The Value of Sensor Networks for Advanced Process Control

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Abstract: We have recently proposed a new formulation of the optimal sensor selection problem for closed-loop partial state information dynamic systems. Although this formulation (a mixed integer convex program) yields to a globally optimal search scheme, the only economic information used is with regard to the capital cost of the sensors. Additionally, we have recently proposed a new stochastic based formulation of the minimally backed-off operating point (MBOP) selection problem. Although this formulation has strong profit based notions (due to its close relations to model predictive control and real-time optimization) and yields to a globally optimal search scheme (due to its convex/reverse-convex form), it assumes a fixed sensor array. Thus, the goal of this work is to combine the two formulations and arrive at a value based sensor network design scheme. In addition to utilizing the capital cost of the sensors, this formulation will incorporate the impact of the sensor network on the feasible set of backed-off operating points and thus the operational profit of the process. We will further show that this new formulation can be cast into a convex/reverse-convex form, and is thus readily solved globally via a branch and bound procedure. The proposed method is then illustrated through a CSTR example.

1. Introduction

The subject of hardware selection has been of interest for some time (see [1], [2], [3] or [4] for extensive reviews of the subject). Clearly, this great interest, by a wide variety of authors, stems from the eventual realization that in spite of a high quality compensating element (i.e., the feedback law) the ultimate limitation on closed-loop performance will be dictated by the quality of hardware elements. Given a set of desired closed-loop performance goals, the typical economic based sensor network design problem ([5], [6], [7], [8]) is to determine the Minimum Capital Cost (MCC) network capable of meeting the pre-specified performance bounds. To date, all existing Sensor Network Design (SND) schemes that include economics are of this class or are equivalent to it. Unfortunately, the required selection of performance bounds is frequently a difficult task. That is, from a financial perspective, the process engineer can only state the cost of an up-grade but may have trouble quantifying the profit that will result. Thus, the goal of this work is to define a control system focused SND scheme that simultaneously incorporates the notion profit.

Without the notion of profit, the MCC-SND problem for closed-loop systems is formulated as follows: Measured but noisy outputs are used by a Kalman filter to generate an estimate of the true system state. This estimate is then used by the controller to coerce the various performance outputs into being close to zero (in deviation variables). Thus, the goal is to achieve small standard deviations for the state and manipulated variables. To capture this notion one should define a set of performance inequalities: $\sigma_{x_i}^2 < \bar{x}_i^2$ and $\sigma_{u_i}^2 < \bar{u}_i^2$ where $\bar{x}_i$ and $\bar{u}_i$ are the maximum acceptable standard deviations. The trade-off concept of this formulation is that while the addition of a sensor will increase the system's capital cost it will also increase the controller's ability to satisfy the performance inequalities. Details of this formulation can be found in [7] and [8]. The main result is to convert the originally found MINLP into a Mixed Integer Convex Program (MICP), from which global solutions can be obtained.

In recent decades, Model Predictive Control (MPC) has become a major vehicle for increasing process profitability, and thus will play a major role our new profit based problem formulation. It has long been recognized that the primary advantage of implementing MPC is the ability to move the steady-state operating point closer to operational constraints, which typically harbor the greatest amount of profit. Unfortunately, operation at the Optimal Steady-State Operating Point (OSSOP) is precluded due to the likelihood of constraint violations in the face of expected disturbances. Thus, the notion of a Backed-off Operating Point (BOP) was introduced by Narraway *et. al*. [9], to allow for disturbance induced variations while preserving much of the economic advantage of the OSSOP. These notions are illustrated in figure 1, where the conservative operating point is desirable from a constraint observance perspective. Using the notion of an Expected Dynamic Operating Region (EDOR), the BOP is selected to balance economic and constraint observance objectives. (The key aspect is to ensure the EDOR does not extend beyond the constraint polytope.) The impact of tuning parameters on the selection of the BOP is that controller design changes can result in modifications of the size and shape of the EDOR. Thus, it is proposed that appropriate controller design will allow the BOP to be moved closer to the OSSOP. By defining a simultaneous controller and BOP selection problem we arrive at the notion of a Minimally Backed-off Operating Point (MBOP) selection problem. As stated above, Narraway *et. al.* [9], first advocated the notion of a BOP. Stochastic versions of the problem were presented in Loeblein and Perkins, [10] van Hessem *et. al.,* [11], Chmielewski and Manthanwar, [12] and Peng *et. al.*,[13]. These define the EDOR based upon the covariance ellipsoid generated by a given confidence interval.

