IMPROVED TECHNIQUE OF MULTI-STAGE COMPENSATION

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Considering marginal control systems, a useful technique, contributing to the method of multi-stage compensation is suggested. A multi-stage compensator, designed according to a number of developed rules, is connected in series with the original control system. It eliminates some properly selected dominant poles of the system's closed-loop transfer function. At the same time it introduces a specifically designed amplification and new dominant poles. This improves the quality of the system's performance in terms of its transient response, stability and accuracy. Rise and settling times, percent maximum overshoot, damping ratio, phase margin and steady-state error are optimised in the process.
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Keywords: system, compensation, marginal, stability, dominant, poles, transient, responses, accuracy, rules.

1. INTRODUCTION

A useful technique is proposed to achieve a high quality performance of n-order process control systems. The principle of the technique emphasizes on development of a number of rules for design of proper compensation. A multi-stage compensator, connected in series with the original control system, eliminates some properly selected dominant poles of the system's high order closed-loop transfer function. At the same time it introduces a specifically designed amplification and new dominant poles. This improves the quality of the system's performance in terms of its transient response, stability and accuracy. Quantities as rise time, percentage overshoot, settling time, damping ratio, phase margin and steady-state error are considerably optimised. The stability of the system is also improved.

The suggested technique is designed for marginal control systems subjected to step input signals. Its application can be easily extended for systems different from marginal and can be used successfully for second, third or higher order process control systems of type 0 and 1. The transfer function of the process is usually determined through testing, physical modelling and approximations. Then the compensation technique is applied, considering that the transfer function is presented as:

$$G_p(s) = \frac{K\prod_{i=1}^{k}(s - z_i)}{s^n \prod_{j=1}^{m}(s - p_j)} \qquad (1)$$

where
- $G_p(s)$ is the *plant transfer function*
- K is the steady-state *gain* of the system
- s is the *Laplace operator*
- k is the number of zeros
- n defines the *type* of the system
- m number of *poles* different from 0
- $n+m$ defines the *order* of the system
- z_i are the *zeros* of the system
- p_j are the *poles* of the system

Poles that are close to the imaginary axis in the left-half s-plane are dominant and are used to design the dynamic performance of a system. The insignificant poles should ensure that the applied compensator transfer function could be realized by physical components. In practice, as suggested by Kuo (1991) and later expanded by Driels (1996), the magnitude of the real part of an insignificant pole is considered at least 10 times larger than that of a dominant pole.

This effect is going to be used in the suggested multi-stage compensation technique.

To meet the ITAE criterion, suggested by Draper (1951), the following system objectives are set:

Damping ratio $\zeta = 0.707$ (2)

Percent maximum overshoot (PMO) $\leq 4\%$ (3)

Settling / to max overshoot time $t_s / t_m \leq 1.4$ (4)

Steady-State error $e_{ss} \leq 1\%$ (type 0 systems) (5)

These objectives will be used for establishing the rules and applying the suggested method.

2.CHARACTER AND PRACTICAL RULES OF THE COMPENSATION TECHNIQUE

2.1 Developed rules of the compensation technique.

The rules of the compensation technique are developed with the aid of the "CODAS" software package. Each one of them is established by optimising the system's specifications using "CODAS" tracking procedures. The method is based on cascade compensation and a unity feedback.

According to the suggested rules the compensator may consist of a multi-stage lead section and/or a lag section, depending on the system type. Additional attenuation and amplification, that is part of the compensator, with factors provided by the rules, should be also applied to bring the system to the desired performance.

The purpose of the rules is to set a design procedure of a compensator that eliminates some properly selected dominant poles, introduces new dominant poles and applies proper amplification.

<u>Rule 1</u>

To optimise ζ, t_s / t_m and the PMO of a type 0 marginal closed-loop system, a cascade multi-stage lead compensation with factors of $\alpha_{1,2,3,...} = 10$ should be applied for a zero-pole cancellation.

The number of the compensating stages N should be one less than the order of the open-loop system, i.e. $N = n + m - 1$.

The most dominant pole of the open-loop system should be left uncompensated.

The current gain should be maintained by an attenuation equal to the product $\alpha_1\alpha_2\alpha_3\ldots$

<u>Rule 2</u>

To optimise a type 0 marginal closed-loop system, a single-stage lag compensation with factor $\beta = 10$ should be applied for a zero-pole cancellation of the uncompensated most dominant pole of the open-loop system.

