OPTIMAL BASE-STATIONS LOCATIONS IN THE LMDS WIRELESS DATA TRANSMISSION SYSTEM

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Abstract: The subject of this paper is the task of designing the LMDS (Local Multipoint Distribution System) wireless broad-band data transmission system. The apparatus of operational research and mathematical programming by using statistical kernel estimators and fuzzy logic is applied to find optimal locations of its base-stations. A procedure which allows to obtain such locations on the base of potential customer distribution and their expected demand, also in the cases of uncertain and non-stationary data, is presented here.

Keywords: wireless broad-band data transmission systems, LMDS, base-stations, operational research, mathematical programming, statistical kernel estimators, fuzzy logic.

1. INTRODUCTION

The LMDS (Local Multipoint Distribution System) is applied by telecommunication operators for wireless broad-band data transmission purposes. Such a system allows to connect the operator’s network node to the buildings in which his customers are located, without necessity to construct an expensive cable infrastructure. Thus, data are transmitted between base-stations distributed across the metropolitan area, and those stations serve regular connections with subscriber stations located within the effective coverage of transceivers belonging to base-stations. Subscriber stations installed on building roofs or facades then transmit data to subscribers located within the coverage area through local, e.g. cable, networks.

An essential factor which often decides about economic justification of the LMDS system implementation is to determine base-station locations so that the highest profit can be achieved, within the available investment funds. Presently, there is no methodology to allow for a general solution of the problem formulated in that way: in practice, heuristic methods are applied, largely based on intuition, or other methods from related fields are adopted; see (Rappaport, 1996; Laibo *et al.*, 2001), also to find rich bibliography. The task of LMDS base-station location planning is not easy due to the requirement to take into account a number of technical conditions, as well as economic ones, also in the situation of data uncertainty and non-stationarity.

Basic technical constrains include the coverage radius of base-station transceivers, as well their maximal bitrate, i.e. the largest total data quantity which can be transmitted in a time unit. What is also required for ensuring data transmission is the line of sight between the base-station and the subscriber-station antennas. For that reason, due to complex land shaping, or such obstacles as tall buildings on the base-station coverage area, there may exist shadow areas, on which it is not possible to transmit between a base-station and the buildings located in such areas. Thus, there is a limited number of sites being well visible due to their elevation, which can be selected as potential locations for base-stations.

In addition to the above-mentioned technical constrains, another problem facing planers is estimation of the future demand. Such estimations are developed on the basis of imprecise and incomplete data concerning potential service users located on a given area. Despite such uncertainty, estimations are indispensable to define a spatial distribution of the predicted demand. This problem gets even more difficult when planning is long-term, especially with non-stationarity of data.

Consequently, the planning task requires a choice of those possible base-station locations which ensure a maximal profit from services, while the number of stations is limited by the availability of funds. This paper will present an algorithm of designing optimal LMDS base-station locations. The method of statistical kernel estimators has been applied for the purpose of describing the spatial distribution of demand for data transmission services. Due to natural uncertainty of demand values, also fuzzy logic elements have been used. In addition, the issue of existence of shadow areas in the coverage areas of base-stations and the problem of their limited bitrates have been taken into account. It is also possible to apply that method with a several-year planning horizon.

This paper summarizes the material which will be published in a full version as paper (Kulczycki & Waglowski, 2004) soon. Complete software applying the algorithm presented here will also be made available.

