

**Effectiveness-Factor Analysis of Membrane Extraction through Parallel-Plate
Mass Exchangers**

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Abstract

Mass transfer of solvent extraction in rectangular membrane modules has been analyzed by the consideration of effectiveness-factor under cocurrent-flow, countercurrent-flow and cross-flow operations. It was found that mass transfer of solvent extraction in rectangular mass exchangers of microporous membrane can be analogous to heat transfer in rectangular heat exchangers. Unlike the calculation of mass-transfer rate by correction-factor analysis, the calculation method by effectiveness-factor analysis developed in present study is easy to conduct without try-and-error and iteration processes.

Keyword: Solvent extraction; Microporous membrane; Rectangular mass exchanger;
Effectiveness factor

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1. Introduction

Mass transfer of solvent extraction in mass exchangers of microporous membrane can be analogous to heat transfer in heat exchangers (Jakob, 1957, Yeh and Huang, 1995). Like ordinary heat exchangers, the mass exchangers of membrane solvent extraction may be extremely different in design and construction, but in principle the only differences are those of relative direction of the two fluids, phases a and b. Accordingly, distinction is made between parallel flow and cross flow. The two fluids in parallel flow may be cocurrent or countercurrent while the direction of two fluids in cross flow may generally be different by 90 angular degrees. It is the purpose of this work to derive the separation theories of membrane solvent extraction with these three types of flow in rectangular mass exchangers with the consideration of effectiveness factor.

2. Theory

Computation of the mass transfer in cocurrent-flow and countercurrent-flow membrane extractors is relatively simple, whereas the theory of the mass exchange in cross-flow system is complicated.

2.1. Mass-transfer coefficients

The equations for calculating mass-transfer rate of solvent extraction through a differential length dx of a membrane sheet, as shown in Fig. 1, are

$$\begin{aligned}dW &= k_a B dx (C_a - C'_a) \\ &= k_m B dx (C'_c - C''_c) \\ &= k_b B dx (C''_b - C_b) \\ &= KB dx (H_{ac} C_a - H_{bc} C_b)\end{aligned}\tag{1}$$

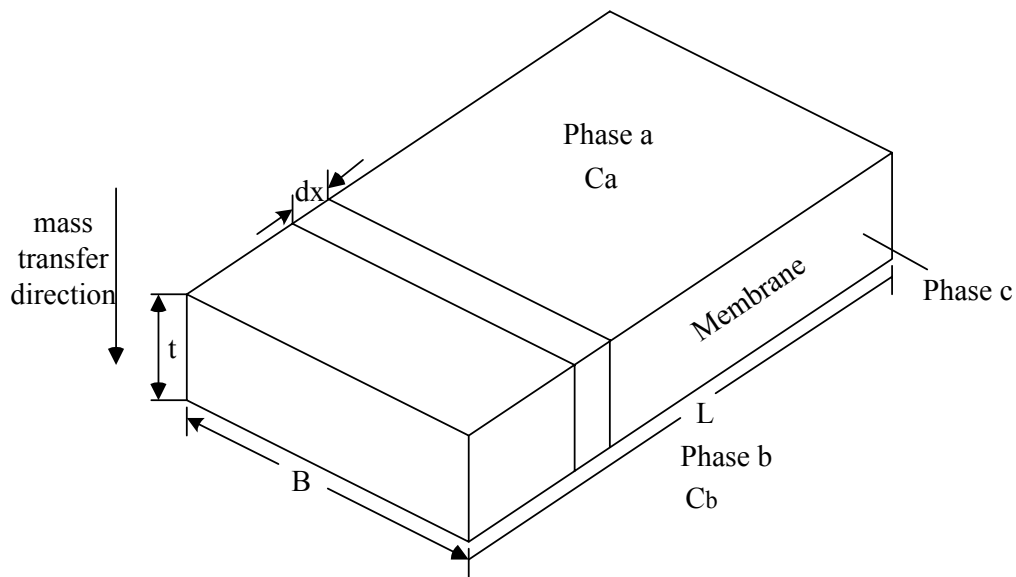
where C_a and C_b denote the bulk concentrations of solute in phases a and b, respectively, H_{ac} and H_{bc} are the distribution coefficients of solute between two different phases, such as

$$H_{ac} = \frac{\text{interface solute concentration in phase c}}{\text{interface solute concentration in phase a}}$$