To optimise the steady-state error e_{ss} the current gain should be increased $\gamma = 10$ times.

<u>Rule 3</u>

If the most significant pole of the open-loop system has a real part close in value to that of an insignificant pole, following Rule 1, Rule 2 is modified. Then a successive two-stage lag compensation with factors $\beta_1 = \beta_2 = 10$ and a factor $\delta = 80$ should be applied.

<u>Rule 4</u>

To optimise ζ, t_s / t_m and the PMO of a type 1 marginal closed-loop system, a cascade multi-stage lead compensation with factors of $\alpha_{1,2,3,\ldots}=10$ should be applied for a zero-pole cancellation.

The number of the compensating stages N should be one less than the order of the open-loop system, i.e. $N = n + m - 1$.

The pure integration, or the most significant pole of the open-loop system should be left uncompensated.

The current gain should be maintained by an attenuation equal to the product $\varepsilon\alpha_1\alpha_2\alpha_3\ldots$, where $\varepsilon = (0.1 \text{ to } 1.27)$.

2.2 Rules design and applications of the suggested compensation technique

By testing different third order transfer functions the applicability of the suggested technique is proved in practice. It can be easily extended to any higher order system.

<u>Case 1.Application of Rules 1 and 2 (Type 0 System)</u>

The suggested application is for a plant with a transfer function of type 0 given in its Bode form:

$$G_P(s) = \frac{70}{(1+0.02s)(1+0.05s)(1+s)} \qquad (6)$$

There are two important steps, establishing the rules of the method. By "CODAS" tracking procedures on the transient response, first, the values of the factors α_1 and α_2 are varied and second, the value of the factor γ is varied, in this way searching for the optimum performance of the compensated system. The results are shown in Figure 1 and Figure 2.

From Figure 1, it can be seen that the set of objectives described in Equations (2), (3) and (4) can be met if $\alpha_1 = \alpha_2 = 10$. The factor $\beta = 10$ is chosen as a realistic figure for the physical realization of the lag compensation stage.

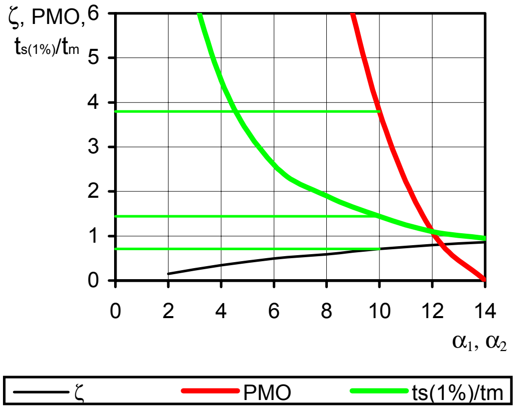

Fig. 1. Results of the tracking procedure for determination of optimum values of α_1 and α_2

Figure 2 shows that the set of the objectives described in Equations (2), (3) and (4) can be met if $\gamma = 10$. In this case the steady-state error is measured as $e_{ss} = 0.14\%$, which satisfies also Equation (5).

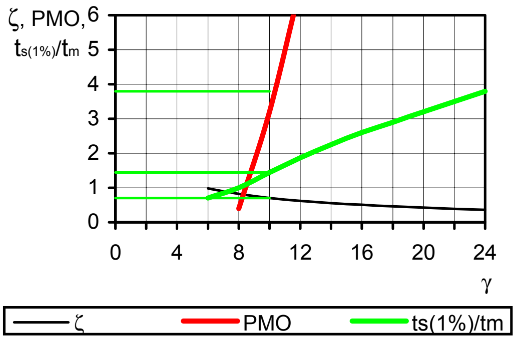

Fig. 2. Results of the tracking procedure for determination of optimum value of γ

Since the plant transfer function from Equation (6) is of a third order, two-stage lead plus one-stage lag compensation is applied. The two less significant poles in Equation (6) are $p_1 = -1/0.02 = -50$ and $p_2 = -1/0.05 = -20$. Then, according to Rule 1, the multi-stage lead section of the compensator should have a transfer function

$$G_c^{'}(s) = \frac{(1+\alpha_1 T_1 s)(1+\alpha_2 T_2 s)}{\alpha_1 \alpha_2 (1+T_1 s)(1+T_2 s)} \qquad (7)$$

or $$G_c^{'}(s) = \frac{(1+0.02s)(1+0.05s)}{100(1+0.002s)(1+0.005)} \qquad (8)$$

Following Rule 1, additional attenuation should be also applied

$$G_c^{''}(s) = \alpha_1 \alpha_2 = 10 \times 10 = 100 \qquad (9)$$

The most significant pole in Equation (6) is $p_3 = -1$. Applying Rule 2, the section of the lag compensation and amplification is presented by

$$G_c^{'''}(s) = \frac{\gamma(1+T_3 s)}{(1+\beta T_3 s)} = \frac{10(1+s)}{(1+10s)} \qquad (10)$$

