2A Decomposition Method for Complex Supply Chain Optimization with Flexible Operation and Delivery Schedule

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Abstract

The world market faces more intensive competition, and as a result, supply chain optimization has gained increasing interests from academic research and business management. Most existing supply chain models follow the basic assumption that the decisions for when manufacturing of products and its delivery are carried out, are known or pre-specified before optimization, and only the amounts of production and delivery are often of interest to optimize.

However, in industrial practices, the selection of schedule for manufacturing and delivery should be strategically determined to maintain economic and sustainable supply chain. Therefore, these decisions should be simultaneously investigated in the decision framework in order to allow flexible and efficient manufacturing activities. However, such considerations increase the complexity of problems to optimize due to increasing numbers of sites, products and time periods. A novel decomposition method is proposed in this study. The whole distribution networks are divided into subsystems and optimized separately, and afterwards a simplified supply chain is optimized. Each subsystem is sequentially optimized until no profit improvement is observed. Case study shows that the proposed method is able to effectively deal with large and complex supply chain problems, involving a couple of thousand of binary variables.

1. Introduction

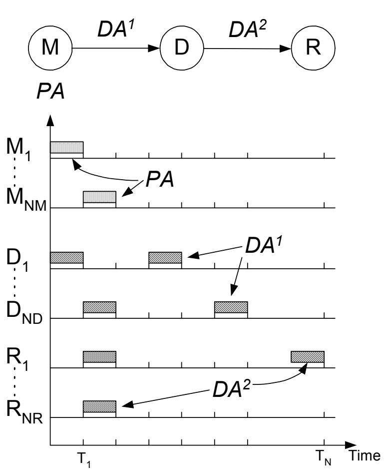

Figure 1. Decision variables of supply chain

Supply chain is a network that runs from raw materials procurement, through production, inventory and warehousing, distribution and delivery, transportation, and order fulfillment. A typical supply chain is composed of manufacturers, distribution centers and retailers. Products produced in manufacturing sites are transported to distribution centers in which the orders from retailers in the corresponding region are placed. The optimization of supply chain focuses on how to utilize all the processes, technology and capability to enhance competitive advantage. The key variables to be decided are the amounts of products that are produced, transported and sold, which are shown in Figure 1.

Although a number of research have been done in the context of supply chain optimization (S. Croom, 2000; N. Shah, 2004), in most researches it is assumed that the time points of all the action are pre-determined, so that only the amounts of products or deliveries are variables to be optimized. The advantage of this assumption is it can reduce the size of problem by less number of variables. However, it often deviates from real practice that the schedules for manufacturing and delivery should be flexible and moreover, require strategic selection of such decisions.

In addition, these decisions should be subject to practical constraints or limitations, such as; a minimum time interval allowed between two consecutive deliveries.

This paper presents an MILP model for supply chain which allows flexible operation and delivery schedule. Because binary variables are introduced in the mathematical formulation, high computational efforts are required to deal with complexity, especially when increasing numbers of manufacturers, distribution centers, retailers, products and time periods lead to a large scale problem. Therefore, a novel decomposition method is proposed based on strategic decomposition which divides the whole distribution networks into subsystems. The optimization framework significantly reduces the computational requirements in complex supply chain.

2. Problem description

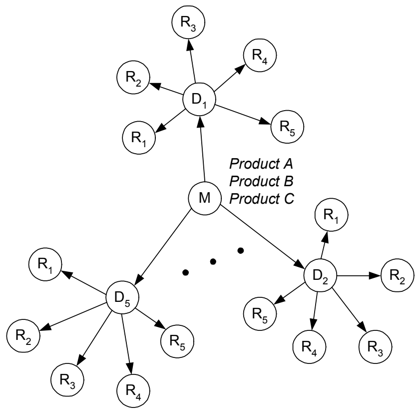

Figure 2. Supply chain networks

Several kinds of product can be produced in one manufacturer by different manufacturing schemes. Each scheme has different processing cost correspondingly, and only one production processing can be organized at a time unless the manufacturer operates multiple manufacturing lines. Moreover, production schedule should consider the processing time and waiting time such as cleaning, etc. Final products are stored in storage facilities before transported to distribution centers, which results in inventory holding costs.

Products are delivered to and stored in Distribution Centers (DC) in different regions until transported to retailers. Consequently, the deliveries and storing lead to transportation and inventory costs. The deliveries also cannot be organized every time period, there has to be minimum time interval between two consecutive deliveries. Until products delivered to retailers are sold to customers, they should be stored in inventories and resulting holding costs occur as well.

As the revenue of supply chain comes from the products sold, the overall profit becomes the revenue deducted from all kinds of costs (processing cost, inventory holding, etc). The objective of this optimization problem is set to find the optimal processing and delivery schedule for achieving maximum overall profit.