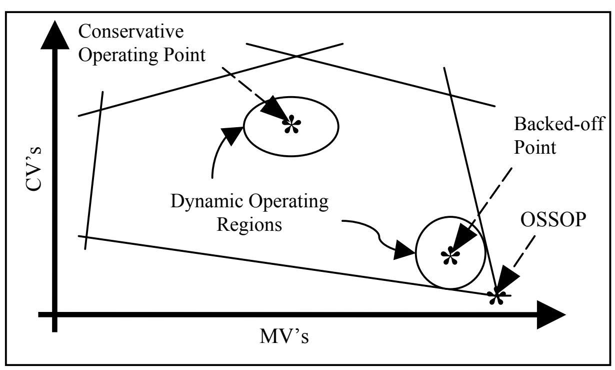

Figure 1: Economics of Operating Point Selection.

Our new formulation aims to combine the capital cost aspects of the sensor selection problem, [8], with the profit based notions of the MBOP selection problem [13]. Starting from the MBOP problem, it is clear that the addition of sensors (i.e., better information to the controller) will allow for reduced back-off, and thus increased profit. Thus, the trade-off question is: When does the cost of adding sensors surpass the expected increase in profit? The computational challenge associated with combining these two problems is to coordinate between the integer constraints of sensor selection and the reverse-convex constraints of MBOP selection. To overcome this issue we will develop a reverse-convex constrained version of the sensor selection problem (i.e., replacing the integer constraints). This will allow for a single branch and bound procedure to solve for the best sensor network while simultaneously finding the MBOP corresponding to that network.

2. Problem Formulation

2.1 Steady-State Optimization

Similar to the MBOP selection problem, our new formulation starts by assuming the existence of a steady-state economic optimizer which operates on a nonlinear dynamic system $\dot{s} = f(s,m,p)$, where s is the state, m is the manipulated input and p is the disturbance. The steady-state optimizer will employ a nonlinear objective along with a set linear inequality constraints of the form: $d_{\min} \le q \le d_{\max}$, where $q = D_x s + D_u u + D_w \bar{p}$. Thus, the steady-state optimization problem is stated as

$$\begin{aligned} &\min_{s,m} \quad g(s,m,\bar{p}) \\ &s.t. \quad 0 = f(s,m,\bar{p}); \quad q = D_x s + D_u u + D_w \bar{p}; \quad d_{\min} \le q \le d_{\max} \end{aligned} \tag{1}$$

where $\bar{p}$ is the measured/expected value of the disturbance and $g(s,m,\bar{p})$ is an annualized profit function in the sense that it has units of \$/yr. The solution to problem 1, the OSSOP, is denoted as s^* and m^*. Next we assume that the actual operation of the plant will occur near the point $(s^*, m^*, \bar{p})$ and develop a linearized dynamic model; $\dot{\tilde{s}} = A\tilde{s} + B\tilde{m} + G\tilde{p}$ where $(\tilde{s}, \tilde{m}, \tilde{p})$ are deviation variables with respect to $(s^*, m^*, \bar{p})$ and A, B and G are partial derivatives of $f(s,m,p)$ evaluated at $(s^*, m^*, \bar{p})$. If we assume p to be equal to $\bar{p}$ $\quad$ (*i.e.*, $\tilde{p} = 0$) then the following equality limits the set of available BOPs: $0 = A\tilde{s}_{ss} + B\tilde{m}_{ss}$. A similar development with respect to the constraints yields the following additional limitation of available BOPs: $\tilde{d}_{\min} \le \tilde{z}_{ss} \le \tilde{d}_{\max}$ where $\tilde{d}_{\min} = d_{\min} - D_x s^* - D_u m^* - D_w \bar{p}, \tilde{d}_{\max} = d_{\max} - D_x s^* - D_u m^* - D_w \bar{p}$ and $\tilde{z}_{ss} = D_x \tilde{s}_{ss} + D\tilde{m}_{ss}$. Since dynamic operation will occur around the point $(\tilde{s}_{ss}, \tilde{m}_{ss}, \bar{p})$, we define new deviation variables and constraints:

$$\dot{x} = Ax + Bu + Gw, \quad z = D_x x + D_u u + D_w w, \quad \tilde{d}_{\min} - \tilde{z}_{ss} \le z \le \tilde{d}_{\max} - \tilde{z}_{ss},$$

where $x = s - s^* - \tilde{s}_{ss}$, $u = m - m^* - \tilde{m}_{ss}$ and $w = p - \bar{p}$.

2.2 Sensor Placement

Given a measurement equation $y = Cx + v$ along with a stochastic framework (w and v being Gaussian white processes with zero mean and covariance Σ_w and Σ_v, respectively), a Kalman filter can be used to generate an estimate, $\hat{x}$, of the true state, x. If we assume that each of the measurement noise terms is independent of the others, then Σ_v will be diagonal and Σ_v^{-1} can be readily defined as $diag\{\beta_i / \bar{\sigma}_{v_i}^2\}$ where $\bar{\sigma}_{v_i}^2$ is the actual variance of sensor i and β_i is a decision variable indicating the presence/absence of sensor i. This definition of Σ_v indicates our method of removing sensors. For example, if $\beta_i \to 0$, then the i^{th} element of Σ_v will become large, indicating that the i^{th} sensor is extremely noisy. In this case, the optimality aspect of the Kalman filter will force it to ignore this noisy sensor and make it appear to be absent. Note that this removal of a sensor is achieved without changing the value or structure of the matrix C.

A direct extension of our previous sensor selection formulation [8] would result in a mixed integer convex program, due to the integer aspects of β_i. However, as discussed in section 1, this integer formulation does not combine well with the reverse-convex aspects of the MBOP selection problem. Thus, we propose the following method of converting the integer constraints to reverse-convex inequalities. We start by defining a scalar function $h : \Re \to \Re$: $h(x) = \begin{cases} m_0 x & \text{if} \quad x < x_0 \\ m_1 x + b_1 & \text{if} \quad x \geq x_0 \end{cases}$ where $m_0 = y_0 / x_0$, $m_1 = (1 - y_0)/(1 - x_0)$ and $b_1 = (y_0 - x_0)/(1 - x_0)$. Selecting $x_0 = 0.85$ and $y_0 = 0.1$ yields a function h as in figure 2 (in the example to follow we used $x_0 = 0.9999$ and $y_0 = 0.0001$). Next we introduce a new set of variables β_c and β_p (each with elements $\beta_{c,i}$ and $\beta_{p,i}$). These will serve to split β into a cost aspect, β_c, and a precision aspect, β_p. That is, $\beta_{c,i}$ will appear in the objective function to reflect the capital cost of sensor *i* and $\beta_{p,i}$ will appear in the Σ_v^{-1} function to reflect the precision of sensor *i*. The two β's are then connected by the inequality $\beta_{p,i} \leq h(\beta_{c,i})$ (see figure 2). This reverse-convex inequality mimics the integer constraint by only allowing the precision aspect to be large (i.e., $\beta_{p,i} > y_0$) if the cost aspect is also large (i.e., if $\beta_{c,i} > x_0$).

