Lattice Boltzmann Simulation of Flow through Three-Dimensional Random Fiber Network with Considering of Quadratic Velocity Term

Xiaoying Rong, & Dewei Qi

Paper Engineering, Chemical Engineering & Imaging, Western Michigan University, Kalamazoo MI 49009

Abstract

Fiber web is modeled as a three-dimensional random cylindrical fiber network. Nonlinear behavior of fluid flowing through the fiber network is numerically simulated by using lattice Boltzmann (LB) method. Nonlinear relationship between the friction factor and the modified Reynolds number is clearly observed and analyzed by using Fochheimer equation, which includes the quadratic term of velocity. We obtain a transition from linear to nonlinear region when the Reynolds numbers are sufficiently high, reflecting inertial effects of flows. The simulated permeability of such fiber network has relatively good agreement with experiment results and finite element simulations.

Introduction

The permeability of porous media is interested in many research and industrial areas, such as paper industries, petroleum industries, environmental, biological processes, and physiological systems. Permeability as a parameter to understand the migration of fluid into porous media has been studied theoretically and experimentally for many years. Modeling the permeability of fibrous porous materials has brought more interests recently.

In two-dimensional simulation, fibrous media were simply modeled as a bank of parallel cylinders. Various ordered arrangements of cylindrical fibers include hexagonal, square and random arrays. The studies involved the calculation of flow along transversal directions of the fibrous network (Happel 1959; Sangani and Acrivos 1982; Drummond and Tahir 1984, Jackson and James 1986). Elliptical cylinders were used by Epstein and Masliyah (1972). They found that the permeability decreases with increasing the flatness of the fibers. Random, squared and hexagonal arrange of two-dimensional fiber networks with band-shaped fibers were studied by Nilsson, *et al*, (1997). The results showed that when the value of fiber aspect ratio is 3.5, the numerical results were consistent with experimental results. Only creeping flow were studied in above simulations.

Koch and Ladd (1997) employed lattice Boltzmann method to simulate the moderate Reynolds number flows through periodic and random arrays of aligned cylinders. The study showed that when the inertial effects were considered, the magnitude of pressure drop per unit length was strongly dependent on the orientation of the axes of the array. The inertial term made transition from linear to quadratic in random arrays. The study also showed that the inertia became smaller at the volume fraction approaching closing packing. Two-dimensional simulation considering inertia was also studied by Andrade, *et al* (1999). The porous media was created by using square plaquettes as obstacles for fluid flow. They showed that the departure from Darcy's law in flow through high porosity percolation structures and at sufficiently high Reynolds numbers inertia became relevant. Forchhermer equation was approved to be valid for low and also a limited range of high Reynolds numbers.

In three-dimensional simulation, modeling of fibrous media becomes much more difficult. Higdon and Ford (1996) used the spectral boundary element formulation to model the permeability of three-dimensional Stokes flow in ordered fibrous media based on simple cubic, body centered cubic and face centered cubic lattices. The volume fraction ranged from extreme dilution to concentrated packing. They obtained the permeability and porosity behavior in a very wide range. Clague and Phillips (1997) investigated the hydraulic permeability for three-dimensional disordered fibrous media with monomodal and bimodal distribution of fiber radius. The fiber network was built as non-overlapped cylindrical fibers for pure collagen and proteoglycan fibers. The results reported were hydraulic permeability vs. diluted fiber volume friction. Koponen, *et al* (1998) employed lattice Boltzmann method in three-dimensional simulation. They applied a gravitational body force to the fluid to simulate the creeping flow. Darcy's law was used to calculate the permeability. The simulated permeability of the web was found to depend exponentially on porosity over a large range of porosity.

Permeability of pulp and paper fibrous media has been measured for different paper grades and different formation. Lindsay and Brady (1993) measured the in-plane and transverse permeability of a large range of paper types which revealed that paper is highly anisotropic material. Nilsson *et al*. (1997) measured 21 different pulp and paper samples with various basis weight, thickness and pulp types. It was found that paper made by hardwood pulp had higher permeability than the paper made by kraft pulp.

Despite the numerous experimental studies, flow through three-dimensional random fibrous porous media has not been well simulated with considering the quadratic term of velocity. In this article, we use lattice Boltzmann method to model and simulate fluid flows through a random fiber network. The fiber network is modeled with equal size and random distribution cylinders. The correlation of pressure drop vs. velocity is studied. The effect of inertia is focused at porosity range from 48% to 72%, a nonlinear behavior between friction factor and modified Reynolds number is clearly observed. The simulated permeabilities are compared with experimental results.

