Flow Characteristics and Particle Size Distribution in Pneumatic Conveying with Attrition

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Introduction

This work presents an analysis of gas-solids flow in a pneumatic conveying line wherein by collisions with the wall, the particles undergo breakage. A measure of particle attrition, the often-undesirable damage to particles, is necessary for design of many fluid-particle operations. For example, proper design of cyclones requires knowledge of the amount and size of fines produced from attrition. Additionally, an estimate of energy consumption from particle breakage is needed to ensure that pneumatic conveying lines operate in the flow regime as designed.

Population balances and kinetic theory are used in this study to describe how the local variables, such as the particle velocity and average particle volume at a specific point in the system, affect the rate of attrition at that point. Particle breakage, which modifies the average size of the particles, affects the characteristics of the multiphase flow. Specifically, the kinetic theory constitutive models proposed by Lun et al. (1984) for the particle phase stress tensor are dependent upon the diameter of the particles. In this work, the typical equations for multiphase flow, conservation of mass, momentum and energy for both the gas and particulate phases (as implemented by Sinclair and Jackson (1989) and others), are coupled with an additional equation for the average particle size.

Model Development

Using the analogy between a perfect gas and the flow of granular materials, Lun et al. (1984) derived constitutive equations for the flow of particles. Lun and coworkers applied the kinetic theory of gases (see Chapman and Cowling 1970) taking into account the inelasticity of typical particle-particle collisions. This model served as the basis for several successful simulations (Sinclair and Jackson 1989, Ding and Gidaspow 1990, Pita and Sundaresan 1993), even though the role of the interstitial fluid was neglected. Ma and Ahmadi's (1988) and Koch's (1990) work incorporated the effect of a low-density carrier fluid (e.g. air) into the kinetic theory formulation. The set of multiphase equations used in this work can be found in Table 1 of Agrawal et al (2002).

Boundary conditions must complete the set of multiphase equations. Hui et al. (1984) equate the limit of shear stress transmitted by the particle phase to the rate of momentum lost to the wall by inelastic particle-wall collisions. The roughness of the wall is described by a specularity coefficient. Jenkins (1992) employs the formalism of kinetic theory to average over all collisions with a solid boundary. Either development allows for the slip of the particle phase relative to the movement of the solid boundary. As it is assumed that particles only break when striking the wall, the collisions with the boundaries must take into account this breakage. In the following section, we derive the effect attrition has on the boundary conditions.

Boundary Condition Development – Population Balances

This development closely follows the work by Jenkins (1992), with additional considerations of the size change of the particle upon impact with a wall. The population balance equation (PBE) for both particle volume (v) and particle velocity $(\mathbf{c})$ as coordinates can be written as:

$$\frac{\partial f_1}{\partial t} + \frac{\partial \dot{V} f_1}{\partial v} + \nabla_{\mathbf{r}} \cdot (\mathbf{c} f_1) + \nabla_{\mathbf{c}} \cdot (\mathbf{F} f_1) = 0 \qquad (1)$$

where $f_1 = f_1(v, \mathbf{c}, \mathbf{r}, t)$, the dependencies have been excluded for brevity in equation (1). $\dot{V}$ is the particle growth rate, and $\mathbf{F}$ is the external forces acting on the particle. In particular, $\mathbf{F}$ includes the drag force and all other fluid-particle interactions, as well as the gravity force.

The population balance equation (1) must be supplemented with an appropriate boundary condition with describes the attrition at the pipe walls, $\mathbf{r}_0 \in \partial\Omega_{\mathbf{r}}$. In analogy to the jump conditions over a singular surface presented by Truesdell and Troupin (1960) two limits are defined. Letting $\phi(v, \mathbf{c}, \mathbf{r}, t)$ be any continuous function, a definite limit as $\mathbf{r}$ approaches a point $\mathbf{r}_0 \in \partial\Omega_{\mathbf{r}}$ (on the pipe wall) exists. The limiting value is dependent upon whether the particle velocity is into the wall (pre-attrition) or away from the wall (post-attrition). Denote the pre-collision limiting value, as $\phi^-(v, \mathbf{c}, \mathbf{r}_0, t)$, and the post-collision limiting value, $\phi^+(v, \mathbf{c}, \mathbf{r}_0, t)$. The discontinuous jump of this arbitrary function is denoted $[\![\phi]\!] = \phi^+ - \phi^-$. Choosing $\phi(v, \mathbf{c}, \mathbf{r}_0, t) = \psi(v, \mathbf{c}, \mathbf{r}_0, t)(\mathbf{g} \cdot \mathbf{n}) f_1(v, \mathbf{c}, \mathbf{r}_0, t)$, the two limits can be evaluated, determined by the physics of the breakage process.

