ON MULTIVARIABLE TRACKING

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Abstract: *The multivariable feedback system tracking error is related to the sensitivity matrix* $\mathbf{S} = [\mathbf{I} + \mathbf{L}]^{-1}$ *and to the uncertainty of plant and instrumentation. The 'popular' highly interacting* 2×2 *distillation column problem is solved in a novel manner.*

Keywords: *Horowitz, sensitivity, tracking error, uncertainty, instrumentation, cross sensitivity, Bristol* (*relative gain*).

1. Introduction.

Eitelberg (2000b) related the SISO feedback system tracking error rigorously to the sensitivity function $S = 1/[1 + L]$ and to the uncertainty of plant and instrumentation. A two design degree of freedom design procedure was presented that guarantees frequency-domain tracking error tolerances despite uncertainties in the feedback and feed-forward components of the system. Boje (2002) extended this work to multivariable systems and further analysed pre-filter design (Boje, 2002 and 2003) but ignored the *practically important instrumentation uncertainty*. The present note rectifies this omission.

A popular (benchmark) 2×2 distillation column example is analysed in the light of the present writer's published work on highly cross sensitive load sharing control.

2. Relative tracking error.

Figure 1 shows a multivariable feedback control structure with *two matrix design degrees of freedom* (2DOF), that is well suited for quantitative step-by-step design. Availability of any measured disturbance or utilisation of the knowledge about the plant operating conditions, $\mathbf{u}_{\mathrm{op}}$ and $\mathbf{y}_{\mathrm{op}}$, would add additional degrees of design freedom. In cases, where lower- and upper-case versions of the same letter are used, the former denotes a time-domain signal and the latter denotes its Laplace transform.

The loop transfer and the system sensitivity matrices are defined respectively as

$$\mathbf{L} = \mathbf{L}(s) = \mathbf{P}(s)\mathbf{G}(s)\mathbf{H}(s); \quad \mathbf{S} = [\mathbf{I} + \mathbf{L}]^{-1} \tag{1}$$

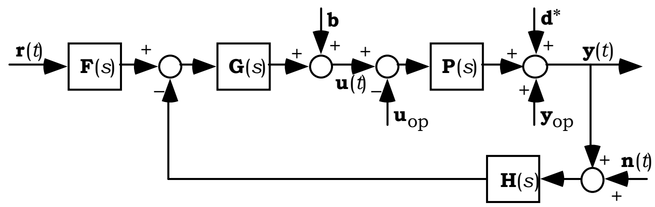

Figure 1: A 2DOF SISO feedback structure with the feed-forward controller **F**, feedback controller **G** and the sensor **H** with sensor noise/error **n**. The transfer matrix **P** describes *small signal* behaviour of the plant around the operating condition $\{\mathbf{u}_{\mathrm{op}}, \mathbf{y}_{\mathrm{op}}\}$. The offset **b** in the controller output **u** is used to neutralise $\mathbf{u}_{\mathrm{op}}$, in case of finite low-frequency loop gains.

The plant output is given by

$$\begin{aligned}\mathbf{Y} = \mathbf{SL}\left(\left[\mathbf{H}^{-1}\mathbf{F}\right]\mathbf{R} - \mathbf{N}\right) \\ + \mathbf{S}\left(\mathbf{D}^* + \mathbf{Y}_{\mathrm{op}} + \mathbf{P}\left(\mathbf{B} - \mathbf{U}_{\mathrm{op}}\right)\right)\end{aligned} \tag{2}$$