Permeability of Porous Media

Single phase flow through microscopically disordered porous media at low Reynolds numbers is described by Darcy's law (Bear 1972). The superficial flow rate $\langle u \rangle$ of a viscous fluid through a porous medium of length *L* is proportional to the applied pressure difference ΔP and inversely proportional to the dynamic viscosity μ.

$$\langle u \rangle = \frac{k}{\mu}\frac{\Delta P}{L} \tag{1}$$

At low Reynolds number where the flow is laminar, viscous forces are predominant, linear Darcy's law is valid. k is the permeability with the unit of length square. However as Reynolds number increases, the inertial forces has to be considered, which describes the transition from viscous forces predominated creeping flow to another inertial forces governed laminar region, and gradually pass to turbulent flow (Bear 1972).

In order to always satisfy Darcy's law in the creeping flow region and to correctly capture the influence of inertia at high Reynolds numbers, the well known Forchhermer equation (Perry 1984) consists of a linear term of viscous component and a power term of inertial component:

$$-\frac{\Delta P}{L} = \alpha\mu\langle u \rangle + \beta\rho\langle u \rangle^2 \tag{2}$$

where α is the viscous coefficient, β is the inertial coefficient, they are both resistance coefficients to describe the physical properties of the porous material. At low Reynolds number, the quadric term of velocity is close to zero, therefore can be ignored, which turns Forchhermer equation to Darcy's law. α^{-1} is defined as the permeability of porous media.

Forchhermer equation can be modified as friction factor and Reynolds number correlation (Andrade, *et al* 1999):

$$f = \frac{1}{\mathrm{Re}'} + 1 \tag{3}$$

where

$$f = -\frac{\Delta P}{L\beta\rho\langle u \rangle^2} \tag{5}$$

$$\mathrm{Re}' = \frac{\beta\rho\langle u \rangle}{\alpha\mu} \tag{6}$$

The formula can be used for calculating the friction factor of a porous media with various geometry and porosity. The universal factors give a good comparison of different porous materials and flow conditions. We will use the Forchhermer equation to analyze the numerical results.

The Lattice-Boltzmann Method

The lattice Bolzmann method has been successfully applied for simulating the interaction between fluid and solid particles (Aidun, 1998; Qi, 1999; Ladd 1994, 2001). The kinetic nature of lattice Boltzmann method enables it to simulate the complex geometry such as fluid flow in porous media.

In lattice-Boltzmann (LB) method, fluid particles reside on the lattice nodes and move to their nearest neighbors along the links with unit spacing in each unit time step. The lattice Boltzmann equation with Bhatanaga-Gross-Krook (BGK) single relaxation time, is given by (Ladd 1994)

$$f_\sigma(\vec{x}_i + \vec{e}_\sigma, t+1) = f_\sigma(\vec{x}_i, t) - \frac{1}{\tau}[f_\sigma(\vec{x}_i, t) - f_\sigma^{eq}(\vec{x}_i, t)] \qquad (7)$$

where $f_\sigma(\vec{x}, t)$ is the fluid particle distribution function for particles with velocity $\vec{e}_\sigma$ at position *x* and time *t*, $f_\sigma^{eq}(\vec{x}, t)$ is the equilibrium distribution function and τ is the single relaxation time.

The simulations described in this paper were performed using the D3Q15 model. It possesses a rest particle state, six links with nearest neighbors, and eight links with next nearest neighbors. Periodic boundary conditions in the flow direction with bounce back on the solid nodes were used. $f_\sigma^{eq}(\vec{x}, t)$ is taken as

$$f_\sigma^{eq}(\vec{x}, t) = \omega_\sigma \rho_f \left\{1 + 3(\vec{e}_\sigma \cdot \vec{u}) + \frac{9}{2}(\vec{e}_\sigma \cdot \vec{u})^2 - \frac{3}{2}(\vec{e}_\sigma \cdot \vec{e}_\sigma)\right\}, \qquad (8)$$

where ρ_f is the density of the fluid, $\vec{u}$ is the velocity. $\sigma = 1$ represents the particles move to the nearest neighbors, $\sigma = 2$ represents the particles move to the second nearest neighbors, $\sigma = 0$ represents the particles rest at the nodes. The weight coefficient ω_σ depends on the discrete velocity set $\vec{e}_\sigma$ and dimensions of space.