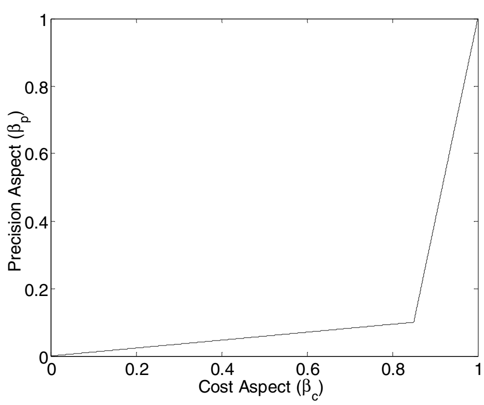

Figure 2: New Reverse Convex Constraint

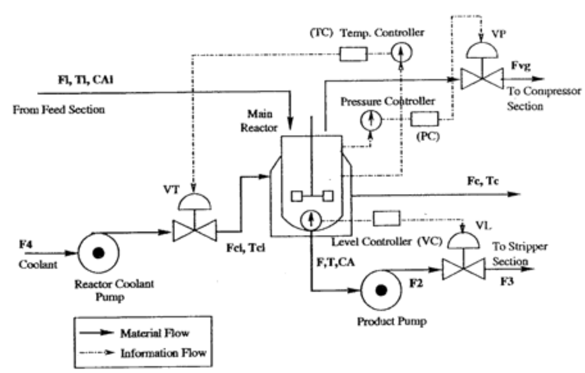

Figure 3: CSTR System of the Example

2.3 Covariance Analysis

Returning to the partial state information control problem, we apply a linear feedback of the form $u = L\hat{x}$ to the linear dynamic system described above. Then, the steady-state covariance of the signal *z* is given by $\Sigma_z = (D_x + D_u L)(\Sigma_x - \Sigma_e)(D_x + D_u L)^T + D_x \Sigma_e D_x^T + D_w \Sigma_w D_w^T$ where Σ_x and Σ_e are the positive semi-definite solutions to

$$A\Sigma_x + \Sigma_x A^T + BL(\Sigma_x - \Sigma_e) + (\Sigma_x - \Sigma_e)L^T B^T + G\Sigma_w G^T = 0$$
$$A\Sigma_e + \Sigma_e A^T - \Sigma_e C^T \Sigma_v^{-1} C \Sigma_e + G\Sigma_w G^T = 0$$

(see [8] for the details of the above derivations). Since the variance of the i^{th} output (denoted ζ_i) is found as the i^{th} diagonal of Σ_z, we define ϕ_i as the i^{th} row of an appropriately sized identity matrix, and conclude that $\zeta_i = \phi_i \Sigma_z \phi_i^T$. Contrasting this with the point-wise in time constraints of section 2.1, $\tilde{d}_{\min,i} - \tilde{z}_{ss,i} \le z_i(t) \le \tilde{d}_{\max,i} - \tilde{z}_{ss,i}$, we propose the following inequality constraints: $\zeta_i \le (\tilde{z}_{ss,i} - \tilde{d}_{\max,i})^2$ and $\zeta_i \le (\tilde{z}_{ss,i} - \tilde{d}_{\min,i})^2$. These reverse-convex constraints serve to ensure that the single standard deviation ellipse defined by Σ_z (and centered at q), will be contained in the box defined by $(d_{\min}, d_{\max})$.

2.4 Simultaneous Formulation

Using the above developments, the simultaneous sensor and MBOP selection problem is now formulated as:

$$\min_{\substack{\tilde{s}_{ss}, \tilde{m}_{ss}, \tilde{z}_{ss}, \\ \zeta, \beta_c, \beta_p, \Sigma_v \ge 0 \\ \Sigma_x \ge 0, \Sigma_e \ge 0}} \quad d_s^T \tilde{s}_{ss} + d_m^T \tilde{m}_{ss} + d_c^T \beta_c$$