2. ESTIMATION OF THE DISTRIBUTION OF SPATIAL DEMAND: STATISTICAL KERNEL ESTIMATORS

Let the *n*-dimensional random variable *X*, with a distribution having the density function *f*, be given. Its kernel estimator $\hat{f}:\mathbb{R}^n \to [0,\infty)$ is calculated on the basis of the *m*-element simple random sample $x_1, x_2, \dots, x_m$, acquired experimentally from the variable *X*, and is defined in its basic form by the formula

$$\hat{f}(x) = \frac{1}{mh^n} \sum_{i=1}^{m} K\left(\frac{x - x_i}{h}\right) \quad , \qquad (1)$$

where the function $K:\mathbb{R}^n \to [0,\infty)$, which is Borelian, radially symmetrical relative to zero, and has a weak global maximum at this point, fulfilling the condition $\int_{\mathbb{R}^n} K(x)\,\mathrm{d}x = 1$, and is called the kernel, whereas the positive coefficient *h* is known as the smoothing parameter. The form of the kernel *K* and the value of the smoothing parameter *h* is selected most often on the basis of the criterion of the minimum mean square error. It turns out that the form of the function *K* has no essential importance from the statistical point of view, and for that reason, it is possible when selecting this function to take into account primarily the properties of the estimator required in the case of a particular problem. Because of the convenience of analytical calculations, the 2-dimensional Cauchy kernel is applied in this paper:

$$K(x) = K([x_1, x_2]^{\mathrm{T}}) = \frac{1}{\pi(x_1{}^2 + x_2{}^2 + 1)^2} \quad . \qquad (2)$$

In particular tasks, additional procedures are used for improving the properties of kernel estimators. In the methodology investigated here, the so-called modification of the smoothing parameter is strongly preferred. For details of the above methodology, see (Kulczycki, 1998; Silverman, 1986; Wand & Jones, 1994). Exemplary applications of kernel estimators are presented in papers (Kulczycki, 2000, 2001, 2002a, b).

In the problem investigated here, the kernel estimator will be used for characterization of the distribution of spatial demand for data transmission services in the area under consideration. The variable *X* is therefore 2-dimensional, i.e. $n = 2$, while its particular coordinates represent longitude and latitude. The kernel estimator with the modification of the smoothing parameter realized by the introduction of the constants $s_i > 0$ for $i = 1,2,\dots,m$, will be applied after additional mapping of the coefficient $w_i > 0$ for $i = 1,2,\dots,m$ to every kernel; therefore,

$$\hat{f}(x) = \frac{1}{h^2 \sum_{i=1}^{m} w_i} \sum_{i=1}^{m} \frac{w_i}{s_i^2} K\left(\frac{x - x_i}{hs_i}\right) \quad . \qquad (3)$$

Finally, having a data base consisting of *m* potential locations of subscriber buildings, where each of them is characterized by its geographical position $x_i = [x_{i1}, x_{i2}]^{\mathrm{T}}$ and the coefficient w_i representing potential demand for data transmission services corresponding to location ($i = 1,2,\dots,m$), one can obtain from formula (3) the kernel estimator describing the density of distribution of the spatial demand for data transmission services on the whole area under consideration. This distribution is properly made continuous owing to the properties of statistical kernel estimators. Moreover, due to averaging aspects of such estimators, it is possible to use a simplified data base, including only the locations of main subscriber buildings, and taking into account in the corresponding coefficients w_i also smaller objects from their neighborhood. That action considerably simplifies the most difficult and expensive phase of the procedure of planning optimal locations of LMDS base-stations investigated in this paper.

3. BASE-STATION SYSTEM PERFORMANCE INDEX

In practice, it is not difficult to identify a limited number of sites for installing base-stations, including e.g. tall buildings and telecommunication towers. Having defined in the previous section the function $\hat{f}$ which characterizes the spatial distribution of demand for data transmission services, one can map for particular locations the values resulting from that function's integration, within the coverage areas of the respective transceivers. In the case of a base-station system, the integral for the whole area covered by the ranges of particular transceivers defines the total demand being also a criterion for the system's quality appraisal.

