have a negative influence in its demand. Cross-product elasticity ($\frac{\partial \ln r_{i,k}}{\partial \ln p_{j,k}} \forall i \neq j,k$) refers to influence of the demand for a product of price changes in other products change. Positive cross-product elasticity is characteristic of substitute (competitive) products, where the increase of the unit price for one product results in an increase for the demand of the substitute product (e.g., coffee and tea are substitute products). Negative cross-product elasticity implies complementary products (e.g., coffee and sugar can be considered complementary products). Cross-node elasticity ($\frac{\partial \ln r_{i,k}}{\partial \ln p_{i,m}} \forall i,k \neq m$) reflects the influence of the price manipulation in neighboring retailer nodes. Cross-node own-product elasticity is positive as the price increase of the same product in a neighboring retailer node leads to increased demand of the product at the given node (i.e., selling the same physical product in two adjacent retail nodes makes the two “locational products” substitutes). Explicit price experimentation by marketing departments or time series of product prices and sales data can be employed to estimate demand elasticity via standard econometric methods

3. Control Strategies for Supply Chain Management

Supply chain management involves a number of decisions to be taken at every time period to meet end-customer requirements. The overall supply chain performance is multi-dimensional and directly or indirectly affected by a number of factors, such as service quality and overall operating costs. Provided that these factors are generally competing with each other, trade-offs and compromises are necessary to achieve the best performance. The flows of products along permissible routes connecting successive echelons correspond to the ordering amounts being directed to the retailer nodes. Inventories at the nodes of the network serve as safety stock to handle the stochastic variation of demand and anticipate for lead time in ordered quantities. The associated transportation costs, inventory costs (assets directed in the production of the inventory), storage costs (operating costs of warehouse facilities and cost from anticipated damage of product while stored) are weighed against customer demand satisfaction (service level) and the generated revenues from the sale of products.

Supply chain management is performed within a two-layered approach. The first layer aims at adjusting the inventory levels with single dedicated controllers for each inventory node and product. The second level of control is a model predictive optimization-based scheme that considers the entire network dynamics including the local product inventory controllers of the first layer and calculates the inventory levels, the pricing policy and the distribution of products in the network that optimize a given performance index for the system over a specified time horizon.

3.1. Inventory Control

Inventory levels should be able to alleviate the effects of disturbances and fluctuations in product demand. On the other hand, inventories translate to higher costs for production and storage. The control objective for the inventory levels requires that inventories remain within acceptable limits while maintaining the optimal operation. Considering that the maintenance of the inventory at target levels is generally of lesser importance than service quality and revenues a second control layer is necessary. The quick adjustment of the inventory levels is achieved through simple feedback controllers that manipulate the incoming flows from upstream network nodes to this end. A single dedicated controller maintains the inventory of each product in each storage node. The relatively fast dynamics of the inventory control at each node are therefore separated from the overall control objectives for the entire network.

A general feedback control law for the inventory of the *i*-th product at node *k* takes the form:

$$mv_{i,k}(t) = \sum_{m=0}^{M} a_{i,k,m} y_{i,k}(t-m) + \sum_{n=1}^{N} b_{i,k,m} mv_{i,k}(t-n) + c \tag{8}$$

The control law is expressed as a weighted sum of past controlled, *y* (e.g., inventory levels), and manipulated variables, *mv*. Coefficients *a*, *b* and *c* can be calculated through a number of methods resulting from the enforcement of different control objectives (e.g., minimum variance controller and so forth). Selection of the number and type of terms allows for appropriate compensation for the associated lead times in the network. Reformulation of Eq 8 results in the discrete PID controller:

$$\begin{aligned} mv_{i,k}(t) &= mv_{i,k}(t-1) \\ &\quad - K_c\left(1+\frac{\Delta t}{t_I}+\frac{t_D}{\Delta t}\right) y_{i,k}(t) + K_c\left(1+2\frac{t_D}{\Delta t}\right) y_{i,k}(t-1) - K_c\left(\frac{t_D}{\Delta t}\right) y_{i.k}(t-2) + \frac{\Delta t}{t_I} y_{sp,i,k} \end{aligned} \tag{9}$$

