Self-Similarity of Particle Size Distribution from Pneumatic Conveying Attrition

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Introduction

An analysis of the size distribution of particles that break when striking the wall in a conveying line is presented. A measure of particle attrition, the often-undesirable damage to particles, is necessary for design of many fluid-particle operations. For example, an estimate of energy consumption from particle breakage is needed to ensure that pneumatic conveying lines operate in the flow regime as designed.

When a single particle collides with the wall, the probability of breakage and the amount attrited is highly dependent upon the particle's mass and impact velocity, as well as its physical properties such as hardness and friability. The characteristics of the multiphase flow surrounding the particle determine its impact velocity and its mean free path in the system.

With all these complexities, the particle size distribution does indeed achieve self-similarity, after enough generations have passed and when scaled appropriately. The probability of breakage is a function of the flow characteristics, such as the fluctuation energy of the particle phase. Additionally, the time between successive impacts is also highly dependent on the multiphase flow – the mean free path is determined by the volume fraction of the particle phase and the fluctuation energy as well. The average velocity of the conveying line converts the time into a distance, which may be very long; self-similarity may not be achieved for tougher particles until many times the length of a typical conveying line.

However, for more brittle particles, the self-similarity of the size distribution provides an opportunity for reducing the complexity of the changing flow. With knowledge of the flow characteristics, the particle size distribution at a certain downstream distance may be estimated by taking advantage of the similarity. That is, the flow profiles of both the particle and fluid phases may be calculated assuming no significant particle size change occurs – representing a significant savings in computational effort. Then, post-simulation, the size change may be estimated using the outputted flow characteristics and the self-similarity.

This work uses Monte Carlo simulation to approximate the particle size distribution after a large number of collisions with the wall. The distribution of impact velocities is based upon flow conditions in a typical conveying line. Then the approximated distributions are compared with a full simulation of coupled mass, momentum, and energy balances with a particle phase that is evolving due to attrition. Evaluations are made between the full simulation and the self-similar approximation.

Self-Similarity of Pure Breakage.

Chapter five of the book by Ramkrishna (2000) describes in detail the theory of self-similarity for particles that undergo pure breakage; that is, no agglomeration or growth. Ramkrishna's analysis shows that self-similarity occurs when the breakage frequency obeys a power law, and the cumulative breakage distribution function can be written as a homogeneous function of the ratio of the parent to child particle sizes.

These conditions are not guaranteed to occur during attrition. The probability that a particle of volume v and velocity $\mathbf{c}$ breaks is estimated by the correlation given by Vogel and Peukert (2003)

$$b(v,\mathbf{c}) = 1 - \exp\left\{-f_{mat} v^{1/3} \mathbf{c}^2\right\}$$

where f_{mat} is a material specific constant. If a Maclaurin series is used to approximate the exp term, the breakage function can be seen to be approximate a power law.

$$b(v,\mathbf{c}) \approx 1 - \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \ldots\right)$$

where here $x = f_{Mat} v^{1/3} \mathbf{c}^2$. Therefore

$$b(v,\mathbf{c}) \approx f_{Mat} v^{1/3} \mathbf{c}^2 + O\left(x^2\right)$$

It appears that if f_{mat} is small, the higher order terms may be insignificant, and one of Ramkrishna's conditions for self-similarity may occur. However, the cumulative breakage distribution most likely will not fulfill the requirements. Nevertheless, it is possible for the distributions to be self-similar; Ramkrishna's requirements are sufficient to ensure self-similarity, but not necessary.

In fact, a self similarity variable, z', may be calculated to test any experimental data for self-similarity.

$$z' = \frac{b(x)t}{b(x_0)}$$

where x_0 is some mean or initial particle size. The test procedure is to plot the cumulative size distributions at different times versus the similarity variable z' and check for collapse of the curves.

In the case of attrition in the pipe, the distance traveled along the pipe is analogous to the time. The velocity of the multiphase flow as well as the frequency with which a particle hits the wall factor into how far particles travel between particle-wall collisions. The frequency of particle wall collisions can be estimated as $\zeta \bar{v}^{-1}\Theta^{1/2}/L$, where ζ is the particle volume fraction, $\bar{v}$ is an appropriate average particle size, L is the pipe radius, and Θ is the fluctuation energy, or granular temperature, of the particle phase.

Monte Carlo Simulation Procedure

In order to check for the possibility of self-similarity, distributions of the particles are simulated then checked using the z' method described above. The distributions are generated using the Monte Carlo technique – each particle and its daughter particles if it should happen to break, undergo several simulated collisions with the wall.