$$\phi^-(v, \mathbf{c}, \mathbf{r}_0, t) = \chi(v, \mathbf{c}, \mathbf{r}_0, t) \psi^-(v, \mathbf{c}, \mathbf{r}_0, t)(\mathbf{g} \cdot \mathbf{n}) f_1^-(v, \mathbf{c}, \mathbf{r}_0, t)$$

$$\phi^+(v, \mathbf{c}, \mathbf{r}_0, t) = \int_v^\infty \int_{\mathbf{g}' \cdot \mathbf{n} \geq 0} \chi(v, \mathbf{c}, \mathbf{r}_0, t) P(v, \mathbf{c} | v', \mathbf{c}') \psi^+(v', \mathbf{c}', \mathbf{r}_0, t)(\mathbf{g}' \cdot \mathbf{n}) f_1^+(v', \mathbf{c}', \mathbf{r}_0, t) \, d\mathbf{c}' dv' \qquad (2)$$

χ is defined as the rate at which particles break. It is related to the radial distribution function at the wall, $g_0(\mathbf{r}_0, t)$, and the probability of breakage, $b(v, \mathbf{c})$, by $\chi = 4 g_0(\mathbf{r}_0, t) b(v, \mathbf{c})$. $\mathrm{r}_0 \in \partial\Omega_{\mathrm{r}}$. Taking $\psi = 1$, the boundary condition for the population balance equation (1) is expressed. That is, the discontinuous jump in the number density function due to breakage at the wall is:

$$[\![(\mathbf{g} \cdot \mathbf{n}) f_1(v, \mathbf{c}, \mathbf{r}_0, t)]\!] = \int_v^\infty \int_{\mathbf{g}' \cdot \mathbf{n} \geq 0} \chi(v', \mathbf{c}', \mathbf{r}_0, t) P(v, \mathbf{c} | v, \mathbf{c})(\mathbf{g}' \cdot \mathbf{n}) f_1(v', \mathbf{c}', \mathbf{r}_0, t) \, d\mathbf{c}' dv'$$
$$- \chi(v, \mathbf{c}, \mathbf{r}_0, t)(\mathbf{g} \cdot \mathbf{n}) f_1(v, \mathbf{c}, \mathbf{r}_0, t) \qquad (3)$$

Define mixed moments of the number density function by:

$$\mu_{n,p}(\mathbf{r}, t) = \int \int v^n \mathbf{c}^p f_1(v, \mathbf{c}, r, t) d\mathbf{c} dv \qquad (4)$$

Taking $\phi(v,\mathbf{c},\mathbf{r}_0,t)=\psi(v,\mathbf{c},\mathbf{r}_0,t)(\mathbf{g}\cdot\mathbf{n})f_1(v,\mathbf{c},\mathbf{r}_0,t)$, and integrating over all particle volumes and over the velocities that strike the wall, the discontinuous jump for any moment can also be found.

$$\left[\!\left[\mu_{n,p}(\mathbf{r}_o,t)\right]\!\right]=\int_0^\infty\int_{\mathbf{g}\cdot\mathbf{n}\le 0}\chi(v,\mathbf{c},\mathbf{r}_0,t)\left\{\int_v^\infty\int_{\mathbf{g}'\cdot\mathbf{n}\ge 0}v'^n\mathbf{c}'^pP(v,\mathbf{c}|v',\mathbf{c}')(\mathbf{g}'\cdot\mathbf{n})f_1(v,\mathbf{c},\mathbf{r}_0,t)\,d\mathbf{c}'dv'-v^n\mathbf{c}^p(\mathbf{g}\cdot\mathbf{n})f_1(v,\mathbf{c},\mathbf{r}_0,t)\right\}d\mathbf{c}dv \tag{5}$$

Certain moments have physical significance. For example, $\left[\!\left[\boldsymbol{\mu}_{1,1}\right]\!\right]=\mathbf{M}$, expresses the change in momentum due to collisions with the wall.

