NON PLANT MODIFYING MULTILOOP QFT REVISITED

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Abstract: The Horowitz and Wang approach to non plant modifying multiloop (NPMM) QFT is revised, introducing as a design factor the high frequency template length of the outer loop. This is a key issue, since it determines the achievable noise reduction of the outer loops against the control effort of the inner loops and thus a balance between both factors is highly desirable.

Keywords: Computer-aided control systems design, Cut-off frequency, Multiloop control, Robust control

1. INTRODUCTION

In this work, the previous approach on NPMM QFT (Horowitz and Wang, 1979*a*; Horowitz and Wang, 1979*b*), is revisited. Attention is concentrated on the simplest multiloop plant structure which contains the essential features of those works (figure 1) in which there is only one

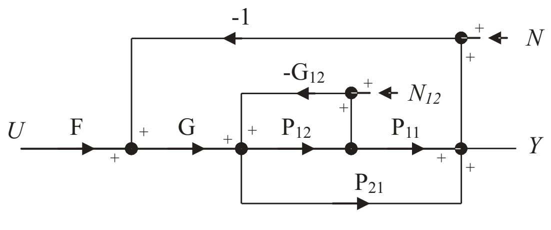

Fig. 1. Plant structure.

internal sensor (and controller associated to it) additional to the usual at the plant output, and there are two parallel branches. Also following (Horowitz and Wang, 1979*a*; Horowitz and Wang, 1979*b*) plants structure is restricted to $P_{ij} = \frac{k_{ij}}{s^{n_{ij}}}$, $k_{ij} \in [k_{ij_n}, k_{ij_x}]$, and the study is restricted to the case in which the poles-zeros excess is the same in both parallel branches, i.e., $n_{21} = n_{11} + n_{12}$. The second restriction has to do with the future extension of the study to more general plant structures, in which the trade-off between inner branches will be also considered: different poles-zeros excess allows no balancing between branches. For simplicity, $n_{21} = 2$ and $n_{11} = n_{12} = 1$ will be used.

In section 2, the basic principles of the design methodology proposed by Horowitz and Wang are summarized and applied to a simple example. Then, it is addressed an important choice of this design methodology, the length of the (vertical) plant template considered for the design of outer the loop, which determines the noise reduction

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achieved. In the original methodology, it is assumed there is only one possibility. In sections 3 and 4, it is shown how this choice is in fact a design factor that the control engineer can use to trade-off between outer loop noise reduction and inner loop control effort, and the rules governing this trade-off are studied. To do so, a simplified formulation of the problem is introduced.

Finally, in section 5, it is presented a MATLAB® based CACSD tool which lets the control engineer interactively choose a certain template reduction or specify an outer loop and check the effect on noise reduction and inner loop control effort.

2. HW DESIGN METHODOLOGY

The design methodology used in (Horowitz and Wang, 1979*a*; Horowitz and Wang, 1979*b*), *Horowitz-Wang design methodology*, *HW* for short, is the evolution of the ideas in (Horowitz, 1993; Horowitz and Sidi, 1973) for the cascaded case. Along this text, subscript 0 denotes *nominal* and, for every plant, the nominal will be chosen at the bottom of the (vertical) template, i.e., $\forall ij$, $k_{ij_0} = k_{ij_n}$. The basic idea in both cases is that an inner loop $L_{12_0} = G_{12}P_{12_0}$ can be used to help an outer loop, $L_0 = G\frac{P_0}{1+L_{12_0}}$, $L_0 = GP_0$ for $L_{12_0} = 0$, with $P_0 = P_{21_0} + P_{11_0}P_{12_0}$, to cope with the uncertainty of inner plants (P_{12}). The way to do this is the following. First it is designed an outer loop which only deals with P_{21} and P_{11} uncertainty, i.e., a fixed value of P_{12} is assumed. Then it is chosen an inner loop L_{12_0} which keeps specifications on L_0 satisfied despite the effect of P_{12} uncertainty.