In order to drive the flow, a pressure difference is imposed between the two faces normal to the axis of the superficial flow by applying a uniform body force to the fluid (Ladd 2001; Guo 2002). The LB method is modified to account for the applied external body forces which adds every time step a fixed amount of momentum on the fluid points:

$$f_\sigma(\vec{x}+\vec{e}_\sigma\Delta t, t+\Delta t) = f_\sigma(\vec{x},t) - \frac{1}{\tau}\left[f_\sigma(x,t) - f_\sigma^{eq}(x,t)\right] + \vec{F}_\sigma \tag{9}$$

The forcing term in the present work is writen as,

$$\vec{F}_\sigma = \rho_f \omega_\sigma \left[3\left(\vec{e}_\sigma \cdot \vec{G}\right)\right], \tag{10}$$

where, ω_σ is determined by equation (8), $\vec{G}$ is pressure gradient parameter. For a spatially uniform force, the second order variation can be neglected (Ladd 2001).

Simulation Results and Discussions

Cylindrical fibers are used to simulate the random network structure of fibrous media. The structure is generated by randomly placing every fiber into the simulation box. With this grown method, the orientation of each fiber is random on *x-y* plane. At *z* direction, fibers were laid with an angle less than ±15-degree. If a fiber meets the nodes occupied by other fibers, these fibers occupy the same nodes. The porosity is calculated by dividing number of nodes occupied by fibers by the total number of lattices nodes. Fiber in this simulation is 25μm in diameter and 1mm in length, fibrous web is simulated at 0.1mm in thickness or *z* direction. The simulation box is 128×128×64. The geometry can be illustrated as:

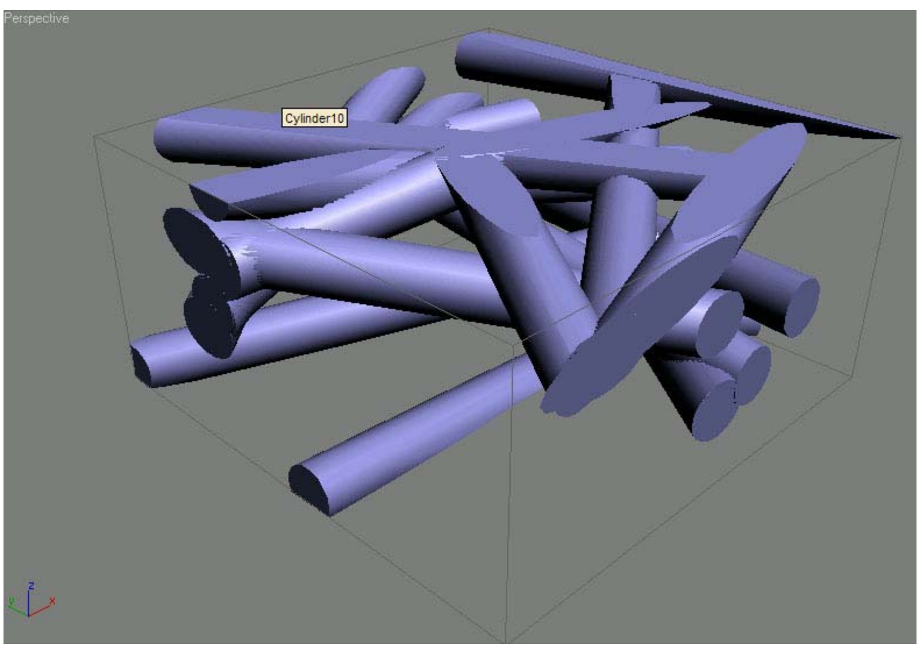

Figure 1: The geometry of simulated fiber network with 18 fibers.

Notice that the porosity depends on the fiber length, diameter of cylindrical fiber, the orientation of fibers. It is evident that this structure is close to fibrous web, e.g. paper handsheet.

In this work, to achieve porosity of 72% needs 18 fibers for random arrangement and 29 fibers for porosity of 63%.

Fluid flows through the fiber network in z direction in order to simulate the transversal permeability. *x* and *y* direction of the simulation box are periodical . Non-slip boundary condition is used at fluid and fiber interfaces. Flow was induced by applying a body force on fluid particles (Ladd 2001).