The jump in the average particle size, a function of the moments of the distribution, can now be written as well:

$$\left[\!\left[\bar{v}_1\right]\!\right]=\left[\!\left[\frac{|\boldsymbol{\mu}_{1,1}|}{|\boldsymbol{\mu}_{0,1}|}\right]\!\right]=\left(\frac{|\boldsymbol{\mu}_{1,1}|}{|\boldsymbol{\mu}_{0,1}|}\right)^+-\left(\frac{|\boldsymbol{\mu}_{1,1}|}{|\boldsymbol{\mu}_{0,1}|}\right)^- \tag{6}$$

Drew and Passman (1999) list the generic jump equations in the following form:

$$\left[\!\left[\rho\Psi(\mathbf{c}-\mathbf{U})+\mathbf{J}\right]\!\right]\cdot\mathbf{n}=m \tag{7}$$

with the definitions of Table 1.

D must also include the extra energy, not required for momentum balance, but available for destruction and dispersion of the fragment debris. It is commonly assumed that

$$D=D_d+D_x=\int E_d+E_x=\int E_d+E_k+E_f \tag{8}$$

where E_d is the dissipation energy in the deformation of the particles, E_k is the kinetic energy of expansion about the mass center, and E_f the energy expended in the fracture and disintegration of the body. Models need to be provided for each of these terms.

Table 1. Terms in the generic jump conditions

Conservation Principle	Ψ	$\mathbf{J}$	m
Mass	1	0	$0=\left[\!\left[\mu_{0,1}\right]\!\right]$
Momentum	$\mathbf{c}$	$\mathbf{T}$	$\mathbf{M}=\left[\!\left[\boldsymbol{\mu}_{1,1}\right]\!\right]$
Energy	$\frac{1}{2}c^2$	$\mathbf{c}\cdot\mathbf{T}-\mathbf{q}$	$D=\frac{1}{2}\left[\!\left[\mu_{2,1}\right]\!\right]$

Unless significant shape-change occurs (specifically, shape change via deformation, not breakage) or the impact is of high enough velocity to generate shock waves, the dissipation has already been accounted for by the coefficient of restitution. The energy of expansion is a function of the diametral velocity of expansion. Specifically, the kinetic energy lost to expansion can be written as

$$E_k=\frac{\beta_k}{8}\chi(v,\mathbf{c},\mathbf{r}_0,t)\int\int\rho_pv'(\mathbf{c}'-\bar{\mathbf{c}}')^2P(v',\mathbf{c}'|v,\mathbf{c})\,d\mathbf{c}'dv' \tag{9}$$

where $\bar{\mathbf{c}}'$ is the mass-center velocity, and β_k is a constant. For example, $\beta_k=0.6$ for a uniformly expanding sphere or $\beta_k=1.0$ for an expanding spherical shell. Lastly, to estimate the energy of fracturing the particle, we let γ_s represent the work per unit area required to create new fragment surface area. In fluids, this is the surface tension, and a similar quantity can be defined for solids. Fracture energy can be written as

$$E_f = \gamma_s \chi(v, \mathbf{c}, \mathbf{r}_0, t)\left(\int\int v'^{2/3} P(v', \mathbf{c}' | v, \mathbf{c}) d\mathbf{c}' dv' - v^{2/3}\right) \quad (10)$$

which represents the extra surface energy that must have been added in order to form all the daughter fragments.

Lastly, a description of the breakage, $P(v', \mathbf{c}' | v, \mathbf{c})$ and the probability of breakage, $b(v, \mathbf{c}, \mathbf{r}_0, t)$, is necessary to complete the description of the system and its boundary conditions.