P_{12} fixed value, $P_{12_F} = \frac{k_{12_F}}{s}$, is chosen in such a way the (vertical) template length of $P' = P_{21} + P_{11}P_{12_F}$, τ_F, is maximum. There are two possibilities:

- *case A*: $\frac{k_{21x}}{k_{21n}} \geq \frac{k_{11x}}{k_{11n}}$; for which $k_{12_F} = k_{12n}$
- *case B*: $\frac{k_{21x}}{k_{21n}} \leq \frac{k_{11x}}{k_{11n}}$; for which $k_{12_F} = k_{12x}$

HW assume minimum phase P. It is also assumed that P uncertainty is such that the control problem is solvable with $G_{12} = 0$, i.e., the outer loop is able to cope with P uncertainty by itself, with no help from the inner loop.

The basic benefit obtained from the inner loop help is the sensor noise effect reduction: the smaller template of P', compared to P, leads to lower height of the universal high frequency boundary (UHFB), which lets the outer loop reach its cut-off frequency, ω_{cu}, at lower frequencies. This effect will be shown by means of a simple example: $k_{ij} \in [10, 100]$, $ij = 11, 12, 21$. The specifications are a $\gamma = 2.3$ dB stability margin and a tracking specification for $\omega \leq 10$ rad/s given by

$$\left|\frac{0.9999}{0.26s^2 + 1.1s + 1}\right| \leq |FT| \leq \left|\frac{1.0001}{0.4s + 1}\right|$$

where $T = \frac{L}{1+L}$. In figure 2 it can be seen a

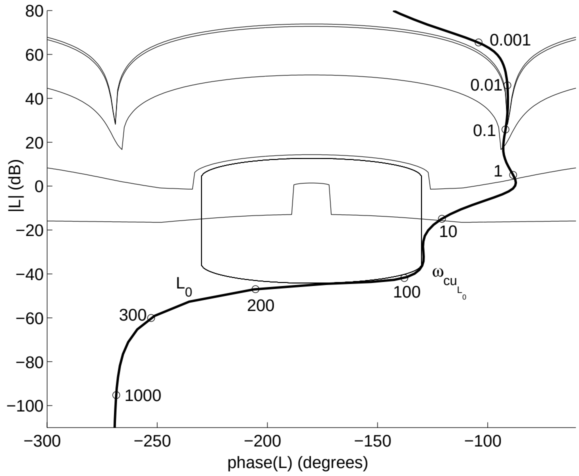

Fig. 2. Loop L_0 and its boundaries.

design of the outer loop coping with the whole uncertainty, L_0, and corresponding boundaries. In figure 3 P_{12} has been fixed and a new outer loop, L_0', designed. The UHFB reduces its length and,

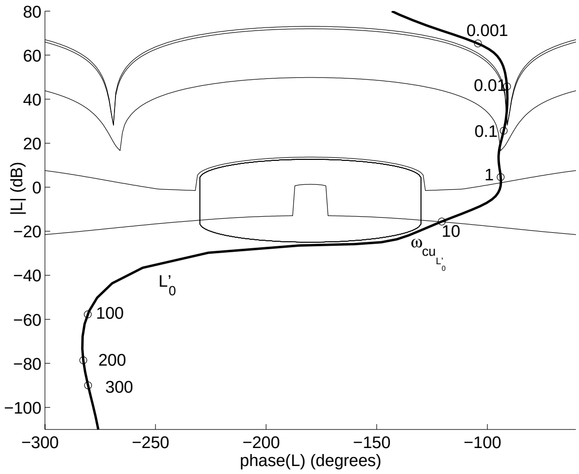

Fig. 3. Loop L_0' and its boundaries.