where $mv_{i,k}(t)$ denotes the value of the manipulated variable for the inventory controller and $y_{sp,i,k}$ the inventory setpoint for the *i*-th product in the *k*-th node that will be calculated through the optimization-based outer control level. K_c is the proportional gain of the controller, t_I is the reset time for the integral mode, t_D is the reset time for the derivative controller mode, and *Dt* is the discrete control interval, which equals the discrete decision time period of the network. The inventory controllers should be tuned for close set-point tracking (e.g., quick response to a new imposed setpoint level) and good disturbance rejection (e.g., tolerate small deviations from setpoint). Large lead times and non-stationary demand variation are key factors that may significantly deteriorate the dynamic performance of the PID controllers. However, the formulation with the free to vary inventory set points retains its robustness to model mismatch and large lead times.

Noting that disturbances in inventory levels are introduced by variations in customer demand, the outgoing flows from each node are basically determined from downstream information and eventually from customer demand. Therefore, the incoming streams to the inventory node are selected as the manipulated variables for the inventory controller. One choice would be to manipulate the total amount of the *i*-th product transferred from all supplying nodes to node *k*:

$$mv_{i,k}(t) = \sum_{k'} x_{i,k',k}(t - L_{k',k}) \quad k \in \{W, D, R\},\ i \in DP \tag{10}$$

Such a selection imposes a constraint on the total amount of incoming products to the particular node as dictated by the control law (Eq 8, 9).

3.2. Optimization-based model-predictive control framework

The control system aims at operating the supply chain at the optimal point despite the influence of demand uncertainty. The control system is required to possess built-in capabilities to recognize the optimal operating policy through meaningful and descriptive cost performance indicators and mechanisms to successfully alleviate the detrimental effects of demand uncertainty and variability. The main objectives of the supply chain network can be summarized as follows: maximize (i) customer satisfaction, and, (ii) profit (gross of manufacturing costs). The first target is attained through the minimization of back-orders over a period of time, while the second target is achieved through the maximization of the network profits (i.e. difference between generated revenues from sales and network costs). Raw material costs are assumed constant per product unit.

Based on the fact that past and present control actions affect the future response of the system, a rolling time horizon is selected. Over the specified time horizon the future behavior of the supply chain is predicted using the described difference model (Eq 1-7). In this model, the state variables are the product inventory levels at the storage nodes, *y*, and the back-orders, *BO*, at the order receiving nodes. The manipulated (control or decision) variables are the product quantities transferred through the network's permissible routes, *x*, the inventory set points, y_{sp}, and product prices, *p*. Finally, the product back-orders, *BO*, and subsequently delivered products, *d*, are matched to the output variables. The inventory target levels (e.g., inventory setpoints) remain constant over the entire time horizon. At each time period the first control action in the calculated sequence is implemented. The effect of unmeasured demand disturbances and model mismatch is computed through comparison of the actual current demand value and the prediction from a stochastic disturbance model for the demand variability. The difference that describes the overall demand uncertainty and system variability is fed back into the model-predictive control scheme at each time period facilitating the corrective action that is required.

The mathematical formulation of the performance index considering simultaneously revenues, back-orders, transportation and inventory costs over the specified time horizon, t_h, takes the following form:

$$\begin{aligned} J = &\sum_{t}^{t+t_h} \sum_{k\in\{R\}} \sum_{i} \left\{\left(p_{i,k}(t) d_{i,k}(t)\right)\right\} - \sum_{t}^{t+t_h} \sum_{k\in\{R\}} \sum_{i} \left\{w_{BO,i,k}\left(BO_{i,k}(t)^2\right)\right\} \\ &- \sum_{t}^{t+t_h} \sum_{k\in\{W,D,R\}} \sum_{i} \left\{w_{T,i,k',k}\left(x_{i,k',k}(t)\right)\right\} - \sum_{t}^{t+t_h} \sum_{k\in\{W,D,R\}} \sum_{i} \left\{w_{Y,i,k',k}\left(y_{i,k}(t)\right)\right\} \\ &- \sum_{t}^{t+T_h} \sum_{k\in\{W,D,R\}} \sum_{i\in DP} \left\{w_{\Delta x,i,k',k}\left(x_{i,k',k}(t) - x_{i,k',k}(t-1)\right)^2\right\} \end{aligned} \tag{11}$$