Let the set of k potential locations of base-stations at sites $x_j = [x_{j1}, x_{j2}]^{\mathrm{T}}$, with $j = 1,2,...,k$, be given. The following notations are introduced:

$$E_j = \int_{C_j} \hat{f}(x)\,\mathrm{d}x \tag{4}$$

$$E_{j_1,j_2,...,j_n} = \int_{C_{j_1} \cap C_{j_2} \cap ... \cap C_{j_n}} \hat{f}(x)\,\mathrm{d}x \quad , \tag{5}$$

where C_j denotes the j-th circle with the center at x_j and the positive radius r_j (representing maximal range of the transceiver mapped to the j-th location), and $j_1, j_2,..., j_n \in \{1,2,...,k\}$ are different, while $2 \le n \le k$. The total demand characterizing the quality of the base-station system, is given by the formula

$$E = \int_{C_1 \cup C_2 \cup ... \cup C_n} \hat{f}(x)\,\mathrm{d}x = \sum_{j=1}^{k} E_j - \sum_{\{j_1,j_2\}} E_{j_1,j_2} + + \sum_{\{j_1,j_2,j_3\}} E_{j_1,j_2,j_3} + \ ... \ + (-1)^k E_{1,2,...,k} \quad . \tag{6}$$

The text given below presents an algorithm calculating the values of formulas (4) and (5), which exhausts the procedure allowing to estimate the demand for data transmission services within the fixed base-station system, in accordance with formula (6), which characterizes the system quality.

Due to the selection of a kernel in the form (2), it is possible to calculate an analytical formula for the integral from the function of the single kernel K_i with the parameters h, s_i and w_i, on the circle C_j, with the radius r_j and the distance d_{ij} between the centers of the circle and the kernel (for $i = 1,2,...,m$ and $j = 1,2,...,k$), expressed by

$$E_j = \frac{1}{2\sum_{i=1}^{m} w_i} \sum_{i=1}^{m} w_i \left(\frac{r_j^2 - d_{ij}^2 - h^2 s_i^2}{\sqrt{r_j^4 + 2[h^2 s_i^2 - d_{ij}^2] r_j^2 + [h^2 s_i^2 + d_{ij}^2]^2}} + 1 \right) \quad . \tag{7}$$

For the purpose of the problem under consideration, the above formula may take on the form

$$E_j = \sum_{i=1}^{m} w_i \left(\frac{r_j^2 - d_{ij}^2 - h^2 s_i^2}{\sqrt{r_j^4 + 2[h^2 s_i^2 - d_{ij}^2] r_j^2 + [h^2 s_i^2 + d_{ij}^2]^2}} + 1 \right) \quad . \tag{8}$$

The values calculated on that basis will be subjected to comparison in the process of search for an optimal element, while multiplication of the performance index by a positive constant does not affect results obtained in such a problem.

Next, the analytical calculation of the integral value, in the case of intersectioning of any number of circles, is practically unexecutable. A practical approximate procedure will therefore be investigated below. First, the case of the kernel K_i and two circles C_{j_1} and C_{j_2} will be considered. Owing to a possibility of renumbering, it can be assumed, without reducing generality, that $r_{j_1} \le r_{j_2}$. Let D_{j_1,j_2} denote the distance between the centers of the circles. One of the following relationships may occur:

(A) $D_{j_1,j_2} \ge r_{j_1} + r_{j_2}$, implying disjunction of the circles or edge contact; then, $E_{j_1,j_2} = 0$;

(B) $D_{j_1,j_2} \ge r_{j_2} - r_{j_2}$, which means that the smaller circle is their intersection; then, $E_{j_1,j_2} = E_{j_2}$, which value may be calculated from formula (8);

(C) neither of the pervious cases occurs; the circle intersection has the shape of a lens; the method of calculating the approximate value of E_{j_1,j_2} is given below.

This method consists in replacing the lens with a circle, for which formula (8) can be applied. By guaranteeing equal fields of the circle and the lens and with proper location of the circle's center, the difference between the values of function (3) on the areas of the lens and of the circle is not large, while the error of integration (having the averaging nature) on them is fairly insignificant. It is worth noticing that the largest values of the error occur when the lens is considerably flattened, or when its field, together with the value E_{j_1,j_2}, is relatively small.