consequently, ω_{cu} for L_0' reduces. Since at high frequency the transfer function from the sensor noise to the plant input, T_N, is given by $T_{N_{L_0}} \approx \frac{L_0}{P_0}$, this ω_{cu} reduction also means a reduction in the bandwidth of sensor noise amplification, $T_{N_{L_0}}$ vs. $T_{N_{L_0'}}$, see figure 4. To get this improvement, an inner loop, L_{12_0}, has to be designed in such a way the design of L_0' is still valid even in the presence of P_{12} uncertainty. This design is done in terms of the satisfaction of certain boundaries relating L_{12_0} with L_0' specifications. The addition of such an inner loop introduces a new noise transfer function to be considered, $T_{N_{L_{12_0}}} = \frac{L_{12_0}}{1+L_{12_0}}\frac{1}{P_{12_0}(1+L_0')}$, transfer function from N_{12} to the plant input, see figure 4. P_{12_0} and L'_0 are fixed, so $T_{N_{L_{12_0}}}$ can be influenced only by the choice of L_{12_0}, the smaller L_{12_0} and its cut-off frequency, the better in terms of $T_{N_{L_{12_0}}}$ noise reduction.

3. NEW APPROACH AND SIMPLIFIED MODEL

The hardness of the boundaries on L_{12_0} to respect L'_0 specifications depends on how much uncertainty is covered by L_{12_0} and, thus, on how much L'_0 has been released from. In HW method, this amount is fixed (cases A and B). In this section this uncertainty reduction is parameterized and the dependence of L_{12_0} boundaries on it studied by connecting L_0 sensor noise reduction and L_{12_0} control effort by means of template length (τ) reduction. Knowledge of the optimum [2] L'_0 for a given τ' is assumed. A simplified formulation of the problem is also defined, in which the UHFB is considered to be rectangular (the rectangle enveloping the real UHFB) and the loops are considered to be piecewise linear in the Nichols chart. In figure 5 optimum outer loops L_0 and L'_0 are represented according to this simplified model. Key frequencies for L_0 are denoted as ω_i, $i = a, b, \ldots$. Their counterpart in L'_0 are denoted as ω'_i when they are different. ρ is the gain margin [3]. Define $\beta_{dB} = \tau_{dB} - \tau'_{dB}$, with $\beta \in [0, \tau_{dB}]$. β_{dB} parameterizes τ reduction.

This model connects τ with the magnitude of the optimum loop fitting the specifications, specifically

[2] Horowitz *optimum* definition (Horowitz, 1993) is used: the optimum loop has minimum high frequency gain and lyes on the boundaries at every frequency.

[3] dB expressed version of a variable is denoted with subscript dB.

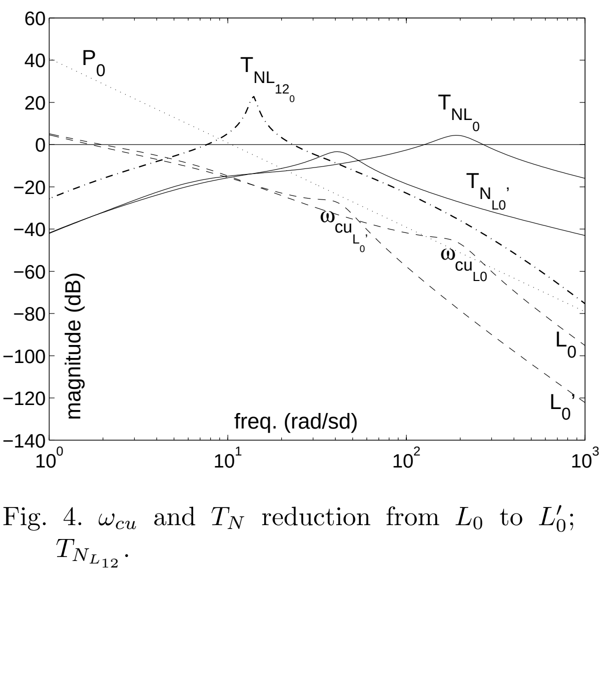

Fig. 4. ω_{cu} and T_N reduction from L_0 to L'_0; $T_{N_{L_{12}}}$.