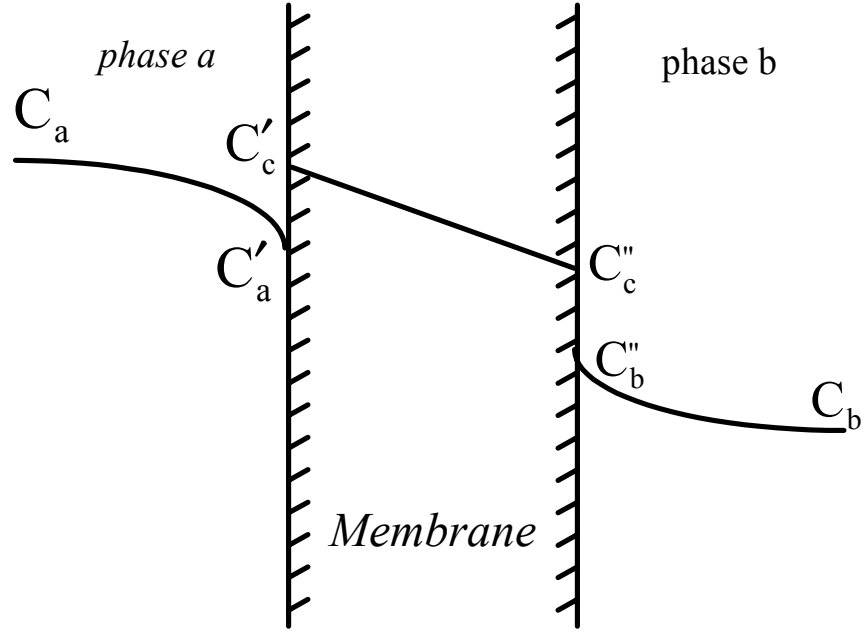
C'_a with C'_c and C''_b with C''_c are two sets of equilibrium concentrations which refer to the conditions at the interface between phases; k_a and k_b are mass-transfer coefficients in the respective phase adjacent to the membrane side, while k_m and K are mass-transfer coefficients within the membrane and the overall mass-transfer coefficient, respectively, B denotes the width of the membrane sheet. The relation between the overall mass-transfer coefficient and three other mass-transfer coefficients can be obtained from Eq. (1) as (Yeh and Huang, 1995)

$$\frac{1}{K} = \frac{H_{ac}}{k_a} + \frac{1}{k_m} + \frac{H_{bc}}{k_b} \quad (2)$$

It should be mentioned that c, the membrane phase, is not necessarily immiscible with phase a or b.



(a) Mass transfer direction



(b) Concentration distributions

Fig. 1. Schematic diagram of membrane solvent extraction.

Since mass transfer within the membrane is due to molecular diffusion alone, the following expression may be used for calculating the mass-transfer coefficient in the membrane (Kiani et al., 1984)

$$k_m = \frac{D_c \varepsilon}{\tau t} \quad (3)$$

where D_c is the diffusivity of solute in phase c, ε and t denote the porosity and thickness of membrane, respectively, and τ is the pore tortuosity of membrane.

Since the mass transfers in both sides of the microporous membrane are convective, k_a and k_b are a function of fluid properties, flow pattern, etc. Some empirical equations for estimating k_a and k_b for microporous hollow fibers were given (Yang and Cussler, 1986; Dahuron and Cussler, 1988; Prasad and Sirkar, 1988). For flow over a microporous membrane sheet (Porter, 1990)

$$k_a = 0.816 \left(\frac{6Q_a D_a^2}{Sh_a^2} \right)^{0.33} \quad (4)$$

$$k_b = 0.816 \left(\frac{6Q_b D_b^2}{Sh_b^2} \right)^{0.33} \quad (5)$$

where Q_a and Q_b denote the volume inflow rates in phases a and b, respectively, D_a and D_b are the mass diffusivities of solute in phases a and b, respectively, S is the overall mass-transfer area of a microporous membrane sheet, h_a and h_b are the heights of flow channel in phases a and b, respectively.