Now the transfer function of the full compensator is

$$G_c(s) = G_c^{'}(s) \times G_c^{''}(s) \times G_c^{'''}(s) \qquad (11)$$

Finally, after applying the full compensation, the transfer function of the open-loop system becomes

$$G(s) = G_c(s) \times G_p(s) \qquad (12)$$

or $$G(s) = \frac{700}{(1+0.002s)(1+0.005s)(1+10s)} \qquad (13)$$

The Bode diagrams in Figure 3 and Figure 4 show the sequence of the method steps.

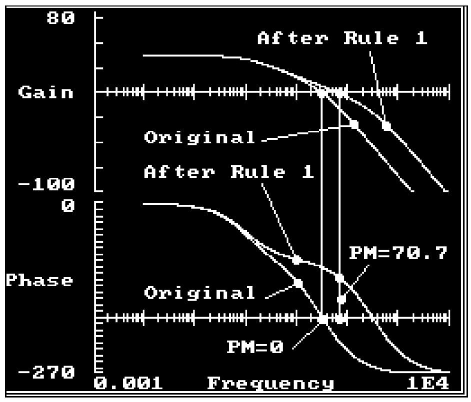

Fig. 3. <u>Case 1.</u> Bode diagrams of the original system and of the system after applying Rule 1

The system's original phase margin is PM = $\Delta\varphi = 0$ and the system is considered unstable. By introducing the lead compensation and attenuation (Rule 1), the phase margin becomes PM = $\Delta\varphi = 70.7$ and the damping ratio is $\zeta = 0.707$. The performance and the stability of the closed-loop system are improved.

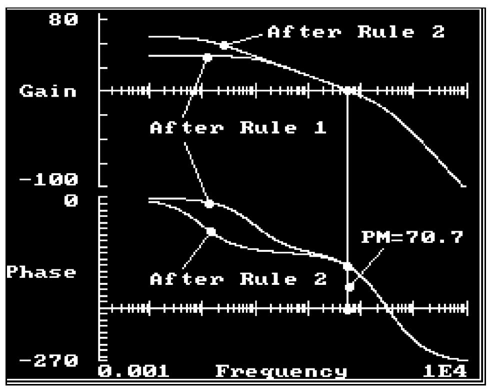

Fig. 4. <u>Case 1.</u> Bode diagrams of the system after applying Rule 1 and Rule 2

The main contribution of the lag compensation and amplification (Rule 2) is eliminating the influence of the most significant pole and reduction of the steady-state error of the closed-loop system.

The system transient responses before and after the full compensation are shown in Figure 5.

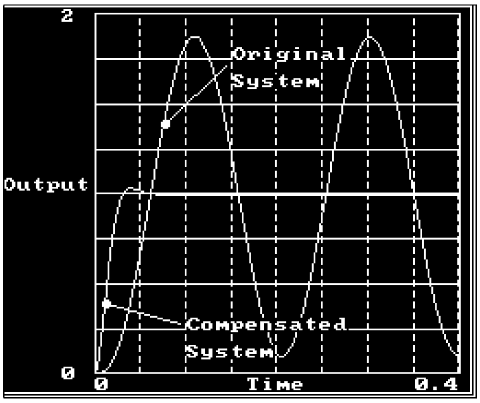

Fig. 5. Case 1. Transient responses of the original and the fully compensated closed-loop system

The real specifications of the compensated system, determined by "CODAS" from the transient response shown in Figure 5, are compared with the objectives for optimal performance, as defined in Equations (2), (3), (4) and (5).

From the summary in Table 1, it is seen that the real transient response in terms of damping ratio, percent maximum overshoot, time ratio and steady-state error is either matching or is better than the one of the set objective.