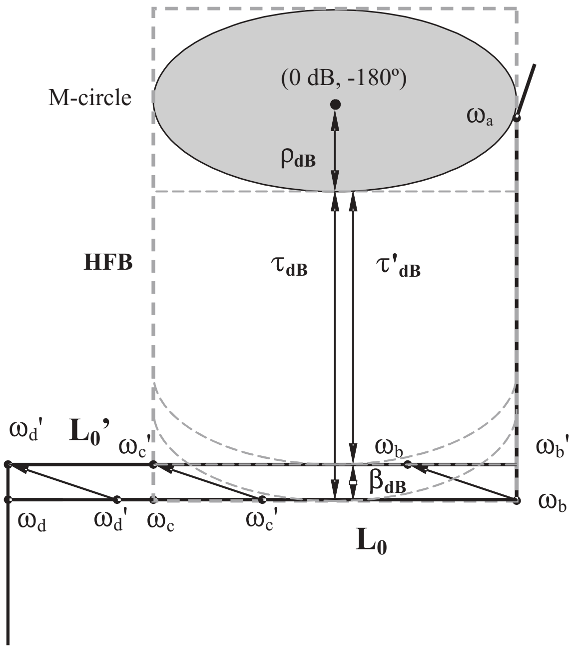

Fig. 5. Simplified model.

in the critical frequency rank in terms of noise reduction: in the interval $[\omega_b, \omega_d]$,

$$|L_0|_{dB} = -(\tau_{dB} + \rho_{dB}), \qquad |L_0| = (\tau\rho)^{-1} \quad (1)$$

and, for any β,

$$|L'_0|_{dB} = -(\tau_{dB} + \rho_{dB} - \beta_{dB}) = |L_0|_{dB} + \beta_{dB} \quad (2)$$

in a certain interval $[\omega'_b, \omega'_d]$ or, in linear terms,

$$|L'_0| = |L_0|\beta, \qquad \beta \in [1, \tau], \qquad \tau > 1 \quad (3)$$

$L'_0 = L_0$ (for $\beta = 1$) is able to handle the whole plant uncertainty by itself (with $L_{12_0} = 0$) in such a way the UHFB is not violated by any point in L'. But for $\beta > 1$, L'_0 needs help from the inner loop to satisfy

$$|L'| < \rho^{-1} \quad (4)$$

for each frequency. Define $\lambda_{12} = \frac{k_{12}}{k_{12_0}}$. From figure 1, $|L'|$ is given by

$$L' = G\frac{P}{1 + \lambda_{12}L_{12_0}} \quad (5)$$

Considering (5) in the nominal case, G, in terms of L'_0, can be obtained

$$G = \frac{L'_0}{P_0}(1 + L_{12_0}) \quad (6)$$

Substituting (6) in (5)

$$L' = L'_0\frac{P}{P_0}\frac{1 + L_{12_0}}{1 + \lambda_{12}L_{12_0}} \quad (7)$$

Defining $k = k_{21} + k_{11}k_{12}$ and using (7) in (4), the condition on L_{12_0} imposed by the fact $\beta > 1$ is that, for any possible value of k_{ij},

$$\left| L'_0\frac{1 + L_{12_0}}{1 + \lambda_{12}L_{12_0}} \right| \frac{k}{k_0} \leq \rho^{-1} \quad (8)$$

Note λ_{12} depends on an uncertain parameter, k_{12}.

In the frequency range of interest, that in which the outer loop is below the UHFB, the instrumental assumption that $\angle L_{12_0} = 0$ will be made. Due to the typical observed shape of L_{12_0} and its boundaries (see figure 6), and according to the