At certain porosity, the simulation data fit to equation (2), and the coefficients α and β are estimated thereafter. Modified Reynolds number Re' and friction factor *f* are calculated by using equation (3) and (4).

The simulated pressure gradient vs. velocity is plotted in **Figure 2**. As shown in the figure, curves with quadratic term of velocity fit the simulation data very well.

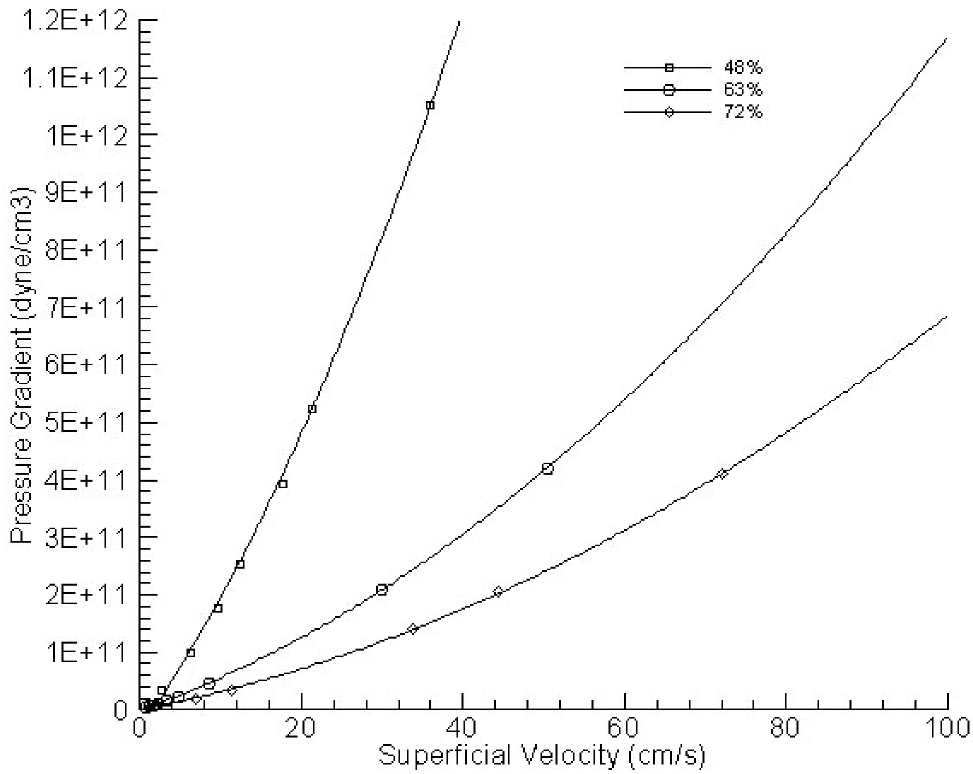

Figure 2: Quadric curves fit the simulation data of pressure gradient vs. superficial velocity.

The curves obtained by using LB method captured the expected tendency and the important transitions. The curving fitting parameters α and β used in equation (3) and (4) are given in **Table 1**.

Table 1: Curve-fit parameters obtained from equation (3) and (4) for random cylindrical fiber network.

Porosity	$\alpha\mu$	$\beta\rho$
48%	2.028 E10	2.740 E8
63%	4.926 E9	6.782 E7
72%	2.680 E9	4.176 E7

As shown in the simulation results, pressure gradients vs. superficial velocity curves are nonlinear after the superficial velocity is higher than 25 cm/s for 72% and 63% porosities. The curve of 48% is more linear in that range.

By plotting the modified friction factor *f* vs. modified Reynolds number Re', we observed the transition zone from linear to nonlinear in terms of modified *f* and Re'. The curves showed in **Figure 3** agree with experiments (Bear 1997). It is clear that the linear to nonlinear transition starts at Re' around 10^{-1}, which is also well agreed with that by Andrade *et al* (1999).