Table 1. Objectives & real results for Case 1

Specifications	Objectives	Real Results	Consideration
ζ	= 0.707	= 0.707	Matching
PMO	≤ 4%	= 3.8%	Better
$t_{s(1\%)}/t_m$	≤ 1.49	= 1.44	Better
$e_{ss}(t)$	< 1%	= 0.14%	Better

Case 2. Application of Rules 1 and 3 (Type 0 System)

Rule 3 can be illustrated for a type 0 marginal control system with a transfer function as shown:

$$G_P(s) = \frac{20}{(1+0.02s)(1+0.01s)(1+0.1s)} \qquad (14)$$

In this case the real part of the most significant pole, $p_3 = -10$, is only 5 times smaller than the real part of the one of the insignificant pole $p_1 = -50$. This implies application of Rule 3.

First, according to Rule 1, two-stage lead compensation and attenuation with factors $\alpha_1 = \alpha_2 = 10$ is applied. Then, following Rule 3, a two-stage lag compensation with factors $\beta_1 = \beta_2 = 10$ and a factor amplification $\delta = 80$ is suggested.

The values of the factors β_1 and β_2 are chosen by the same considerations as in Rule 2. Using "CODAS" tracking procedures on the transient response, the value of the factor δ is varied, searching for the optimum performance of the compensated system.

It can be seen from Figure 6 that the set of objectives described in Equations (2), (3) and (4) can be met if $\delta = 80$. In this case the steady-state error is measured to be $e_{ss} = 0.06\%$, also satisfying Equation (5).

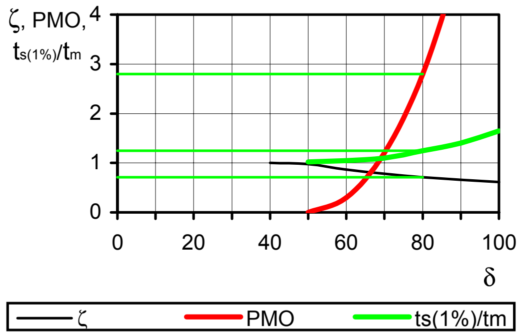

Fig. 6. Results of the tracking procedure for determination of optimum value of δ

Using a similar sequence as in Case 1, applying Equation (7), the two-stage lead stage is presented by

$$G_c^{'}(s) = \frac{(1+0.02s)(1+0.01s)}{100(1+0.002s)(1+0.001)} \qquad (15)$$

The attenuation $G_c^{''}(s)$ is the same as in Equation (9). Then applying Rule 3, the two-stage lag compensation and amplification is presented by

$$G_c^{'''}(s) = \frac{\delta(1+T_3 s)(1+T_4 s)}{(1+\beta_1 T_3 s)(1+\beta_2 T_4 s)} \qquad (16)$$

or

$$G_c^{'''}(s) = \frac{80(1+0.1s)(1+s)}{(1+s)(1+10s)} \qquad (17)$$

Now, applying the full compensation, considering Equations (11) and (12), the transfer function of the open-loop system becomes

$$G(s) = \frac{1600}{(1+0.002s)(1+0.001s)(1+10s)} \qquad (18)$$

The effect of the application of the method can be seen from the Bode diagrams shown in Figure 7. The phase margin of the system and hence the damping ratio and stability are improved considerably.

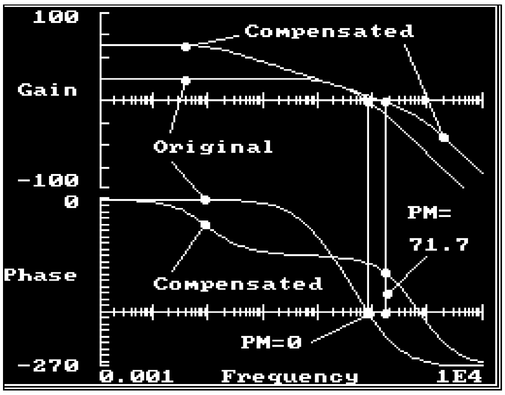

Fig. 7. Case 2. Bode diagrams of the system before and after applying the full compensation

The closed-loop system transient responses before and after full compensation are shown in Figure 8.

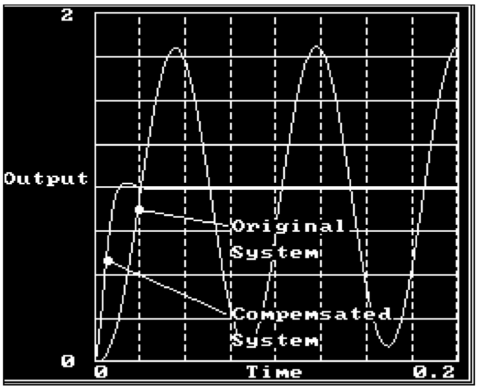

Fig. 8. Case 2. Transient responses of the closed-loop system before and after the full compensation

From the summary in Table 2, it is seen that the objectives are met. The real results for the compensated control system are either close or better than the set specifications.