Performance index, *J*, comprises a term accounting for the network revenues, a term penalizing back-orders at all retailer nodes, two terms accounting for the transportation and inventory costs, and a quadratic term that penalizes changes between successive time periods in the distributed products along the transportation routes. The weighting factors, $w_{Y,i,k}$ reflect the inventory storage costs and inventory assets per unit product, $w_{x,i,k',k}$ account for the transportation cost per unit product for route (*k*¢,*k*), $w_{BO,i,k}$ correspond to the penalty imposed on unsatisfied demand and are estimated based on the impact service level has on the company reputation and future demand and $w_{Dx,i,k',k}$, are associated with the penalty on the rate of change for the transferred amount of the *i*-th product through route (k',k). Such a term tends to eliminate abrupt and aggressive control actions and subsequently, safeguard the network from saturation and undesired excessive variability in the transported products induced by sudden demand changes. In addition, transportation activities are usually preferred to resume a somewhat constant level rather than fluctuate from one time period to another. The implementation of the move suppression term would affect control performance leading to a more sluggish dynamic response. Furthermore, variability induced by the stochastic nature of demand will be passed to the pricing policy (i.e. greater variation of the product prices between successive time periods may be observed). Even though, factors $w_{Y,i,k}$, $w_{T,i,k',k}$ and $w_{BO,i,k}$ are cost related that can be estimated with a relatively good accuracy, factors $w_{Dx,i,k',k}$ are judged and selected mainly on grounds of desirable achieved performance and variability manipulation. The weighting factors in Eq 11 also reflect the relative importance between the controlled (back-orders and inventories) and manipulated (transported products) variables.

The overall problem thus takes the following form:

$$\begin{aligned} &\underset{p,x,y,y_{sp}}{\text{Max}} \quad J \\ &\text{s.t.} \quad \text{Supply chain model Eq } 1-7 \\ &\qquad \text{Feedback inventory controllers for each node and product Eq 7 or 6} \end{aligned} \tag{12}$$

At each time period the calculated optimal decision control variables are implemented and the actual demand and back-orders are recorded. Forecast equations employing a stochastic ARIMA model for the product demand that is identified from historical demand data calculate the forecasts for future product demand over the entire span of the rolling horizon.