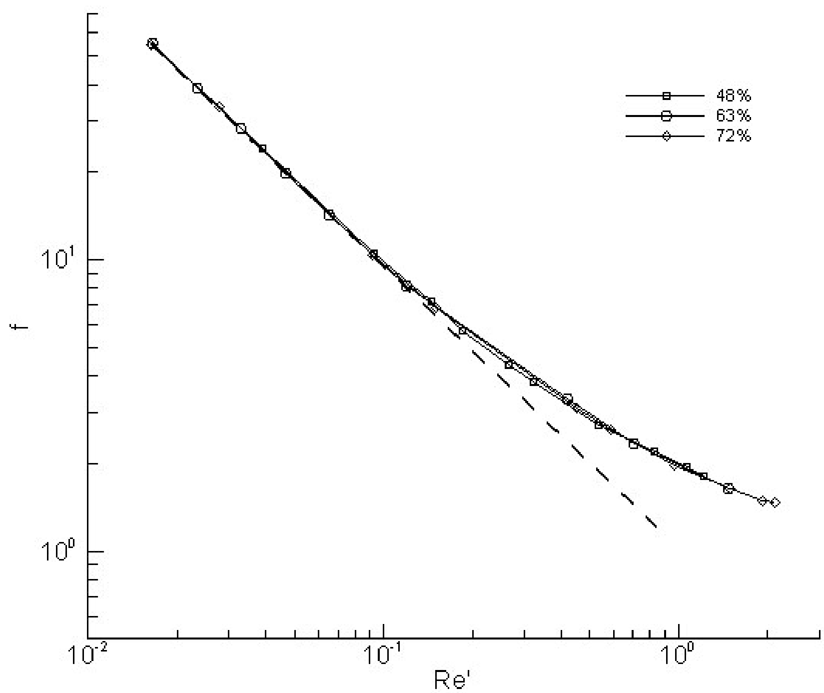

Figure 3: Modified friction factor and Reynolds number show the transaction zone from linear to nonlinear of random fiber network. Solid lines are the fit to the Forchheimer equation, dash line is the fit to Darcy's law at low Re'.

The calculated permeabilities α^{-1} for random fiber network are compared with the experimental results of transversal permeability for hardwood handsheets (Lindsay *et al* 1993) and permeailities measured by gas permeation of newsprint at different porosities (Nillson *et al* 1997). The comparison is given in **Figure 4**. The simulation results showed the good agreement with those by Lindsay *et al* (1993).

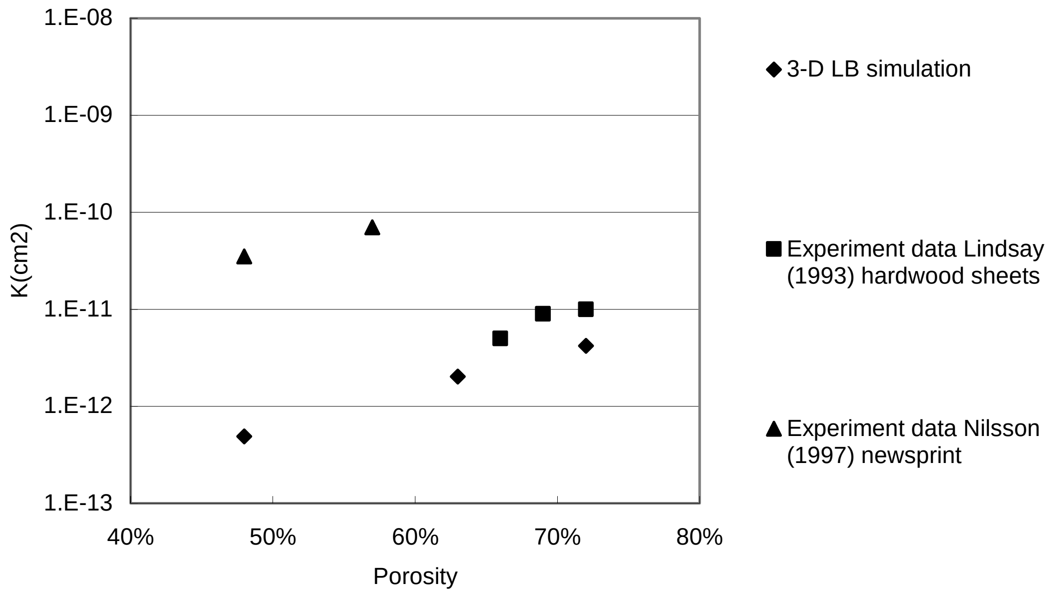

Figure 4: Comparison of permeabilities for three-dimensional LB simulation and experimental data.

The further conclusion to be drawn form the data in Figure 4 is that the permeability calculated by considering the quadric velocity term has the reasonable accuracy of predicting the permeability of paper fiber network. The fiber network model we built well estimated the fluid transportation of hardwood sheets.