Let $\hat{D}_{j_1,j_2}$ mean the distance between the points of intersection of the circles C_{j_1} and C_{j_2}; according to the assumptions of case (C): $\hat{D}_{j_1,j_2} > 0$. The calculation of the value $\hat{D}_{j_1,j_2}$ is not difficult based on non-complex procedures of analytical geometry. The field of the lens L_{j_1,j_2} in defined, in the case of a flat lens, i.e. when $r_{j_2} \ge \sqrt{r_{j_1}^2 + D_{j_1,j_2}^2}$, by formula

$$L_{j_1,j_2} = \frac{\hat{D}_{j_1,j_2}}{2} \left(\sqrt{r_{j_1}^2 - \left(\frac{\hat{D}_{j_1,j_2}}{2} \right)^2} + \sqrt{r_{j_2}^2 - \left(\frac{\hat{D}_{j_1,j_2}}{2} \right)^2} \right) + r_{j_1}^2 \arcsin\left(\frac{\hat{D}_{j_1,j_2}}{2 r_{j_1}} \right) + r_{j_2}^2 \arcsin\left(\frac{\hat{D}_{j_1,j_2}}{2 r_{j_2}} \right) - \hat{D}_{j_1,j_2} D_{j_1,j_2} \quad , \tag{9}$$

however, in the case of a convex lens, i.e. when $r_{j_2} < \sqrt{r_{j_1}^2 + D_{j_1,j_2}^2}$, by formula

$$L_{j_1,j_2} = \frac{\hat{D}_{j_1,j_2}}{2} \left(\sqrt{r_{j_1}^2 - \left(\frac{\hat{D}_{j_1,j_2}}{2} \right)^2} + \sqrt{r_{j_2}^2 - \left(\frac{\hat{D}_{j_1,j_2}}{2} \right)^2} \right) + r_{j_1}^2 \arcsin\left(\frac{\hat{D}_{j_1,j_2}}{2 r_{j_1}} \right) - r_{j_2}^2 \arcsin\left(\frac{\hat{D}_{j_1,j_2}}{2 r_{j_2}} \right) + \hat{D}_{j_1,j_2} D_{j_1,j_2} \quad , \tag{10}$$

Upon calculation of the lens field L_{j_1,j_2} from formulas (9) or (10), one can easily calculate the radius of the substitute circle r_{j_1,j_2}:

$$r_{j_1,j_2} = \sqrt{\frac{L_{j_1,j_2}}{\pi}} \quad . \tag{11}$$

Its center is defined as follows. The straight line crossing the centers of the circles C_{j_1} and C_{j_2} is also crossing each of them at two different points, one on each lens edge. Let the center between those points be the center of the substitute circle. Calculation of its coordinates is not difficult using the analytical geometry methods. Once the center and the radius of the substitute circle are known, it is possible to calculate the value of E_{j_1,j_2} based on formula (8).

The above procedure may be easily generalized in the recurrent manner in the cases of intersection of any number of circles. Upon circle ordering in accordance with the increasing radius size, it is necessary to calculate the substitute circle parameters for the lens obtained from the first pair, followed by subsequent iterations for the substitute circle and subsequently considered ones, repeating such iterations until the list of circles is exhausted. The result is a substitute circle for the area being intersection part of all the circles under consideration. It is possible to apply formula (8) to the resulting circle.

The above completes a basic calculation algorithm necessary to apply formula (6), allowing to characterize the quality of the given base-station system. This algorithm can be easy modified to take into account shadow areas and limited bitrates of base-stations.

Thus, as it was mentioned in the introduction, within the area theoretically covered by transceivers occurs the shadow area in which transmission is impossible due to uneven land or obstacles, e.g. tall buildings. In practice, shadow area are often approxi-mated by simple geometric figures, while those figures are treated as circles, or, generally, circle unions. With this assumption, the algorithm developed above allows for easy calculation of the integral from the density function of the spatial distribution of the demand for data transmission services on shadow areas, in analogy to formula (6), followed by subtraction of that value from the index calculated previously.