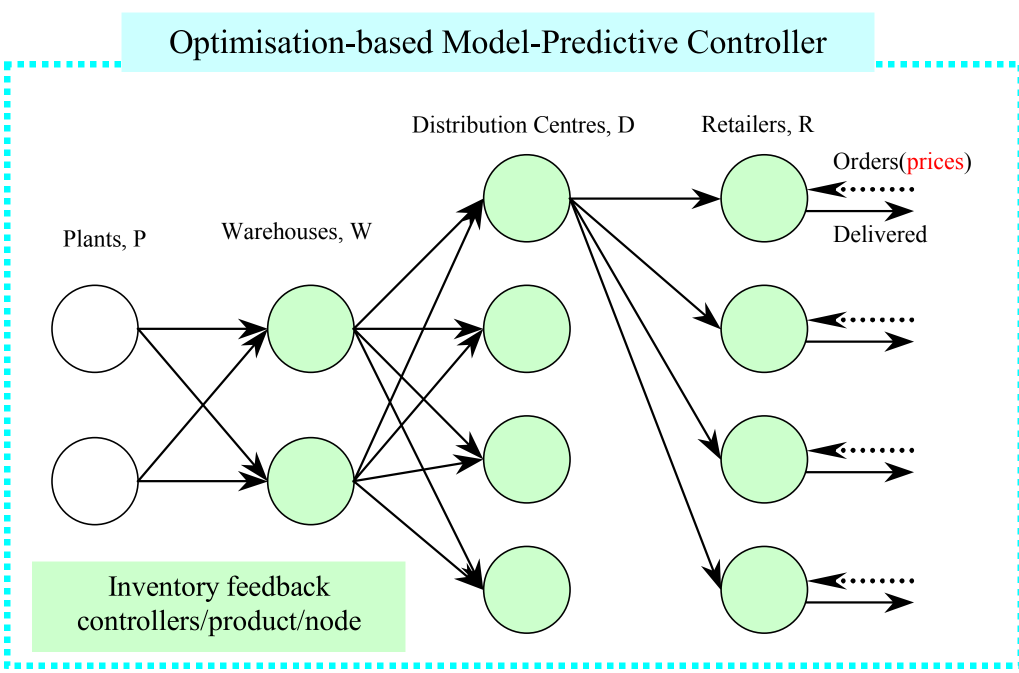


Figure 1. Schematic of the supply chain control structure

Eq (11) contains a number of non-convex bilinear terms that result in multiple local optima. The system is solved to global optimality by replacing the non-convex terms by proper under-relaxation approximations[8] and solving a sequence of linear and non-linear (non-convex) problems according to the algorithm of Adjiman *et al.*[9]

4. Results and Discussion

A four-echelon supply chain system is used in the simulated examples. The supply chain network is consisted of two production nodes, two warehouse nodes, two distribution centers, and four retailer nodes. All possible connections between immediately successive echelons are permitted. Two substitute products or product families are being distributed through the network. Inventory setpoints, maximum storage capacities at every node, and transportation cost data for each supplying route are reported in Table 1. For each node, a set of low-cost routes that carry the bulk of the supply was generally available. An additional set of alternative but much more expensive routes were also available and used occasionally to eliminate accumulating back-orders when the regular routes reached their saturation level. The use of the expensive alternative connecting routes mainly depends on the balance between the apparent cost for the accumulated back-orders and the transportation cost along these routes.

Table 1. Control data

	Warehouse	Distribution Center	Retailer
Max inventory level	250	150	80
Max product inventory setpoint	40	20	10
Transportation cost (also used in the move suppression term)	Plant to Warehouse	Warehouse to Distribution Center	Distribution Center to Retailer
	$\begin{bmatrix} 0.05 & 0.20 \\ 0.20 & 0.05 \end{bmatrix}$	$\begin{bmatrix} 0.05 & 0.05 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.05 & 0.05 \end{bmatrix}$	$\begin{bmatrix} 0.05\times\mathbf{1}_4 & 0.2\times\mathbf{1}_4 & 0.5\times\mathbf{1}_4 & 0.5\times\mathbf{1}_4 \\ 0.2\times\mathbf{1}_4 & 0.05\times\mathbf{1}_4 & 0.5\times\mathbf{1}_4 & 0.5\times\mathbf{1}_4 \\ 0.5\times\mathbf{1}_4 & 0.5\times\mathbf{1}_4 & 0.05\times\mathbf{1}_4 & 0.2\times\mathbf{1}_4 \\ 0.5\times\mathbf{1}_4 & 0.5\times\mathbf{1}_4 & 0.2\times\mathbf{1}_4 & 0.05\times\mathbf{1}_4 \end{bmatrix}$
Inventory weights	$2\ 10^{-3}$	$2\ 10^{-3}$	$2\ 10^{-3}$
Move suppression weight	10.0	10.0	10.0
Back-order weights	-	-	1.0
Own-product demand elacticity	-	-	-1.0
Cross-product same node elasticty	-	-	0.08
Own-product cross-node elasticity	-	-	0.08
Cross-product cross-node elasticity	-	-	0.0
	Inventory controller tuning $\{Kc, \tau_I, \tau_D\}$		
Case 1: L=[4 3 2]	{0.30, 8.0, 1.85}	{0.30, 8.0, 1.85}	{0.30, 8.0, 1.85}
Case 2: L=[6 5 4]	{0.30, 8.0, 1.85}	{0.30, 8.0, 1.85}	{0.30, 8.0, 1.85}