3. Model formulation

In respect of minimum time interval constraints, binary variables are used to represent them. For example, if ΔT_1 is the minimum time interval between two consecutive productions in manufacturer, the constraint is formulated as following inequality:

$$\sum_{t}^{t+\Delta T_1}\sum_{p} Process_{m,p,t} \leq 1 \qquad Process_{m,p,t} \in \{0,1\} \tag{1}$$

If the binary variable, $Process_{m,p,t}$, is 1, the production of product p starts at time period t in manufacturer m. Otherwise no production is organized.

Similarly, the constraints for delivery from manufacturer to distribution centers and from distribution centers to retailers could be formulated as well:

$$\sum_{t}^{t+\Delta T_2} Deliver_{m,d,p,t} \le 1 \qquad Deliver_{m,d,p,t} \in \{0,1\} \tag{2}$$

$$\sum_{t}^{t+\Delta T_3} Deliver_{d,r,p,t} \le 1 \qquad Deliver_{d,r,p,t} \in \{0,1\} \tag{3}$$

where, $Deliver_{m,d,p,t}$ is the binary variable representing whether product p is delivered from manufacturer m to distribution center d at time t, and $Deliver_{d,r,p,t}$ is the binary variable representing whether product p is delivered from distribution center d to retailer r at time t. Obviously, all these processing and transportation are subject to capacity constraints:

$$PA_{m,p,t} \le Cap_{m,p} \times Process_{m,p,t} \tag{4}$$

$$DA^1_{m,d,p,t} \le TrnCap_{m,d,p} \times Deliver_{m,d,p,t} \tag{5}$$

$$DA^2_{d,r,p,t} \le TrnCap_{d,r,p} \times Deliver_{d,r,p,t} \tag{6}$$

Where $PA_{m,p,t}$ is the amount of product p produced in manufacturer m at time period t; $DA^1_{m,d,p,t}$ is the amount of product p transported from manufacturer m to distribution centre d at time t; $DA^2_{d,r,p,t}$ is the amount of product p transported from distribution centre d to retailer r at time t; $Cap_{m,p}$ is processing capacity of product p in manufacture m and $TrnCap_{m,d,p}$, $TrnCap_{d,r,p}$ are transportation capacities respectively.

Objective function:

Maximize $$Z = INC - PC - HC - TC - Penalty \tag{7}$$

Subject to Constraints (1-6) and following constraints:

Mass balance constraints: $$Inv_{i,p,t+1} = Inv_{i,p,t} + \sum Pin_{i,p,t} - \sum Pout_{i,p,t} \tag{8}$$

Income: $$INC = \sum_{r,p,t} Sales_{r,p,t} \times Price_{r,p} \tag{9}$$

Processing cost: $$PC = \sum_{m,p,t} PA_{m,p,t} \times UPC_{m,p} \tag{10}$$

Inventory holding cost:

$$HC = \sum_{p,t} \left(\sum_{m} InvM_{m,p,t} \times UHC_{m,p} + \sum_{d} InvD_{d,p,t} \times UHC_{d,p} + \sum_{r} InvR_{r,p,t} \times UHC_{r,p} \right) \tag{11}$$

Transporting cost:

$$TC = \sum_{p,t} \left(\sum_{m} \sum_{d} TN_{m,d,p,t} \times UTC_{m,d,p} + \sum_{d} \sum_{r} TN_{d,r,p,t} \times UTC_{d,r,p} \right) \tag{12}$$

Back order penalty cost: $$Penalty = \sum_{r,p,t} BO_{r,p,t} \times UPen_{r,p} \tag{13}$$

4. A decomposition approach

In real supply chain optimization problems, the large numbers of manufacturers, distribution centers, retailers, products and time periods result in increasing complexity of the problems. The numbers of both continuous and binary variables could be a couple of thousands. Such a huge number of binary variables cause difficulties in solving the whole problem at one time. In order to deal with these difficulties, a decomposition method is proposed.

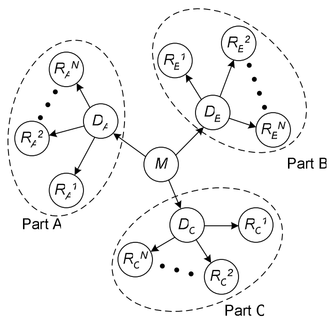

Figure 3. Two subproblems of supply chain

In general supply chain operations, when products are delivered from manufacturer, there are tradeoffs between different downstream parts. The total amount of products is fixed, so that if more products are delivered to one part, less can be delivered to other parts. However, after products are transported to distribution centre, the subsequent actions including storage, delivery to retailers and sales are usually organized within such a region that is separated from others, which means the interactions between different parts only lie in the deliveries from manufacturer and afterwards these parts are relatively independent. Hence, it is assumed in this paper that the distribution centre and retailers assigned to it in different regions can be regarded as a subsystem, such as part A, B and C in Figure 3. From this point of view, the whole problem can be decomposed into two subproblems.