The performance index of the particular base-station system, defined formula (6), represents the capability of meeting the total demand for teletransmission services provided within the system's transceiver coverage. However, on especially attractive city areas, the coverage demand may not be met due to limited transceiver bitrates. In the following, the procedure allowing to account for limited bitrates of particular base-stations will be presented.

Let $b_j > 0$ with $j = 1,2,...,k^*$, mean maximal bitrates of particular transceivers belonging to a subsystem of k^* base-stations with connected coverage area. The set, being the union of the areas within the base-stations' coverage, is divided by the circles constituting coverage edges of particular transceivers into a number of subsets with nonempty interior (the maximal possible number is $2^{k^*} - 1$). Those sets, denoted further as Z_i, will be numbered with the index $i = 1,2,...,I$. Using the algorithm presented previously, the approximate value of the integral $\int_{Z_i} \hat{f}(x)\,\mathrm{d}x$, for each $i = 1,2,...,I$, can be calculated.

Let now the matrix A with dimension $k^* \times I$ and nonnegative elements, be given. Particular rows of the matrix are connected with subsequent base-stations of the system under consideration, while columns are connected with particular subsets Z_i. If the *i*-th subset is outside of the range of the *j*-th station, one should assume that $a_{j,i} = 0$. The following performance index will be considered, and the decision variables will be all the elements of the matrix *A* whose value was not assumed above as zero (the respective set will be denoted below as $\{a^*_{j,i}\}$):

$$\max_{\{a^*_{j,i}\}} \sum_{\substack{j=1,2,...,k^* \\ i=1,2,...,I}} a_{j,i} \quad , \tag{12}$$

with the boundaries

$$a_{j,i} \geq 0 \quad \text{for} \quad j = 1,2,...,k^* \quad \text{and} \quad i = 1,2,...,I \tag{13}$$

$$\sum_{i=1}^{I} a_{j,i} \leq b_j \quad \text{for} \quad j = 1,2,...,k^* \tag{14}$$

$$\sum_{j=1}^{k^*} a_{j,i} \leq \int_{C_i} \hat{f}(x)\,\mathrm{d}x \quad \text{for} \quad i = 1,2,...,I \quad . \tag{15}$$

That is a typical linear optimization task. By arranging the elements of the set $\{a^*_{j,i}\}$ in a vector, it may be transformed to a canonical form and solved by the generally available simplex method. Each of the elements $a_{j,i}$, obtained in accordance with the above procedure, indicates which portion of the demand from the area Z_i should be served by the station *j* in order to meet the largest possible demand for telecommunication services for the given base-station system, taking into account limited bitrates of the respective transceivers.

4. SELECTION OF THE OPTIMAL BASE-STATION SYSTEM

Once the base-station system performance index has been worked out in accordance with previous section, one may start resolving the basic task of the present paper, i.e. the selection of the optimal base-station system. For that purpose, the methods origin from operational research will be applied.

The utilization of radio frequencies made available to the telecommunication operator requires application of devices, with essentially different functional parameters. In the model presented here, a possibility of selection, in each potential location, of one possible version of transceivers, from among p options, while $p \in \mathbb{N}$, is assumed. Particular versions are represented with the following positive parameters: r_i – a coverage radius, b_i – a maximal bitrate, and c_i – the cost of equipment and its installation, where $i = 1,2,..., p$. The case when no equipment is installed at a location is reflected by $i = 0$ and $c_0 = 0$.