2.2. Outlet concentrations in parallel-flow systems

The schematic diagrams in Figs. 2 and 3 may serve to explain the nomenclature to be employed for cocurrent and countercurrent flows, respectively. Each system consists of two channels, for the fluids a and b, respectively, which are separated by a microporous membrane sheet through which solute is extracted and transferred perpendicularly to its exposed surfaces.

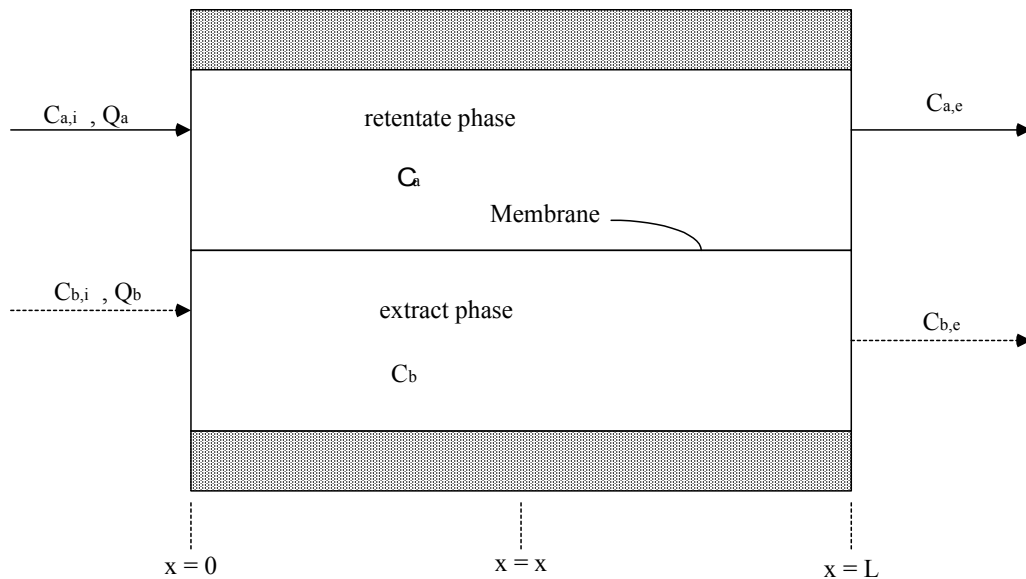


Fig. 2. Cocurrent-flow membrane extractor

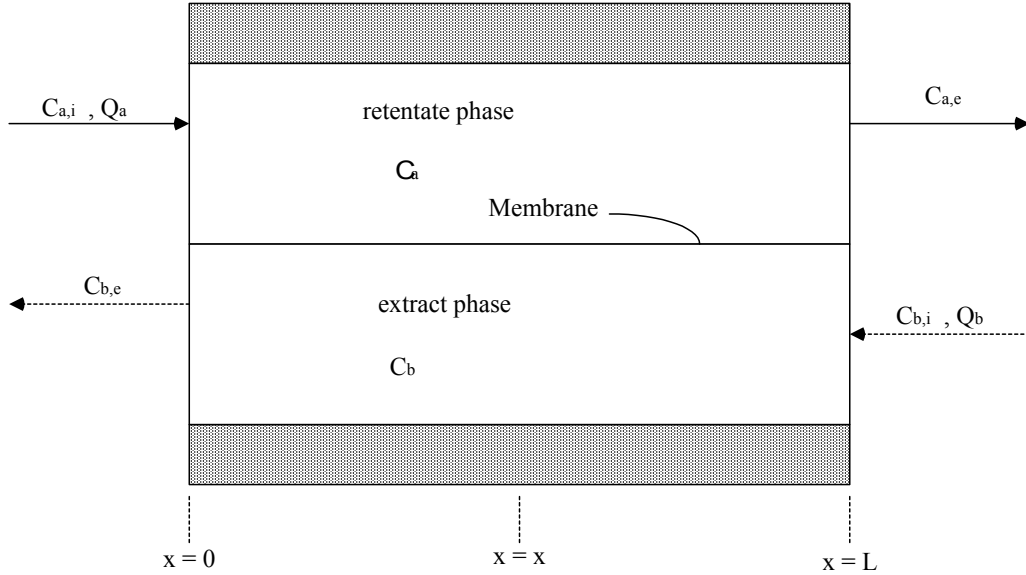


Fig. 3. Countercurrent-flow membrane extractor

We are following the procedure of Underwood (Jacob, 1957). The assumptions made in this analysis are: steady state, no chemical reaction, uniform concentrations and velocities over the cross section of flow, constant rates of flows, constant mass-transfer coefficients and constant distribution coefficients.

A mass balance for the solute in phase a of a differential length dx :

$$-Q_a dC_a = KB(H_{ac}C_a - H_{bc}C_b)dx \quad (6)$$

Similarly, in phase b:

$$Q_b dC_b = KS(H_{ac}C_a - H_{bc}C_b)dx, \text{ cocurrent flow} \quad (7)$$

$$Q_b dC_b = -KS(H_{ac}C_a - H_{bc}C_b)dx, \text{ countercurrent flow} \quad (8)$$

2.2.1. Cocurrent flow

The concentration distributions of solute in phases a and b for cocurrent-flow systems can be obtained by solving Eqs. (6) and (7) simultaneously with the use of the following boundary conditions:

B.C. 1 and 2:

$$C_a = C_{a,i} \quad \text{and} \quad C_b = C_{b,i} \quad \text{at} \quad x = 0 \quad (9),(10)$$

$$C_a = C_{a,e} \quad \text{and} \quad C_b = C_{b,e} \quad \text{at} \quad x = L \quad (11),(12)$$