$$\begin{aligned}
s.t.\quad & 0 = A\tilde{s}_{ss} + B\tilde{m}_{ss}; \quad \tilde{z}_{ss} = D_x \tilde{s}_{ss} + D_u \tilde{m}_{ss}, \quad \tilde{d}_{\min} \le \tilde{z}_{ss} \le \tilde{d}_{\max} \\
& \zeta_i < (\tilde{z}_{ss,i} - \tilde{d}_{\min,i})^2, \quad \zeta_i < (\tilde{z}_{ss,i} - \tilde{d}_{\max,i})^2, \quad i = 1 \cdots n_z \\
& 0 \le \beta_{c,i} \le 1, \quad 0 \le \beta_{p,i} \le 1 \quad i = 1 \cdots n_\beta \\
& \beta_{p,i} \le h(\beta_{c,i}) \quad i = 1 \cdots n_\beta \quad \Sigma_v^{-1} = diag\{\beta_{p,i} / \bar{\sigma}_{v_i}^2\} \\
& \zeta_i = \phi_i [(D_x + D_u L)(\Sigma_x - \Sigma_e)(D_x + D_u L)^T + D_x \Sigma_e D_x^T + D_w \Sigma_w D_w^T] \phi_i^T, \quad i = 1 \cdots n_z \\
& 0 = A\Sigma_x + \Sigma_x A^T + G\Sigma_w G^T + BL(\Sigma_x - \Sigma_e) + (\Sigma_x - \Sigma_e) L^T B^T \\
& 0 = A\Sigma_e + \Sigma_e A^T - \Sigma_e C^T \Sigma_v^{-1} C \Sigma_e + + G\Sigma_w G^T
\end{aligned}$$

where d_s and d_m represent partial derivatives of $g(s,m,p)$ with respect to s and m evaluated at the point s^*, m^* and $\bar{p}$. Additionally, d_c is a vector indicating the annualized cost associated with each selecting sensor. This annualized cost includes purchase, installation, maintenance and replacement costs (replacements being at periods equal to the average lifespan of the sensor).

Theorem 1 $\exists$ stabilizing L, $\Sigma_x \ge 0$, $\Sigma_e \ge 0$ and ζ_i

$$\begin{aligned}
s.t.\quad & A\Sigma_x + \Sigma_x A^T + G\Sigma_w G^T + BL(\Sigma_x - \Sigma_e) + (\Sigma_x - \Sigma_e) L^T B^T = 0 \\
& A\Sigma_e + \Sigma_e A^T - \Sigma_e C^T \Sigma_v^{-1} C \Sigma_e + + G\Sigma_w G^T = 0 \\
& \zeta_i = \phi_i [(D_x + D_u L)(\Sigma_x - \Sigma_e)(D_x + D_u L)^T + D_x \Sigma_e D_x^T + D_w \Sigma_w D_w^T] \phi_i^T, \quad i = 1 \cdots n_z
\end{aligned}$$

and $\zeta_i < \tilde{z}_i^2, \quad i = 1 \cdots n_z$ if and only if $\exists\, Y, X > 0, W > 0,$ and ζ_i.

$$s.t.\quad (AX + BY) + (AX + BY)^T + G\Sigma_w G^T < 0$$

$$\begin{bmatrix} -WA - A^T W + C^T \Sigma_v^{-1} C & (*) \\ GW & \Sigma_w^{-1} \end{bmatrix} > 0 \qquad \begin{bmatrix} \zeta_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & (*) & (*) \\ (D_x X + D_u Y)^T \phi_i^T & X & (*) \\ D_w^T \phi_i^T & I & W \end{bmatrix} > 0 \quad i = 1 \cdots n_z,$$

and $\zeta_i < \tilde{z}_i^2, \quad i = 1 \cdots n_z$

The proof of this theorem is a simple extension of a theorem presented in [8]. Using this theorem we can exactly convert a portion of the nonlinear constraints of problem 2 (namely the last 3) into a Linear Matrix Inequality (LMI) form. (For details concerning the computational aspects of LMI constrained problems please see [14] or [15]). The end result is a linear objective problem possessing a set of convex constraints along with a set of reverse-convex constraints. Details concerning a branch and bound, globally optimal search scheme for this class of problems can be found in [13].