Let the k-dimensional decision vector

$$[g_1, g_2,..., g_k]^{\mathrm{T}} \tag{16}$$

be given. Particular coordinates represent potential base-station locations, and assume the values $g_j \in \{0,1,..., p\}$ for $j = 1,2,..., k$. To be more precise: if the j-th coordinate is 0, i.e. $g_j = 0$, it means that the transceiver installation at the j-th location is not planed; however, if that coordinate takes on the value i from the range 1,2,...,p, it means that the i-th version of such devices is installed at the j-th location. The optimization task consists here in searching for the maximum of the expression

$$\max_{g_1, g_2,..., g_k} E([g_1, g_2,..., g_k]^{\mathrm{T}}) \quad , \tag{17}$$

with the boundary

$$\sum_{j=1}^{k} c_{g_j} \le \tilde{C} \quad , \tag{18}$$

where the positive number $\tilde{C}$ means the maximal amount of available funds, and $E([g_1, g_2,..., g_k]^{\mathrm{T}})$ denotes the value of function (6) for a system of transceivers distributed in accordance with the value of the decision vector $[g_1, g_2,..., g_k]^{\mathrm{T}}$.

Let the k-level decision tree be given: particular levels represent subsequent potential base-station locations. Decision tree nodes are assigned subsequently one of possible values $g_j \in \{0,1,..., p\}$ for $j = 1,2,..., k$; if the j-th level is assigned the value g_j, the node represents the case in which the g_j-th version of a transceiver is installed at the j-th location. That also implies assigning to that node the cost c_{g_j} of a given version of a transceiver, which is necessary to verify boundary (18). The solution of the problem under consideration consists in the determination of a path from the first level node to the k-th level node, described by the vector $[g_1, g_2,..., g_k]^{\mathrm{T}}$, for which the function E reaches the maximum, and boundary (18) is fulfilled. To solve so formulated a task, a classical method of division and boundaries has been applied.

An important element affecting the rate of calculation is effective "closing" those nodes from which a better path then previously found cannot be generated. Let the numbering of particular transceiver versions be such that $c_0 \le c_1 \le ... \le c_p$, while the numbering of tree levels such that $\int_{C(x_1, r_{\max})} \hat{f}(x)\,\mathrm{d}x \ge \int_{C(x_2, r_{\max})} \hat{f}(x)\,\mathrm{d}x \ge ... \ge \int_{C(x_k, r_{\max})} \hat{f}(x)\,\mathrm{d}x$, i.e. according to the demand level met by a given location, for the transceiver version with the largest range $r_{\max}$. If the node under consideration is located in layer $j \in \{1,2,..., k-1\}$, it will not be difficult to calculate the number $J \in \mathbb{N}$ stating how many cheapest transceivers may be installed within available funds:

$$J = \mathrm{int}\left(\frac{\tilde{C} - \sum_{i=1}^{j} c_{g_i}}{c_{g_1}} \right) \quad , \tag{18}$$

where int(a) denotes the integer part of the number $a \in \mathbb{R}$. Let a fragment of the path above level j describe the partial decision vector $[g_1, g_2,..., g_j]^{\mathrm{T}}$. If the value E, characterizing according to formula (6) the base-station quality for the decision vector

$$[g_1, g_2,..., g_j, \underbrace{g_{r_{\max}},..., g_{r_{\max}}}_{J \text{ factors}}, 0,...,0]^{\mathrm{T}} \quad , \tag{19}$$

where $g_{r_{\max}}$ representing the version of transceivers with the largest coverage, is smaller then or equal to the maximum of previously calculated value E, then such a node should be closed because decision vectors of the form of $[g_1, g_2,..., g_j, \text{any}]^{\mathrm{T}}$ may not produce a better path then the one found.

5. LONG-TERM PLANNING HORIZON

The task previously considered was stationary in nature. However, one can expect increased transmission to current customers and inclusion of new customers with time. Also, a gradual increase of funds can be expected owing to current income and growing operator's creditworthiness. In addition, the parameters of transceivers are also changed. After signing new agreements and expanding urban infrastructure, new base-station locations will become available. The methodology presented in this paper allows to accounting easily for the time factor and, in particular, all the above-mentioned task aspects.