The outlet concentrations are

$$\begin{aligned} \zeta_{a,e} &= -(a/b)\zeta_{b,e} \\ &= [a/(a+b)] [e^{-(a+b)} - 1] \end{aligned} \quad (13)$$

where

$$\zeta_{a,e} = \frac{(H_{ac} C_{a,e} - H_{ac} C_{a,i})}{(H_{ac} C_{a,i} - H_{bc} C_{b,i})} \quad (14)$$

$$\zeta_{b,e} = \frac{(H_{bc} C_{b,e} - H_{bc} C_{b,i})}{(H_{ac} C_{a,i} - H_{bc} C_{b,i})} \quad (15)$$

$$a = (KSH_{ac})/Q_a \quad (16)$$

$$b = (KSH_{bc})/Q_b \quad (17)$$

It is obviously seen from Eq. (13) that both $\zeta_{a,e}$ and $\zeta_{b,e}$ are functions of a and b only and that though these outlet concentrations depend on the overall mass-transfer area S but do not depend either on the length L , or on the width B , of a microporous membrane sheet.

2.2.2. Countercurrent flow

The concentration distributions of solute in phases a and b for countercurrent-flow systems can be obtained by solving Eqs. (6) and (8) simultaneously with the use of the following boundary conditions:

B.C.3:

$$C_a = C_{a,i} \quad \text{and} \quad C_b = C_{b,e} \quad \text{at} \quad x = 0 \quad (18),(19)$$

B.C.4:

$$C_a = C_{a,e} \quad \text{and} \quad C_b = C_{b,i} \quad \text{at} \quad x = L \quad (20),(21)$$

The outlet concentrations are

$$\begin{aligned} \zeta_{a,e} &= -(a/b)\zeta_{b,e} \\ &= [e^{(b-a)} - 1]/[1 - (b/a)e^{(b-a)}] \end{aligned} \quad (22)$$

It is also seen from Eq. (22) that $\zeta_{a,e}$ and $\zeta_{b,e}$ depend on S but not on either L or B.

Further, the relation between the outlet concentrations of two fluids in countercurrent flow is the same as that in cocurrent flow, as shown in Eqs. (13) and (22).