For this particular supply chain structure the optimization problem consists of 104 variables, 72 equality constraints and 14 total resource and capacity inequality constraints per time period. The total problem size is proportional to the size of the selected rolling horizon. The solution of the non-convex problem in Eq 12 and its convex underestimators is obtained using MINOS 5.5[10].

4.1. Effect of pricing policy

The effect of price manipulation on the overall performance of the supply chain network was investigated through a series of simulations under the presence of stochastic demand variation. The two main scenarios that were examined involved the calculation of the performance index for (i) a case that product prices were allowed to vary in each time period for the entire time horizon and (ii) a case were the product prices ware held constant over the time horizon. Comparison of these two scenarios shows the gain from continuous price manipulation. Figure 2 compares the achieved performance index value for these two cases, (i) and (ii). As anticipated, case (i) led to superior performance index values as more degrees of freedom were available in the problem. Similar trends were evident for different demand elasticity values. In general, the more elastic demand was becoming the more were the benefits on the improvement of the performance index from the price manipulation.

Figure 2 also shows the effects of different lead times and length of control horizon. Larger lead times require larger inventories and therefore higher inventory and transportation costs. The positive effects of a larger time horizon (20 time periods vs 10 time periods) are evident through the significant improvement of the performance index. However, in the presence of significant model mismatch a large time horizon may not be as successful.

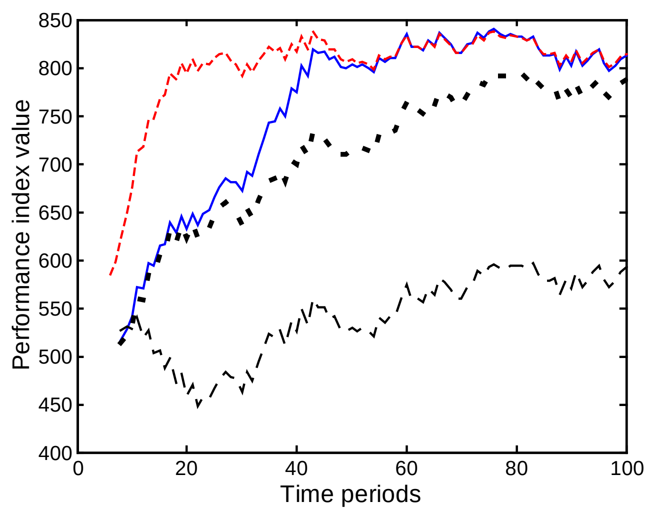
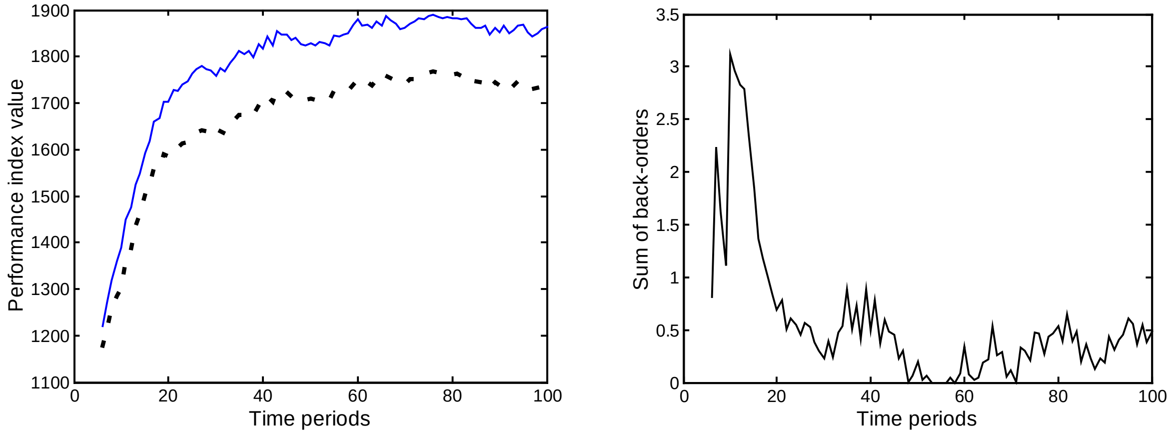