Subproblem 1: Distribution centre and retailers

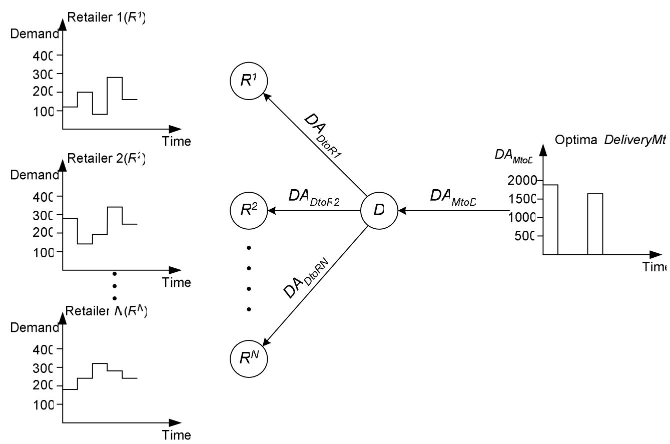

Figure 4. Optimization of subproblem 1

Subproblem 1 is an optimization of a subsystem composed of a distribution centre and the retailers assigned to it. This subsystem receives products from manufacturer and sells them to customers. Operation costs of this subsystem are delivery and inventory costs and the revenue comes from the sales. At current stage, this subsystem is isolated from other parts of the supply chain, so the achievable best performance of this part is the maximization of local profit in the subsystem. Hence given demands information, a sub-problem subject to operation constraints is optimized to obtain the optimal delivery schedules of DA_{DtoR} and DA_{MtoD} and the maximum profit (denoted as P_D).

It should be mentioned that for the whole supply chain problem, even maximum capacity of production might not meet all the demands due to capacity constraints, including back order penalty at some retailers. However, such circumstances are not considered in subproblem 1 as there are no constraints for the possible delivery amount of DA_{MtoD}, and the back order penalties are taken into account in subsystem 2 which is explained in detail later.

Subproblem 2: Manufacturer and distribution centre

Subproblem 2 consists of manufacturer and distribution centers. After optimal DA_{MtoD} schedules are obtained from optimization of subproblem 1, they are considered as demands placed directly on manufacturer.

When the subproblem 1 is optimized, it is assumed that the delivery from manufacturer to distribution centre has no constraints, (i.e. No matter what the amount is, it always can be fulfilled.) However, when subsystem 2 is optimized, the production capacity is taken into account, so some deliveries might not be fully fulfilled. It results in back order penalty for manufacturer and included in the subproblem 2 optimization. Hence the possible back order penalties at retailers of the whole supply chain problem have been transferred to subproblem 2.

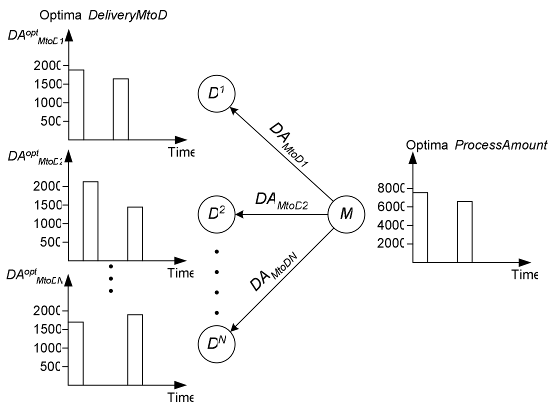

Figure 5. Optimization of subproblem 2

There is no revenue for this subproblem and the objective is to minimize the cost, which is denoted as C.

Because the whole supply chain problem has been divided into two subproblems and these two subproblems have been optimized separately, the overall profit obtained by combining subproblem 1 and 2 should be a lower boundary of the global optimal solution.

$$P_o = \sum_D P_D - C$$

where P_o is the overall profit for the supply chain; P_D is the maximum profit of subsystem 1;C is the minimum cost subproblem 2.

It should be noted that when DA_{MtoD} cannot be fully fulfilled, the downstream actions have to be re-optimized to incorporate such product shortage situation. An iteration mechanism is proposed as shown in Figure 6.