2.3. Outlet concentrations in cross-flow system

Here we only consider the simplest type of cross-flow systems in which the flow directions of the two fluids are perpendicular, instead of being oblique, to each other. The schematic diagram in Fig. 4 may serve to explain the nomenclature to be employed for cross flow. This system consists of two channels, for the fluids a and b, respectively, which are separated by a microporous membrane sheet through which solute is extracted and transferred perpendicularly to its exposed surfaces.

In addition to all the assumptions made in the previous section, here for simplicity, we also assume uniform concentrations in the cross-sections of flow channel. This is valid when C_a and C_b do not vary sensitively with y and x , respectively. Therefore, $C_a(x, y) \approx C_a(x)$ and $C_b(x, y) \approx C_b(y)$. By taking the mass balances for membrane solvent extraction through a differential area $dx dy$ in a cross-flow mass exchanger, two differential equations for solute concentrations are obtained

$$-Q_a dC_a dy = KB(H_{ac}C_a - H_{bc}C_b) dx dy \quad (23)$$

$$Q_b dC_b dx = KL(H_{ac}C_a - H_{bc}C_b) dx dy \quad (24)$$

Eqs. (23) and (24) can be solved simultaneously by the use of the following boundary

conditions:

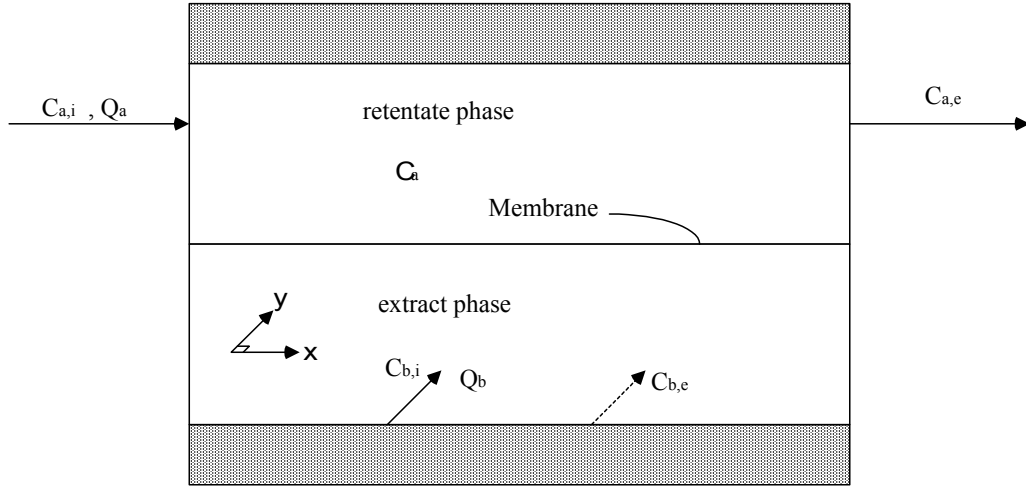


Fig. 4. Cross-flow membrane extractor

B.C.5:

$$C_a = C_{a,i} \text{ at } x = 0 \text{ and } C_a = C_{a,e} \text{ at } x = L \quad (25),(26)$$

B.C.6:

$$C_b = C_{b,i} \text{ at } y = 0 \text{ and } C_b = C_{b,e} \text{ at } y = B \quad (27),(28)$$

The outlet concentrations are (Yeh and Chen, 2000)

$$\begin{aligned} \zeta_{a,e} &= -(a/b)\zeta_{b,e} \\ &= -\left(\frac{1}{1-e^{-a}} + \frac{(b/a)}{1-e^{-b}} - \frac{1}{a}\right)^{-1} \end{aligned} \quad (29)$$

This relation between $\zeta_{a,e}$ and $\zeta_{b,e}$ are exactly the same as those in Eqs. (13) and (22).

2.4. Total mass-transfer rate and effectiveness factor

The total mass-transfer rates for any flow type can be calculated from a mass balance over the entire mass exchanger

$$W = Q_a (C_{a,i} - C_{a,e}) \quad (30)$$

$$= Q_b (C_{b,e} - C_{b,i}) \quad (31)$$

in which the outlet concentrations, $C_{a,e}$ and $C_{b,e}$ can be calculated from Eqs. (14)

and (15) once $\zeta_{a,e}$ and $\zeta_{b,e}$ are known.