Breakage Models for Attrition

Based on the equipartition of energies principle, the velocity distribution of the fragments should approach a Maxwellian form. Livesey (1980) showed that for as few as $N = 5$ fragments, the distribution of velocities is indeed normal. Computationally, the same fragment velocity distribution was found by Kun and Herrmann (1996). Kun and Herrmann found that the width of the Maxwellian velocity distribution can be written as a function of the pre-collision particle radius and velocity.

$$P_c(c_i' | c_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{1}{2}\left(\frac{c_i'}{\sigma_i}\right)^2\right] \quad (11)$$

where

$$\sigma_i = \sigma_{c,i} v^{\frac{1}{3}\alpha_i} |\mathbf{c}|^{\beta_i} \quad (12)$$

Their values for α and β were given for 2-D particle-particle collisions; however the scaling relationship should remain the same, albeit with new values for the exponents.

If impact energy were low, erosive-type breakage most probably occurs. Also known as abrasive attrition, in this case the parent particle is disintegrated into two parts: the fines and a single daughter particle, which is only slightly smaller than the parent particle. Recent work by Ghadiri and Zhang (2002a,b) theoretically develops a relationship between the impact velocity, the particle size, and the physical properties of the particle and the mass of the attrited material. This relationship is successfully employed to describe experimental data of attrition of melt-grown crystals (MgO, NaCl, and KCl).

Specifically, ξ, the fractional loss per impact, is proportional to the kinetic energy of impact, and varies linearly with particle radius.

$$\xi = \xi_c \frac{\rho_p |\mathbf{c}|^2 v^{1/3} H}{K_c^2} \quad (13)$$

H is the particle hardness, and K_c is the fracture toughness, also known as the critical stress intensity factor. For plane stress, K_c is a function of other measurable physical properties, such as the Young's modulus (E), the fracture surface energy (γ_s), and Poisson's ratio (ν_{Poi}).

$E\gamma_s = K_c^2\left(1-\nu_{Poi}^2\right)$

H/K_c^2 is a ratio that represents the material resistance to plastic flow and fracture.

There are two simple models that may be appropriate. The first is simple binary breakage into its two daughter particles:

$$2P_v\left(v'|v\right) \propto \delta\left(v'-(1-\xi)v\right)+\delta\left(v'-\xi v\right) \tag{14}$$

In the second model, a power law is employed to describe the fines, similar to the kernel investigated by Ziff (1991):

$$(\eta+1)P_v\left(v'|v\right) \propto \delta\left(v'-(1-\xi)v\right)+\frac{1}{\xi v}\frac{2\eta}{4-\eta}\left(1-\left(\frac{v'}{\xi v}\right)^{-\frac{\eta-4}{\eta-2}}\right)U\left(\xi v-v'\right) \quad \eta \neq 2,4 \tag{15}$$

In this model, the chipped off mass is η particles described by a power law model. Several prior investigators have observed power-law behavior in the resulting particle size distributions from attrition (e.g. Kun and Herrmann 1996).

The first model is easier to implement, but the second model may be more realistic in describing actual fracture dynamics. In addition, an appropriate model for the number of particles broken off, $\eta(v,|\mathbf{c}|)$, may be needed as different impact speeds and sizes will most likely yield different numbers of daughter particles.

With the knowledge that the above distribution for velocity $P_{\mathbf{c}}\left(\mathbf{c}'|\mathbf{c}\right)$ conserves momentum, and the distribution for particle volume $P_v\left(v'|v\right)$ conserves mass, we let the total distribution be described by the product of the two:

$$P\left(v',\mathbf{c}'|v,\mathbf{c}\right) = P_v\left(v'|v\right)P_{\mathbf{c}}\left(\mathbf{c}'|\mathbf{c}\right) \tag{16}$$

The probability of breakage can be estimated from the single-impact breakage correlations of Vogel and Peukert (2003):

$$b\left(v,\mathbf{c}\right) = 1-\exp\left\{-f_{mat}v^{1/3}\mathbf{c}^2\right\} \tag{17}$$

where f_{mat} is a material specific constant. While Ghadiri and Zhang's (2002a,b) analysis is based on plastic breakage, and Vogel and Peukert's (2003) analysis is based on elastic fracture, the functional forms of both correlations are obvious. This similarity has been discussed by Weichert (1988). Therefore f_{mat} and ξ_c are related, and may be determined from the same single-impact experiment.

The above breakage distribution and breakage probability is then inserted into the expressions for the $[\![\cdot]\!]$ terms and excess energy dissipation D_x, equation (8).