Figure 2. Effect of pricing manipulation on the performance index (i) solid line – variable product prices for each time period, (ii) dotted line – constant product price for the entire control horizon, (iii) dashed line – effect of lead time (L=[6,5,4]) on (i) (L=[4,3,2]), and (iv) dashed-dotted line – effect of length of control horizon (20 time periods) on (i) (10 time periods)



(a) (b)

Figure 3. (a) Effect of pricing manipulation on the performance index (i) solid line – variable product prices for each time period, (ii) dotted line – constant product price for the entire control horizon. (b) Sum of back-orders over time. (combined deterministic – step change – and stochastic demand variation)

The performance during a combined deterministic (step change for product A) and stochastic demand variation is shown in Figure 3. The price manipulation every time period during the control horizon successfully anticipated the increased demand for product A by increasing the corresponding price. Accumulation of back-orders was unavoidable in the case where product prices were kept constant at a single price level during the control horizon.

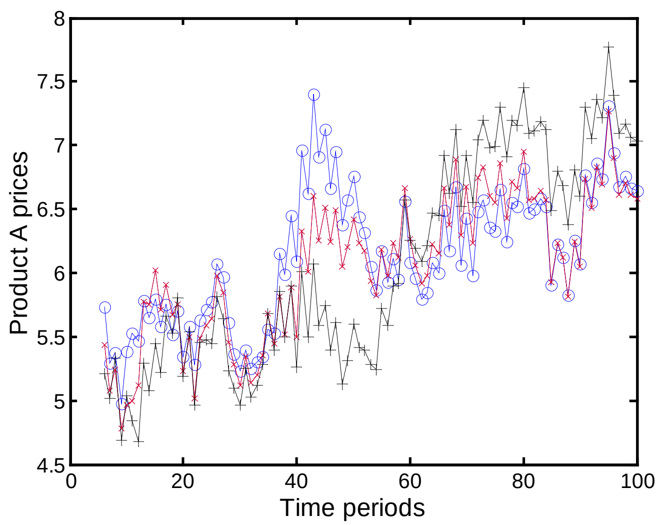


Figure 4. Effect of transportation suppression factor on price variability. (i) x – no suppression term $w_{\Delta x} = 0$, (ii) + – $w_{\Delta x} = 10\ w_T$, (iii) o – $w_{\Delta x} = 100\ w_T$.

The impact of the transportation suppression factor term on the variability of the product prices is shown in Figure 4. Such a term passes conveys the stochastic demand variability to the product prices as the corresponding weight increases. A heavy penalty on the changes in the product quantities transferred increases the use of the product prices to alleviate the stochastic demand variation. In the specific example the observed variance for the price of product A was 0.287, 0.314 and 0.684 for transportation suppression factor equal to 0, $10w_T$, and $100w_T$, respectively. Whereas large price variability may not be always desirable (e.g., frequent and large price changes may confuse the market) the selection of the weights may be used to adaptively direct the behavior of the system.

5. Conclusions

A two-layered optimization-based control approach for multi-product, multi-echelon supply chain networks was presented. The control strategy applies multivariable model-predictive control principles to the entire network while maintaining the safety inventory levels through the use of dedicated feedback controllers for each product and storage node. The optimization-based controller aims at maximizing revenues and customer satisfaction with the least operating costs. Product prices act as an additional instrument, which interact with inventory policies, to achieve the highest level of service for incoming demand.

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