Table 2. Objectives & Real Results for Case 2

Specifications	Objectives	Real Results	Consideration
ζ	= 0.707	= 0.717	Close
PMO	≤ 4%	= 2.8%	Better
$t_{s(1\%)}/t_m$	≤ 1.49	= 1.25	Better
$e_{ss}(t)$	< 1%	= 0.06%	Better

Case 3. Application of Rule 4 (Type 1 System)

The suggested application is for a plant with a marginal transfer function of type 1, presented in its Bode form

$$G_P(s) = \frac{70}{s(1+0.02s)(1+0.05s)} \qquad (19)$$

Applying Rule 4, the values of the factors α_1 and α_2 are determined similarly as in Rule 1. By introducing an additional adjustment factor ε, the real attenuation becomes $\varepsilon\alpha_1\alpha_2$. Using a "CODAS" tracking procedure ε is determined for different marginal control systems of type 1. To keep $\zeta = 0.707$, when the ratio of the less significant to the most significant pole of the plant transfer function, $r = p_1/p_2$, varies from 50 to 1, the value of ε may vary from 0.1 to 1.27, as shown in Figure 9. If the ratio is $r = 2.5$, as in the case of Equation (19), then $\varepsilon = 1$. If $r < 2.5$, the attenuation should be adjusted within the limits $\varepsilon = (1$ to $1.27)$. If $r > 2.5$, then the adjustment should be within $\varepsilon = (0.1$ to $1)$.

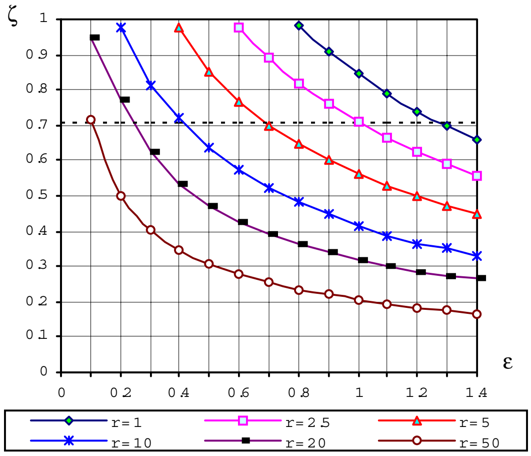

Fig. 9. Relationship between the damping ratio ζ and the factor ε for different poles ratios $r = p_1/p_2$

In this case, first, a two-stage lead compensation and attenuation is used. The two poles of Equation (19), to be cancelled, are $p_1 = -50$ and $p_2 = -20$. The lead compensation and attenuation employ transfer functions like those shown in Equations (7), (9). The factors used are $\alpha_1 = \alpha_2 = 10$ and $\varepsilon = 1$. Then, after applying Equation (12), the transfer function of the open-loop system becomes

$$G(s) = \frac{70}{s(1+0.002s)(1+0.005s)} \qquad (20)$$

The open-loop system Bode diagrams, before and after applying the technique are shown in Figure 10.

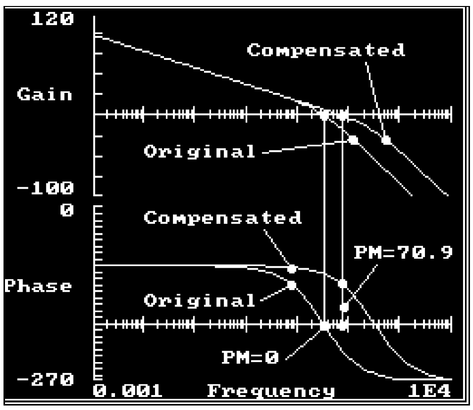

Fig. 10. Case 3. Bode diagrams of the system before and after applying the full compensation

The transient responses of the closed-loop system before and after applying the compensation are shown in Figure 11.