If the project is considered within $T \in \mathbb{N} \setminus \{0,1\}$ time sections (in practice such periods most often refer to particular years, with $T = 2$ or $T = 3$), decision vector (15) should be generalized to

$$[g_{1,t=1}, g_{2,t=1}, ..., g_{k_1,t=1}, \quad g_{1,t=2}, g_{2,t=2}, ..., g_{k_1,t=2}, \quad ... \quad , \quad g_{1,t=T}, g_{2,t=T}, ..., g_{k_1,t=T}]^{\mathrm{T}} \tag{21}$$

and boundary (18) assumes a form of T independent conditions

$$\sum_{j=1}^{k_t} c_{g_j,t} \le \tilde{C}_t \quad \text{for} \quad t = 1,2,...,T \quad , \tag{22}$$

where the parameter $t = 1,2,...,T$ characterizes particular time sections, and all the quantities using in optimization task are correspondingly indexed by t, characterizing conditions possibly different for particular time sections.

6. FUZZY NATURE OF DEMAND

The coefficients w_i for $i = 1,2,...,m$ introduced in formula (3) represent the demand for teletransmission services assigned to particular subscriber-station locations. Their value is estimated in practice by an expert opinion expressed verbally, often based on intuitional premises. Consequently, the description of the predicted demand for teletransmission services by a subscriber station will require fuzzy logic elements. What should also be taken into account is a specific nature of the task under consideration: a lot of fuzzy numbers (equal to the number of subscriber stations m) necessary to identify and to use in subsequent analysis. In that situation, especially suitable are the fuzzy numbers of the type *L-R* (Kacprzyk, 1986).

Thus, for each of m locations of subscriber buildings, the coefficient w_i representing a potential demand for data teletransmission services, introduced in formula (3), can be generalized to the three-parameter fuzzy number *L-R* suitable for identification and calculations $\mathcal{W}_i = (w_i, \alpha_i, \beta_i)$, where $w_i - \alpha_i \ge 0$ for every $i = 1,2,...,m$. In a special case, $\mathcal{W}_i = (w_i, 0, 0)$ may represent the real (nonfuzzy) number w_i. In this situation, the performance index of the base-station system under consideration (6) has a form of linear combination of three-parameter fuzzy numbers $\mathcal{W}_i$, and, therefore, it also becomes a three-parameter fuzzy number $\mathcal{E}$. To allow for comparison of qualities of particular base-station systems, the methodology of preference theory (Fodor & Roubens, 1994) will be applied. The preference function P of the fuzzy number $\mathcal{E}$, with the bounded support of the membership function, will be adopted in the form resulting from the decision-making practice

$$P(\mathcal{E}) = \delta \frac{\int_{\min \operatorname{supp} \mu_{\mathcal{E}}}^{\max \operatorname{supp} \mu_{\mathcal{E}}} x \mu_{\mathcal{E}}(x)\, \mathrm{d}x}{\int_{\min \operatorname{supp} \mu_{\mathcal{E}}}^{\max \operatorname{supp} \mu_{\mathcal{E}}} \mu_{\mathcal{E}}(x)\, \mathrm{d}x} + (1-\delta) \min \operatorname{supp} \mu_{\mathcal{E}} \quad , \tag{23}$$

where $\delta \in [0,1]$, $\mu_{\mathcal{E}}$ means the membership function of the fuzzy number $\mathcal{E}$, while $\operatorname{supp} \mu_{\mathcal{E}}$ denotes its support. The value of the membership function is therefore a linear combination with weights δ and $1-\delta$ of the average value of the fuzzy number and the minimum value of its support. The average number corresponds to the Bayes decision rule and expresses a "realistic" operation, while the minimum value of the membership function support results from the minimax rule and represents the "pessimistic" point of view. The parameter δ determines therefore the company's strategy in the range from realistic one (assuming average of the predicted demand) for $\delta = 1$, to pessimistic one (assuming the lowest level of the predicted demand) for $\delta = 0$. When clear preferences are missing, the value $\delta = 0{,}5$ can be proposed.

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