Lastly, to close the equations a model for the number distribution itself is needed. Ideally, we would like for the physics of the problem to completely determine the distribution, but in this simplified analysis, an appropriately straightforward number distribution is assumed.

$$f_1\left(v,\mathbf{c},\mathbf{r},t\right) = \delta\left(v-\bar{v}_1(\mathbf{r},t)\right)\frac{n(\mathbf{r},t)}{\left(2\pi\Theta(\mathbf{r},t)\right)^{3/2}}\exp\left(\frac{-C^2(\mathbf{r},t)}{2\Theta(\mathbf{r},t)}\right) \tag{18}$$

where $\mathbf{C}$ is the fluctuation velocity: $\mathbf{C} \equiv \mathbf{c}-\mathbf{c}_0$ and Θ the granular temperature, both as defined in previous works.

Results and Discussion

In this section, a few preliminary one-dimensional results will be displayed. Hamilton et al. (2003) discuss the assumptions necessary to implement a one-dimensional simplification of the system. Primarily, the ratio of particle-wall collisions to particle-particle collisions must be small for a pseudo-steady-state assumption to be valid.

All simulations were carried out glass spheres in air at room temperature. The gas pressure drop was $dp/dz = -20000\text{Pa/m}$ and the mass flux was set at $\dot{G} = 890\text{ kg solid/m}^2 \cdot \text{s}$. The particle properties were: $\rho_p = 2500\text{kg/m}^3$, initial particle diameter of $d_p = 0.00015\text{m}$, particle-particle restitution coefficient $e_p = 1.0$, and particle-wall restitution coefficient $e_w = 0.9$. The particle material resistance was set to $H/K_c^2 = 0.01\ 1/\left(\text{Pa}\cdot\text{m}^2\right)$, and all particles were allowed to have a 50% chance to break – the effects of implementing equation (17) has not be explored yet. The conveying line's properties were: radius, $R = 0.015\text{m}$ and length, $L = 40\text{m}$.

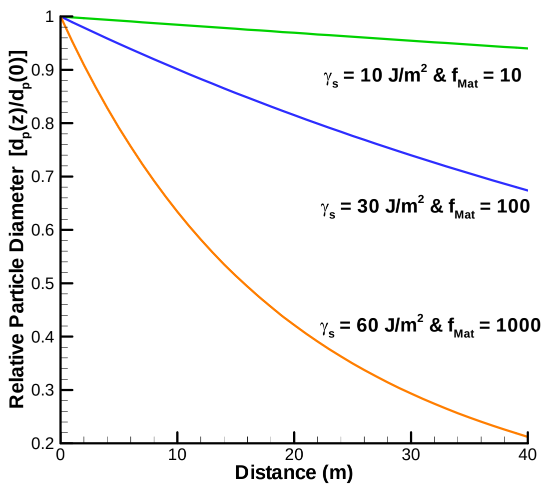

Figure 1. Particle diameter vs. downstream distance.

Figure 1 displays how the particle diameter reduces as the gas-solid mixture flows downstream. Figures 2 and 3 demonstrates how the attrition affects the flow profiles. Both of these figures show the differences in the profiles at the entrance to the conveying line. As the attrition propensity constant and surface energy increases, the granular temperature decreases across the entire span of the pipe as the energy is being consumed by the breakage. The velocity of the particle phase increases as from kinetic theory the shear viscosity of the particle phase is proportional to the square root of the granular temperature, $\mu_{part,shear} \propto \sqrt{\Theta}$. Therefore, as the granular temperature decreases, the particle phase is effectively less viscous and flows more easily.

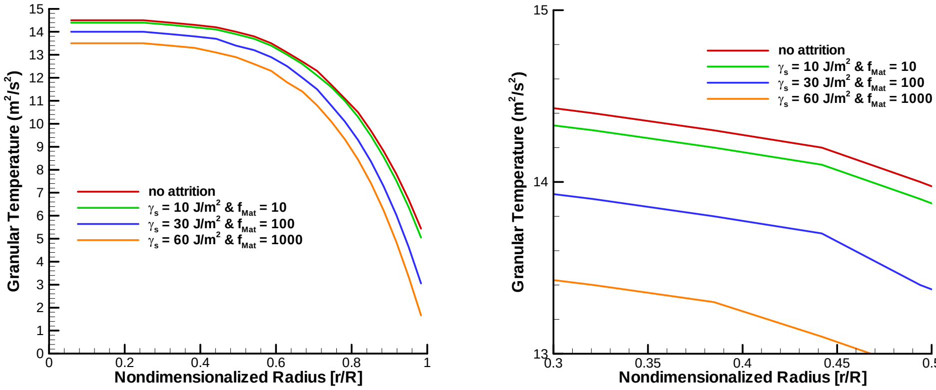

Figure 2. Radial distribution of granular temperature. The right graph is a zoomed in portion to show details.