The separation efficiency of a membrane solvent extractor may be defined as

$$\eta_{eff} = \frac{\text{actual mass-transfer rate}}{\text{maximum possible mass-transfer rate}}$$

Accordingly, with the use of Eqs. (14), (15), (30) and (31) we may define the effectiveness factor as

$$\begin{aligned} \eta_{eff} &= \frac{W}{KS(H_{ac}C_{a,i} - H_{bc}C_{b,i})} \\ &= \frac{-\zeta_{a,e}}{a} = \frac{\zeta_{b,e}}{b} \end{aligned} \quad (32)$$

Figs. 5-7 show the effectiveness factors for cocurrent flow, countercurrent flow and cross flow, respectively. Once η_{eff} is determined, the total mass-transfer rate can be easily estimated by

$$W = KS\eta_{eff}(H_{ac}C_{a,i} - H_{bc}C_{b,i}) \quad (33)$$

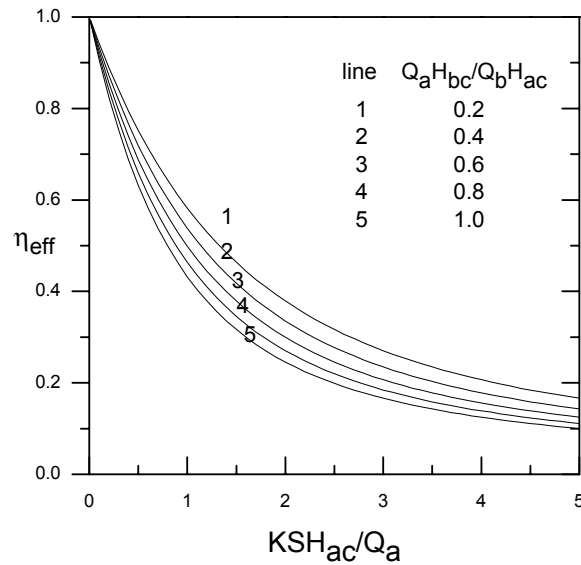


Fig. 5. Effectiveness factor vs a for the cocurrent-flow system

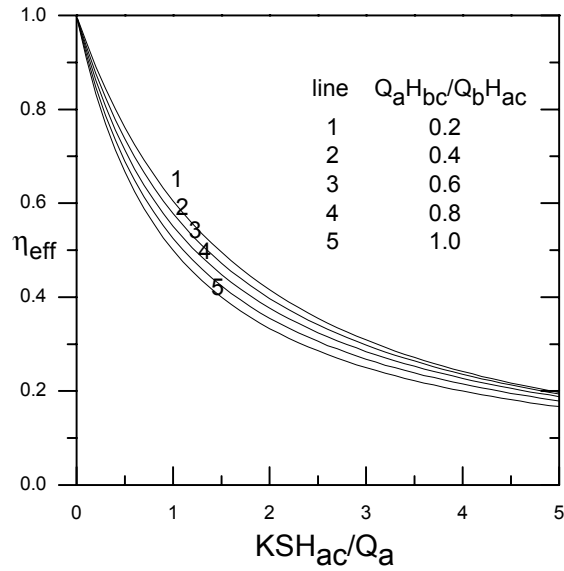


Fig. 6. Effectiveness factor vs a for the countercurrent-flow system

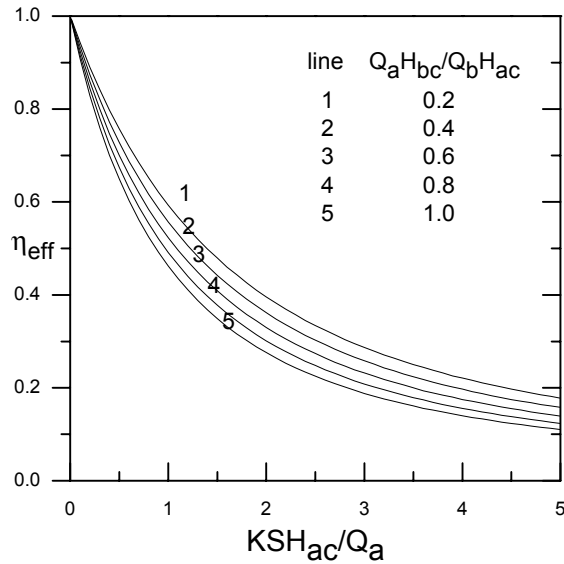


Fig. 7. Effectiveness factor vs a for the cross-flow system

3. Discussion and Conclusion

The mass transfer of membrane solvent extraction through rectangular mass exchangers has been discussed with the consideration of effectiveness factor, under cocurrent-flow, countercurrent-flow and cross-flow operations. Once the solute outlet

concentrations were obtained, the overall mass-transfer rates and separations efficiencies for all types of flow were calculated from the appropriate equations.

Figs. 5-7 are plotted for η_{eff} vs a for cocurrent-flow, countercurrent-flow and cross-flow operations, respectively, under various values of b/a . The term b/a is proportional to the ratio of the flow rate in fluid a (raffinate phase) to that in fluid b (extract phase), (Q_a/Q_b) . The separation efficiency increases with Q_a and Q_b as well as with Q_b/Q_a for the system of interest. The effect of Q_b/Q_a on the separation efficiency will become insignificant when KSH_{ac} is very small or as Q_a is very large. Further, the separation efficiencies for three types of flow turn to the same value when Q_a and Q_b becomes very large, or when KSH_{ac}/Q_a or KSH_{bc}/Q_b approaches zero.