1) The particle's distribution of velocity is assumed normal, and is characterized by the granular temperature. This distribution is sampled via a random number, and the particle's velocity for this collision is calculated.
2) Using the particle's properties such as hardness, density, etc. the probability of breakage when this particle of volume v and velocity $\mathbf{c}$ hits

the wall can be calculated. Another random number is used to poll whether this particle breaks.

3) The sizes of the resulting particles are calculated, then each daughter particle may undergo further collisions by repeating from 1) above

The procedure is repeated for thousands of particles, in order to ensure a statistically significant number of particles and collisions occur. Then, the entire distribution of particle sizes after any number of collisions can be found.

An Example Result and Conclusions

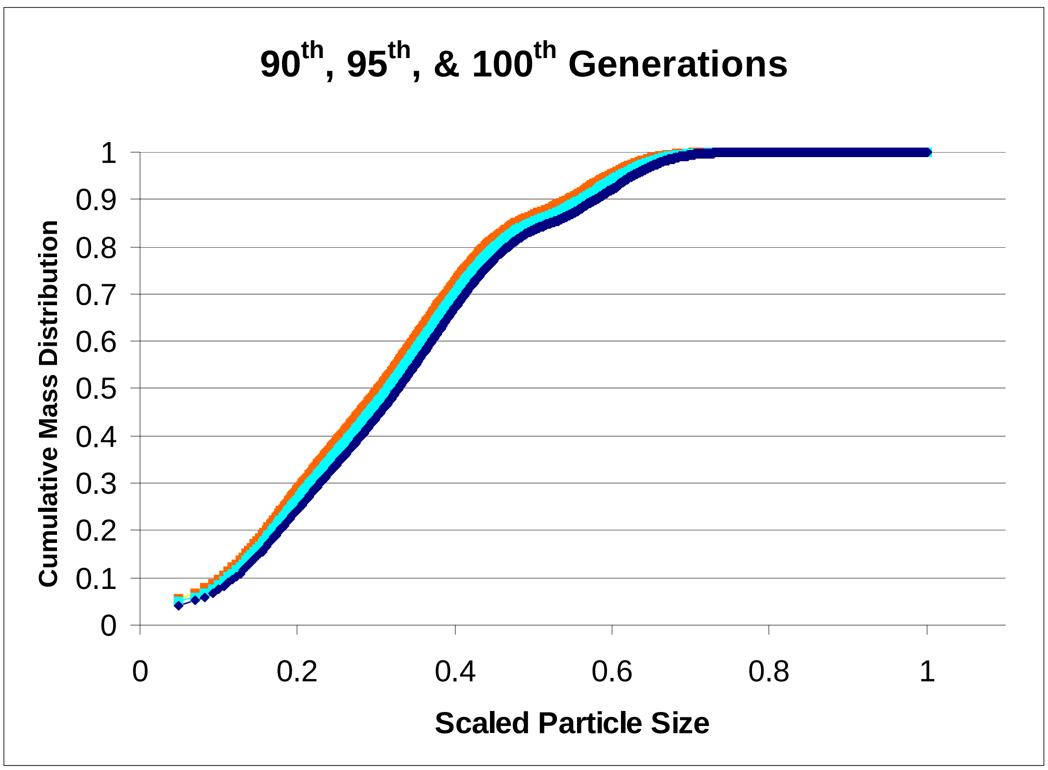

Figure 1. Cumulative Mass Distributions versus the Similarity Variable.

Figure 1 shows the cumulative mass distributions after every particle has undergone 90, 95 and 100 collisions (named the 90^{th}, 95^{th}, or 100^{th} generation) for one specific particle density and hardness and one specific particle phase granular temperature . As seen in the figure, there is significant collapse of the curves, indicating the distributions are approaching a self-similar form after this large number of collisions.

These Monte Carlo simulated curves need to be compared with a more detailed simulation to know how they compare with the actual flow situation. The simulation method of Hamilton et al. (2004) is currently limited to one or two particle sizes, not the entire distribution of sizes, but will be extended in the near future.

In effect, the Monte Carlo simulation method is carried out completely independently of the multiphase flow simulation. If this independence can be exploited, there is a possibility the fully coupled simulation may not be needed to predict the exiting distribution of sizes, thereby enjoying significant computational savings. The granular temperature and volume fraction and average velocity of the multiphase mixture can be estimated from a simulation carried out without breakage, and then imputed into the Monte Carlo simulation.

References

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