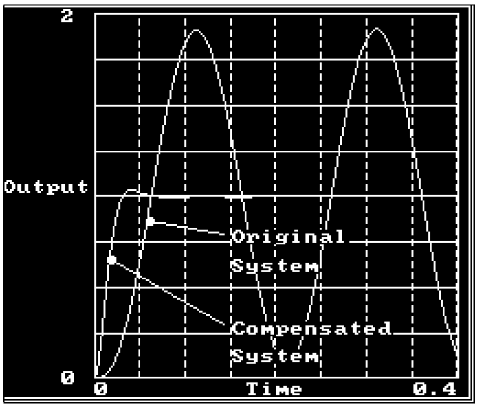

Fig. 11. Case 3. Transient responses of the closed-loop system before and after the full compensation

The real results of the compensated system are compared with the objectives for optimal performance and summarized in Table 3. It can be seen that again the objectives are met.

Table 3. Objectives & Real Results for Case3

Specifications	Objectives	Real Results	Consideration
ζ	= 0.707	= 0.709	Close
PMO	≤ 4%	= 3.3%	Better
$t_{s(1\%)}/t_m$	≤ 1.49	= 1.4	Better
$e_{ss}(t)$	=0%	= 0%	Matching

2.3 Application of the compensation technique with changed objectives.

Some process control systems may require a very fast response, compromising with the PMO and $t_{s(1\%)}/t_m$ values. In practice the damping ratio may be modified, to be within the range of $\zeta = 0.30$ to 0.45. Then all the rules of the proposed method will stand, but additional amplification by a factor ϕ = 2 is required. The value of ϕ is determined by a "CODAS" tracking procedure for different transfer functions. For example, the system described by Equation (20) has $\zeta = 0.707$. Increasing its gain two times secures a damping ratio ζ = 0.423 and faster response, which is illustrated in Figure 12.

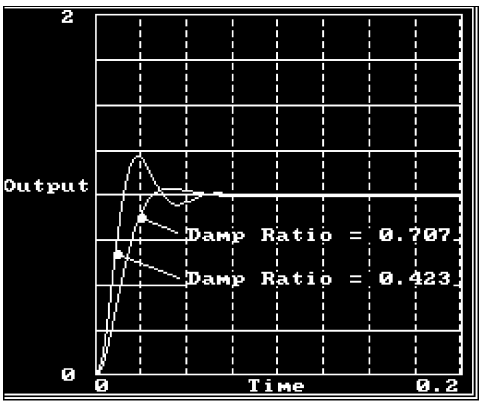

Fig. 12. Comparison between the original and a faster transient response

2.4 Application of the compensation technique for systems different from marginal.

The suggested technique can be also used for systems that are different from marginal. Any system can become marginal by preliminary tuning of its original steady-state gain *K*. For example, a system with a transfer function as shown in Equation (21), is not marginal, but its performance is unacceptable due to large oscillations. Tuning the original gain *K* =50 to *K'* = 70 and then applying Rule 1 and Rule 2 of the method, brings the system to the desirable performance as shown in Figure 13.

$$G_P(s) = \frac{50}{(1+0.02s)(1+0.05s)(1+s)} \qquad (21)$$

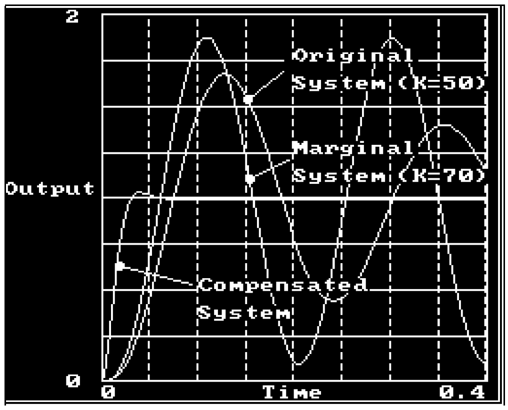

Fig. 13. Application of the compensation technique for systems different from marginal

3. CONCLUSIONS

Although the suggested method of multi-stage compensation is based on some known theoretical procedures, like the zero-pole cancellation, the lead and lag compensation, combining and analysing them, results in development of some new ideas. The originality of the suggested technique is based on the statement of a number of rules, which are applied in a predetermined sequence. The compensation equipment consists of three major parts. Its lead section eliminates all the insignificant poles of the plant's transfer function and introduces new properly designed poles. This improves the transient response specifications, especially the damping ratio of the system. The lag section eliminates the most significant pole of the plant's transfer function and along with the amplifying section improves further the transient response and reduces considerably the steady-state error.

4.REFERENCES

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