It was also found in previous work that mass transfer of solvent extraction in rectangular mass exchangers of microporous membrane can be analogous to heat transfer in rectangular heat exchangers by correction-factor analysis. However, the calculation of mass-transfer rate by correction-factor analysis should be carried out by iteration method coupled with the assumption of outlet concentration, while the calculation method by effectiveness-factor analysis developed in present study is easy to conduct without try-and-error and iteration processes. It is shown that with the order of magnitude of separation efficiencies in three flow-type devices, most solute is extracted in the countercurrent-flow arrangement, the least in cocurrent flow. The effectiveness-factor approach can be extended for application to a hollow-fiber system, to a multipass membrane system, to a recycled membrane system, or to different mass-transfer devices with or without chemical reaction.

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Notation

a, b	dimensionless group defined by Eq. (16), (17)
B	membrane width, m
C'_a, C'_c	equilibrium concentration of solute at the interface of phase a and phase c, kg mol m^{-3}
C''_b, C''_c	equilibrium concentration of solute at the interface of phase b and phase c, kg mol m^{-3}
C_a, C_b	bulk solute concentration in phase a, in phase b, kg mol m^{-3}
$C_{a,i}, C_{b,i}$	inlet solute concentration in phase a, in phase b, kg mol m^{-3}
$C_{a,e}, C_{b,e}$	outlet solute concentration in phase a, in phase b, kg mol m^{-3}
D_a, D_b, D_c	solute diffusivity in phase a, in phase b, in phase c, $\text{m}^2 \text{s}^{-1}$
H_{ab}, H_{ac}, H_{bc}	distribution coefficient between phase a and phase b, between phase a and phase c, between phase b and phase c
K	overall mass-transfer coefficient, m s^{-1}
k_a, k_b, k_m	mass-transfer coefficient in phase a, in phase b, in the membrane, m s^{-1}
L	effective length of a mass exchanger, m
Q_a, Q_b	volume flow rate in phase a, in phase b, $\text{m}^3 \text{s}^{-1}$
S	overall mass-transfer area of a microporous membrane sheet, BL,

	m^2
t	membrane thickness, cm
W	total mass-transfer rate in a mass exchanger, kg mol s^{-1}
x, y	rectangular coordinates, m

Greek letters

ζ_a, ζ_b	normalised solute concentration change, in phase a, in phase b, compared to the maximum possible concentration change
$\zeta_{a,e}, \zeta_{b,e}$	outlet value of ζ_a, ζ_b
ϵ	porosity of membrane
τ	pore tortuosity of membrane
η_{eff}	effectiveness factor

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