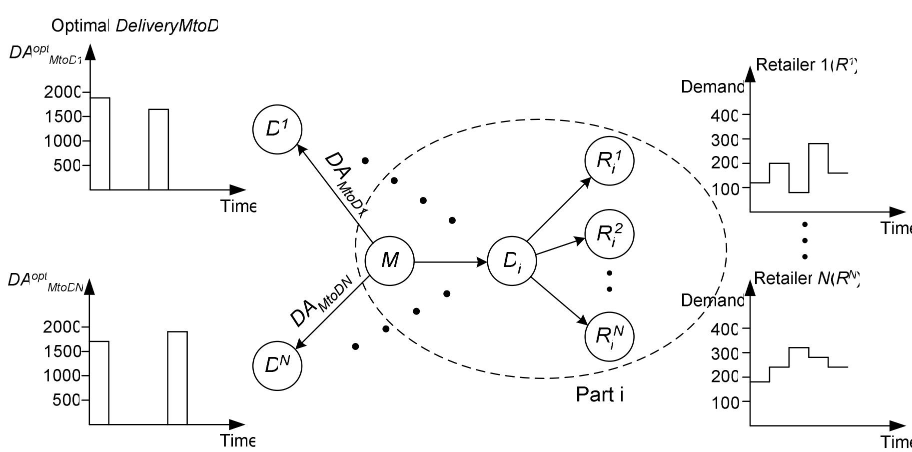

Figure 6. Combination of subproblems 1 and 2

A simplified supply chain composed of manufacturer, one distribution centre and assigned retailers, is optimized, while the deliveries to other distribution centers remains as direct demands to the manufacturer, in such a way other distribution centers are pseudo-connected with the simplified subsystems. Then, each subsystem is sequentially optimized. The optimization of subsystems provides the optimal delivery schedule, including the delivery from distribution centre to retailers, as well as that from the manufacturer to distribution centre. If the solution provides profit improvement, the schedule for manufacturing and deliveries are updated. This procedure is repeated until no improvement in profit is observed during the iteration. The overall framework of proposed method is illustrated in Figure 7.

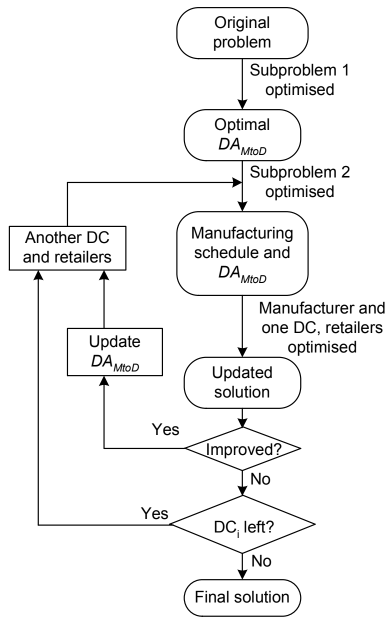

Figure 7. Framework of decomposed method

5. Case Study

A problem studied has the features as listed below.

30 Time periods
1 Manufacturer
3 Products
5 Distribution centers
5 Retailers for each distribution centre

For the purpose of comparison, an overall model developed on the basis of model proposed by Z. Zhou (2000) and J. Bok (2000) for whole supply chain is used. Both overall model and decomposed model are formulated and solved using the CPLEX solver accessed via GAMS on a PC of AMD Athlon XP 2500+ with 512 MB memory. The results of example are shown in Table 1.

Table 1. The results of example

	Computation time	Profit (MM$)
Decomposed method	4 Hours	219.8
Overall model	120 Hours	212.4

Decomposed method obtains a solution of 219.8 MM$ within approximately 4 hours of computation. Overall model is found to be extremely computationally intensive. After more than 120 hours when resource limit of solver is exceeded, the available feasible solution is around 212.4 MM$ that is still worse than decomposed method. Also it should be pointed out that the best possible solution shown by GAMS with overall model is 222 MM$. If take this figure as global optimal solution, that relative error of the solution obtained by decomposed method is near 1%. Compared with far less computational effort, it can be seen that proposed method is quite efficient.

Table 2. Comparison of different cases*

	Profit (MM$)
Case 1	157.9
Case 2	211.5
Case 3	219.8

* 1% relative gap is used as termination condition

Meanwhile, the effect of optimization on manufacturing schedules is investigated. In Case 1, manufacturing schedule is predetermined and full production capacity is used. In Case 2, manufacturing schedule is still predetermined as Case 1, but amount of production and deliveries are to be optimized. The results are shown in Table 2 together with proposed model as Case 3.

As can be predicted, because Case 1 has no consideration for distribution activities and demand, overproduction is inevitable and excessive holding cost makes profit extremely worse. Of course in real practice, such operation would never occur. From Case 2, it can be found that the consideration of demand and delivery could improve the overall performance. Regarding proposed model in Case 3, the manufacturing schedule obtained is different from previous cases. This is because not only demand but also distribution networks are included in the optimization. Such simultaneous optimization achieves a far better overall profit than other cases.

6. Conclusion

In order to deal with the complex supply chain optimization problem with flexible schedules, a decomposed method has been presented. The method optimizes two subproblems of supply chain, based on strategic decomposition of overall model, through the iteration procedure which systematically considers the interactions between two subproblems, and provides efficient searching for optimal solution. The method has proven to be effective and robust to solve large and complex supply chain optimization.

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