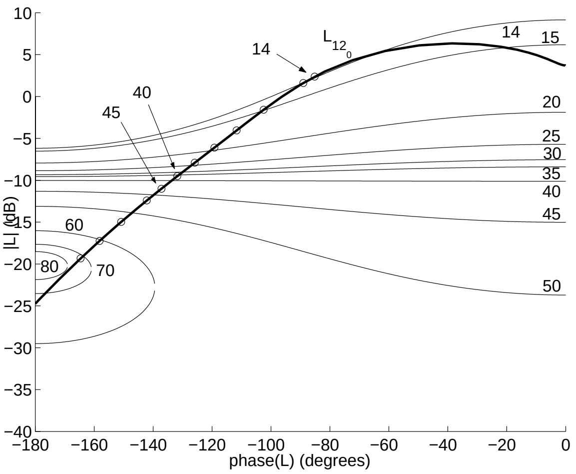

Fig. 6. L_{12_0} and its boundaries.

authors' experience, knowing $|L_{12_0}|$ at $0°$ phase gives a very good idea of the boundaries at every phase, so studying this particular L_{12_0} still will provide good information about demands on L_{12_0}. The boundaries on L_{12_0} are tipically:

- Non-existing for $\omega \in [0, \approx \omega_b]$ rad/s, $\omega_b \approx 14$ rad/s in the example.
- Single-valued, lower bounds, for $\omega \in [\approx \omega_b, \approx \omega_d]$ rad/s, $50 < \omega_d < 60$ in the example.
- Double-valued for $\omega \in [\approx \omega_d, \infty]$ rad/s.

In the first frequency range L_{12_0} can be arbitrarily small. In the second frequency range there will be certain boundaries which obligues L_{12_0} to reach its maximum magnitude, in the example, there is only one boundary producing this effect, for $\omega = 14$ rad/s. Arrows have been used to point the frequencies at which L_{12_0} is closer to the boundary comparing to other frequencies in this range. At those frequencies the outer loop is closer to the UHFB, this is the reason why the boundaries for L_{12_0} at these frequencies are more demanding.

Applying (3) and (1) in (8), and using $\tau = \frac{k_x}{k_0}$, $k_x = k_{21_x} + k_{11_x} k_{12_x}$, it is obtained

$$\beta \frac{1 + L_{12_0}}{1 + \lambda_{12} L_{12_0}} \frac{k}{k_x} \leq 1 \tag{9}$$

From (9) it can be computed the maximum achievable β, β_{MAX}, assuming $L_{12_0} \to \infty$ (i.e., L_{12_0} helps as much as possible L'_0)

$$\beta \leq \frac{k_x}{k} \lambda_{12} \tag{10}$$

Rewriting the right member in (10) in terms of uncertain parameters and computing its minimum

$$\beta_{MAX} = \frac{k_x}{k_a} \tag{11}$$

where $k_a = k_{21x} + k_{11x} k_{12n}$. This value is determined by (11) irrespective of the case (A or B). HW method would use the same value in case A, but a different one in case B, $\beta = \frac{k_{21n} + k_{11n} k_{12x}}{k_{21n} + k_{11n} k_{12n}} < \beta_{MAX}$. As a result:

- in case B, HW method is always conservative. For instance, for $k_{11_x} = 200$ (case B), $\beta_{MAX} \approx 18.27$, whereas using $k_{12_F} = k_{12_x}$ yields $\beta = 9.18$.
- in case A, it chooses the biggest possible uncertainty reduction.

In both cases any different possibility is ignored. In next section the cost of this uncertainty reduction in terms of L_{12_0} control effort is analyzed, and it is introduced a tool which lets the control engineer choose a trade-off between both factors.

4. INNER-OUTER LOOP TRADE-OFF

Minimum L_{12_0} needed in order not to violate the UHFB is deduced from (9)

$$L_{12_0} \geq \frac{\beta \frac{k}{k_x} - 1}{\lambda_{12} - \beta \frac{k}{k_x}} \tag{12}$$

Define

$$\sigma = \frac{\beta - 1}{\beta_{MAX} - 1} \tag{13}$$

i.e., σ represents linearly, in the rank $[0, 1]$, β evolution in $[1, \beta_{MAX}]$. It represents τ proportional reduction with respect to the maximum possible reduction. (12) is rewritten in terms of σ

$$L_{12_0} \geq \frac{\sigma \left(\frac{k}{k_a} - \frac{k}{k_x} \right) + \frac{k}{k_x} - 1}{\lambda_{12} - \sigma \left(\frac{k}{k_a} - \frac{k}{k_x} \right) - \frac{k}{k_x}} \tag{14}$$