3. Example

Consider the reactor system of figure 4 (details about this system can be found in [16]). We start by linearizing the nonlinear dynamic model around the nominal operating point. The resulting linear model $\dot{s} = As + Bm + Gp$ has 5 states and 5 inputs (3 manipulated and 2 disturbance) $s = [C_A \ T \ T_c \ V \ P]^T$, $m = [F_c \ F \ F_{vg}]^T$ and $p = [F_i \ C_{Ai}]^T$. From this model we determine the set of possible steady-state operating points defined by the equalities $0 = As + Bm + g_1 \overline{p}_1$. (In this steady-state phase we allowed the nominal value of the inlet flow rate (denoted $\overline{p}_1$) to be selected by the economic optimizer. This was not required, but it did make the example more interesting. When we get to the dynamic phase, the actual inlet flow rate will vary stochastically around this selected point.) Combining the above equalities with a set of upper and lower inequality bounds on each variable (see the dashed line of figures 4 - 7) we fined the set of feasible steady-state operating points. Next we define a profit function:

$$g(C_A, F_c, F, F_{vg}) = M_{an}[\alpha_1 (C_{Ai} - C_A)F - \alpha_2 F_c - \alpha_3 F_{vg}]$$

where α_1=\$0.375/mole B, α_2=\$ 0.015/ft^3 of cooling water, α_3=\$ 0.00225/ ft^3 of vapor pumped and M_{an}=8760 hr/yr. The profit function is then linearized around the nominal operating point to yield $g \cong g(\overline{C}_A, \overline{F}_c, \overline{F}, \overline{F}_{vg}) + c_g{}^T [s^T \ m^T \ \overline{p}_1]^T$, where $c_g{}^T$ is the partial derivative of the profit function evaluated at the nominal conditions. This along with the set of feasible steady-state operating points is used to define a steady-state optimizing LP (which maximizes profit or minimizes negative profit). The solution to this LP (the OSSOP) is indicated by the * points in figures 4 - 7 and has a profit g^* = \$47,370/yr. This solution represents the amount of profit one would yield if zero cost, perfect sensors were available and no disturbances acted on the system.