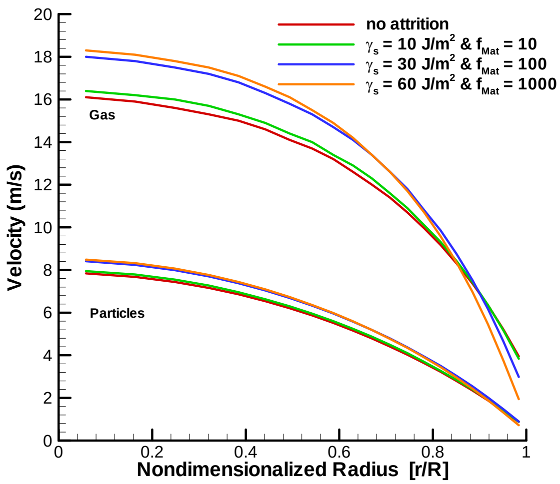

Figure 3. Velocity profiles of gas and particle phases.

Figure 4 compares the theoretical minimum amount of power necessary to convey the mixture compared with the breakage parameters. The minimum power necessary is a function of the gas pressure drop, and velocity of the phases (Collando et al. 2002). The figure shows that a maximum amount of energy appears – that is, even in a worst case with a large amount of attrition only a certain amount of energy will need to be inputted. In other words, there is only a certain amount of total energy available, and the system distributes the energy between breakup of the particles and conveying the mixture. Finally, Figure 5 shows how increases in the number of daughter particles increase the necessary power.

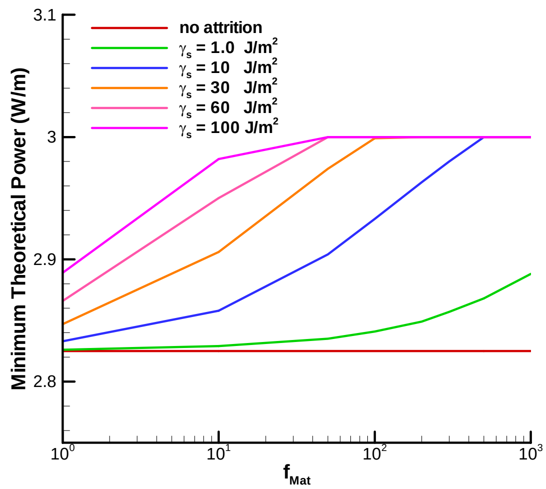

Figure 4. Theoretical power as a function of breakage parameters.

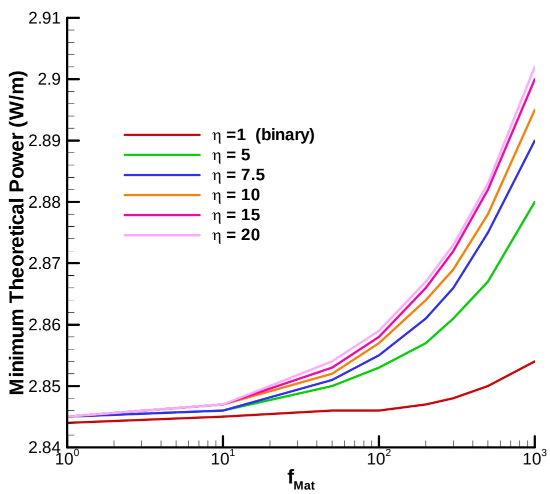

Figure 5. Theoretical power for increasing number of daughter particles. $\gamma_s = 30\text{J/m}^2$.

This work has shown how using population balance methods, an analysis of attrition in a pipe may be carried out. Attrition significantly affects the characteristics of the multiphase flow, primarily through the boundary conditions.

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