The right hand member of (14) is maximum for $k = k_x$, so this condition can be rewritten as

$$L_{12_0} \geq f(\sigma) = \frac{\sigma \left(\frac{k_x}{k_a} - 1 \right)}{\lambda_{12_x} - 1 - \sigma \left(\frac{k_x}{k_a} - 1 \right)} \tag{15}$$

$f(\sigma)$ gives the connection between uncertainty reduction and L_{12_0} control effort. In particular, it defines L_{12_0} needed to get maximum τ reduction ($\sigma = 1$)

$$L_{12_0} \geq f(1) = \frac{k_x - k_a}{k_a \lambda_{12_x} k_x} = \frac{k_{11x} - k_{12n}}{k_{21x}} \tag{16}$$

In figure 7 it can be seen an example of $f(\sigma)$ in which it is clearly advisable to trade-off using its information, instead of just getting as much as possible from L_{12_0}.

5. CACSD TOOL

This software is just the first stone in the final goal of a toolbox for the design of QFT multiloop

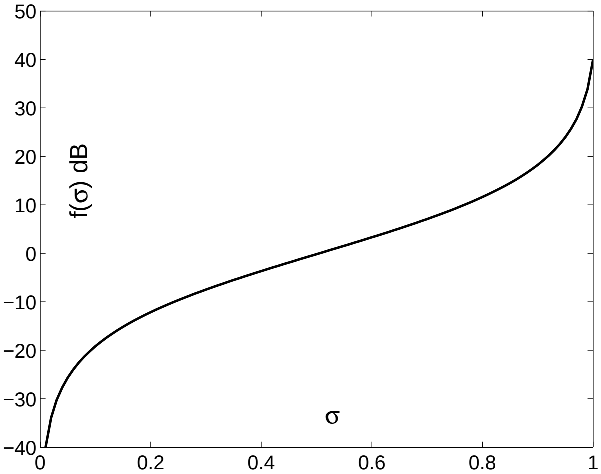

Fig. 7. $f(\sigma)$ example, with $k_{21} \in [1, 10]$ and $k_{11}, k_{12} \in [10, 100]$.

systems. The user introduces a multiloop plant like the one used in this paper, together with an outer loop design for $\beta = 1$, L_0, and another for $\beta = \tau$, L_0^*.

In the window in figure 8 the user can design another loop, L_0', in two different ways: a) directly manipulating its poles and zeros (symbols x and *dot*, respectively); and b) linearly interpolating the poles and zeros of L_0 and L_0^* by means of the slider, where these loops are denoted as $L01$ and $L02$ respectively. The value chosen in the slider is approximately equivalent to σ as defined in (13).

Every time a L_0' is chosen, windows corresponding to figures 9 and 10 are updated. In figure 9 the three loops are represented in the Nichols chart, and corresponding UHFB's are shown. Dots used for the loops are placed at fixed frequencies, allowing to observe the evolution of cut-off frequency. Figure 10 is an active version of figure 4, with wide lines representing L_0' and $T_{N_{L_0'}}$, and thin ones representing L_0, L_0^* and their associated noises.

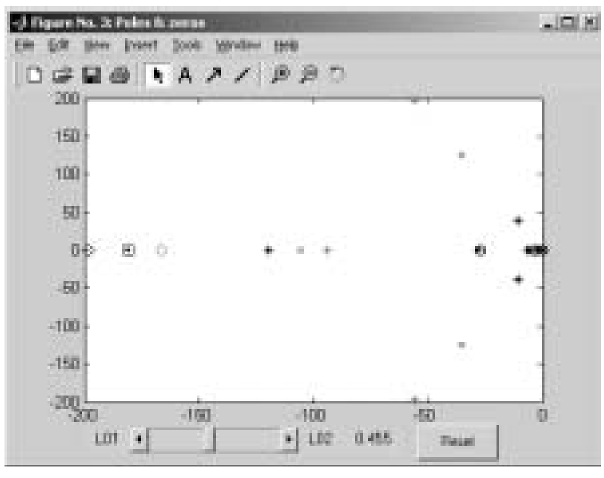

Fig. 8. CACSD tool, windows for interactive L_0' design.