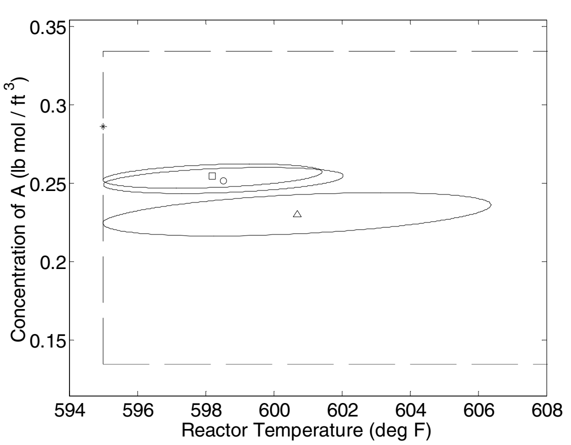

Figure 4: C_A vs. T

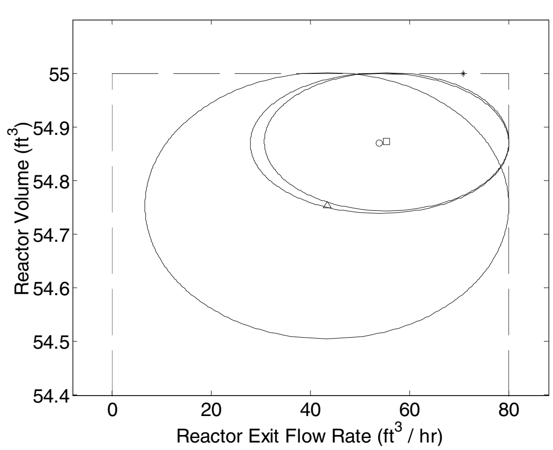

Figure 5: V vs. F

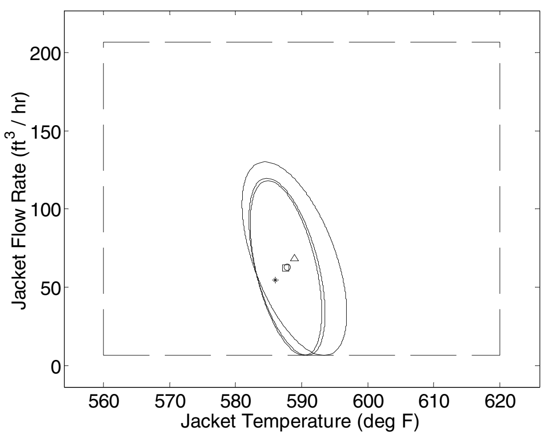

Figure 6: F_c vs. T_c

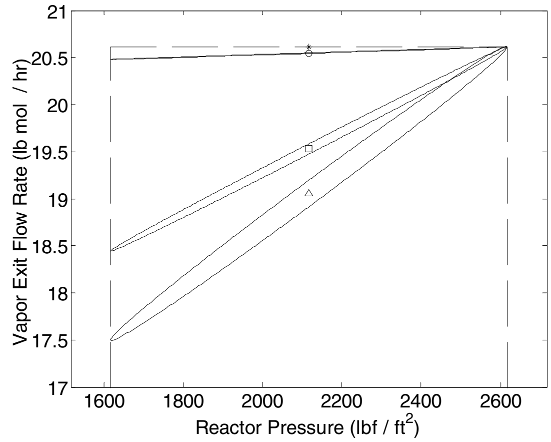

Figure 7: F_{vg} vs. P

Table 1: Profits and Values of Upgrade Configurations

No	New Sensors	Profit ($/yr)	Value ($/yr)	Sensor Costs ($/yr)	Value - Sensor Costs ($/yr)
1	C_A, P	36,030	7,060	2,000	5,060
2	C_A, T_c, P	36,600	7,630	3,000	4,630
3	C_A, T, P	36,590	7,610	3,000	4,620
4	C_A, V, P	36,060	7,090	3,000	4,080
5	C_A	33,840	4,870	1,000	3,870
6	P	33,470	4,500	1,000	3,500
7	T, P	34,390	5,420	2,000	3,420
8	C_A, T, T_c, V, P	37,060	8,090	5,000	3,080
9	T, T_c, V, P	35,120	6,150	4,000	2,140
10	none	28,970	0	0	0

Returning to the system dynamics, we start by assuming the disturbance inputs F_i and C_{Ai} will have standard deviation values equal to 0.1 and 0.01, respectively. Next, we assume that the existing sensor network consists of 4 sensors, at C_A, T, V and P, each with a precision of 2%. If we then solve the MBOP problem using this existing network, we find the expected profit to be $28,970/yr. The MBOP and EDOR resulting from the existing sensor network is indicated by the triangle points of figures 4 - 7). Next we assume that new 1% precision sensors are available at each state (i.e., at C_A, T, T_c, V and P). However, if a new sensor is placed then the old one must be removed. The placement of a new sensor will have an annualized cost of $1000/yr, and there will be no annualized cost due to leaving an old sensor in place. If we now place new sensor at all 5 locations and apply the MBOP selection method, we find that the profit will increase to $37,060/yr, due to our ability to move of the steady-state operating point closer to the OSSOP (this solution is indicated by the square points of figures 4 - 7). If we then subtract this amount from the profit of the existing network, we find that the value of the 5 sensor configuration is $8,090/yr, and the increase in profit (value minus sensor cost) is $3,080/yr. Application of a branch and bound, global search scheme to our simultaneous sensor and MBOP selection problem indicates that replacement of sensors at $C_{A},$ and P will yield greatest increase in profit (see the circle points of figures 4 - 7). The profit/value figures for this and other configurations can be found in table 1.

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