Conclusions

In this paper, we numerically study the nonlinear behavior of fluid flowing through a three-dimensional random fiber network in a porosity range from 48% to 72% by using LB method. We found that the curves of pressure gradients vs. superficial velocity are nonlinear after the superficial velocity is higher than 25 cm/s for 72% and 63 % porosities. The curve at 48% porosity is more linear. It is shown that the inertia effect is important at relatively high Reynolds numbers. The relationship between modified Reynolds number and friction factor are in excellent agreement with the Forchhermer equation. The results of permeability in the fiber network have good agreement with the experimental data (Landsay and Brady 1993).

References

Aidun, C. K., Lu, Y. and Ding, E., *Direct Analysis of Particulate Suspensions with Inertia Using the Discrete Boltzmann Equation*, J. Fluid Mech. **373**, 287 (1998)

Andrade, J. S. Jr., Costa, U. M., S. Almeida, M. P. Makse, H. A., and Stanley, H. E. *Inertial Effects on Fluid Flow through Disordered Porous Media*, Physical Review Letters, **82**, 5249 (1999)

Bear, J. *Dynamics of Fluid in Porous Media* (Dover, New York, 1972)

Chen, S., and Doolen, G. D., *Lattice Boltzmann Method for Fluid Flows*, Annu. Re. Fluid Mech., **30**, 329 (1998)

Clague, D. S. and Phillips, R. J., *A Numerical Calculation of the Hydraulic Permeability of Three-Dimensional Disordered Fibrous Media*, Phys. Fluids, **9**(6), 1562 (1997)

Drummond, J. E., Tahir, M. I., *Laminar Viscous Flow through Regular Arrays of Parallel Solid Cylinders*, Int. J. Multiphase Flow, **10**, 515 (1984)

Epstein, N., Masliyah, J. H., *Creeping Flow through Clusters of Spheroids and Elliptical Cylinders*, Chem. Eng. J. **3**, 169 (1972)

Guo, Z. L., Zheng, C. G. and Shi, B. C., *Discrete Lattice Effects on the Forcing Term in the Lattice Boltzmann Method*, Phys. Re. E, **65**, 046308 (2002)

Happel, J., *Viscous Flow Relative to Arrays of Cylinders,* AIChE J. **5**, 174 (1959)

Hingdon, J. J. L., Ford, G. D., *Permeability of Three-Dimensional Models of Fibrous Porous Media*, J. Fluid Mech., **308**, 341 (1996)

Koponen, A. D., Kandhai, E. Hellén, Alava, M., Hoekstra, A., Kataja, M., Niskanen, K., *Permeability of Three Dimensional Random Fiber Webs*, Physical Review Letters, **80**, 716 (1998)

Ladd, A. J. C., *Numerical Simulations of Particle Suspensions via a Discretized Boltzmann Equation. Part 1. Theoretical Foundation*, J. Fluid Mech., **271**, 285 (1994)

Ladd, A. J. C., Verberg, R., *Lattice Boltzmann Simulations of Particle-Fluid Suspensions,* J. Statistical Physics, **104**, 1191 (2001)

Lindsay, J. D. and Brady, P. H., *Studies of Anisotropic Permeability with Applications to Water Removal in Fibrous Webs. Part 2: Water Removal and Other Factors Affecting Permeability*, Tappi J. 76, 167 (1993b)

Nilsson, L. and Stenstrom, S., *A Study of the Permeability of Pulp and Paper*, Int. J. Multiphase Fluid, **23**, 131 (1997)

Perry, J. H., *Chemical Engineer's Handbook*, 6th ed., McGraw-Hill (1984)

Qi, D. W., *Lattice-Boltzmann Simulations of Particles in Non-Zero-Reynolds-Number Flow*, J. Fluid Mech., **385**, 41 (1999)

Qian, Y., PhD thesis, de l'Universite, (Pierre et Marie Curie 1990).

Qian, Y., D. d'Humieres, and P. Lallemand, *Lattice BGK Models for Navier-Stokes Equation*, Europhys. Lett., **17**, 479 (1992)

Sangani, A. S., Acrivos, A., *Slow Flow Past Periodic Arrays of Cylinders with Application to Heat Transfer*, Int. J. Multiphase Flow, **8**, 193 (1982)