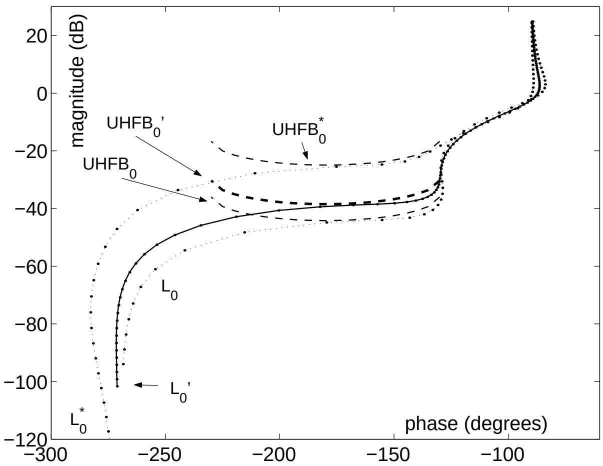

Fig. 9. CACSD tool, loops and UHFB's, Nichols chart.

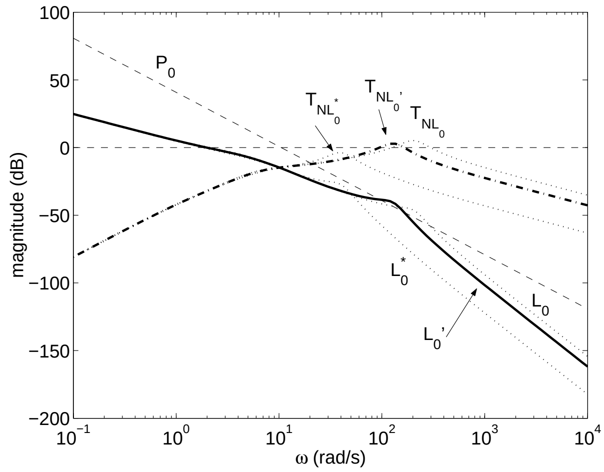

Fig. 10. CACSD tool, loops and associated noises.

Another window shows $f(\sigma)$ curve (see figure 7), highlighting actual σ and corresponding $f(\sigma)$, so that demands on L_{12_0} for the selected loop are also known and, thus, trade-off between L_{12_0} and L_0' can be interactively chosen.

6. CONCLUSIONS

In this work the NPMM QFT problem has been revisited. The simplest case has been focused and a simplified formulation of the problem has been developed, formulation which allows to mathematically express and analyze the problem, contrasting the previous approach. Some of the simplifying assumptions are strongly based on previous experience of the authors. Specifically, this formulation has been used to show how a parameter, previously considered as fixed, can be used to trade-off between inner and outer loops. A CACSD tool, which lets the control engineer interactively choose this trade-off, and observe the noise reduction achieved, is also presented.

This work aim is to constitute the first step leading to formalization of NPMM QFT, which provides a deeper understanding of involved phenomena, having as a final goal the development of a general theory which covers any system. Research is being assisted by ad hoc developed CACSD tools, like (Cervera *et al.*, 2001), which are also conceived to constitute in the future a multiloop QFT systems design toolbox.

Throughout this work, minimum phase plants have been used, following (Horowitz and Wang, 1979*a*; Horowitz and Wang, 1979*b*). This noise reduction scheme could also be used with NMP plants, in fact can help to get a lower crossover frequency, as in figure 4. If the inner plant, P_{12}, is NMP, then there is a limitation in what can be done by the inner loop, question which has not been addressed in this work and deserves a detailed analysis.

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