Normally, the primary control requirement is to achieve $\mathbf{Y} \to \mathbf{R}$ — in some suitable sense — where **r** is the reference for the plant output **y** in the true sense of the word. However, in some control literature (as reflected in Boje, 2002), the plant output is expected to behave as an output of some given model $\mathbf{T}_{\mathrm{m}}(s)$ when excited by the same reference, $\mathbf{Y_r} = \mathbf{T}_{\mathrm{m}}\mathbf{R}$. In the context of 2DOF feedback system design, there is no practical need for this complication, because the output $\mathbf{Y_r}$ of this model is really the reference ($\mathbf{r}(t)$ in Figure 1) for the system output. In other words, this model $\mathbf{T}_{\mathrm{m}}$ can be implemented in front of a suitably designed pre-filter **F**. Nevertheless, in order to remain formally compatible with the work of Boje (2002), the tracking error vector is defined as

$$\mathbf{E} = \mathbf{Y_r} - \mathbf{Y} = \mathbf{T}_{\mathrm{m}}\mathbf{R} - \mathbf{Y} \tag{3}$$

Compatibility to the original formulation in Eitelberg (2000b) is attained by setting $\mathbf{T}_m = \mathbf{I}$. Ignoring all additive uncertainty ($\mathbf{N}$, $\mathbf{D}^*$, $\mathbf{Y}_{op}$, and $\mathbf{B}$–$\mathbf{U}_{op}$), the tracking error vector becomes

$$\mathbf{E} = \mathbf{E}^r\mathbf{R} \quad \text{with} \quad \mathbf{E}^r = \mathbf{T}_m - \mathbf{SL}\left[\mathbf{H}^{-1}\mathbf{F}\right] \tag{4}$$

For each component Y_j of the system output vector $\mathbf{Y}$, the relative error would be

$$\frac{E_j(s)}{Y_{rj}(s)} = \sum_\mu E^r_{j\mu}R_\mu \Big/ \sum_\nu T_{mj\nu}R_\nu \tag{5}$$

This was not considered by Boje (2002). Instead, $\mathbf{E}^r$ was called the 'relative error' and magnitude specifications for the individual elements $E^r_{\mu\nu}(s)$ of the error transfer matrix $\mathbf{E}^r(s)$ were considered further. This is followed here.

The uncertainty of the instrumentation is described here by

$$\left[\mathbf{H}^{-1}\mathbf{F}\right] = \left[\mathbf{H}^{-1}\mathbf{F}\right]_n(\mathbf{I} + \Delta) \tag{6}$$

The subscript 'n' in eq. (6) denotes '*nominal*' in the usual engineering sense — the most important, probable or expected value. The matrix Δ denotes the relative uncertainty of $[\mathbf{H}^{-1}\mathbf{F}]$ with respect to its nominal. The usually larger uncertainty of the plant (in addition to that of $\mathbf{H}$ and $\mathbf{G}$) leads to a corresponding loop transfer uncertainty $\mathbf{L} \in \{\mathbf{L}_i\}$. For any $\mathbf{L}_k \in \{\mathbf{L}_i\}$, the corresponding error transfer matrix can be written as

$$\begin{aligned}\mathbf{E}^r_k &= \mathbf{E}^r_{nk} - \left(\mathbf{T}_m - \mathbf{E}^r_{nk}\right)\Delta \quad \text{with} \\ \mathbf{E}^r_{nk} &= \mathbf{T}_m - \mathbf{S}_k\mathbf{L}_k\left[\mathbf{H}^{-1}\mathbf{F}\right]_n\end{aligned} \tag{7}$$

The difference between two nominal (with respect to instrumentation) error transfer matrices can be expressed as

$$\mathbf{E}^r_{nk} - \mathbf{E}^r_{ni} = \mathbf{S}_i\left(\mathbf{L}_i\mathbf{L}_k^{-1} - \mathbf{I}\right)\left(\mathbf{T}_m - \mathbf{E}^r_{nk}\right) \tag{8}$$

Hence, combining equations (7) and (8) yields

$$\begin{aligned}\mathbf{E}^r_k = \mathbf{S}_i&\left(\mathbf{L}_i\mathbf{L}_k^{-1} - \mathbf{I}\right)\left(\mathbf{T}_m - \mathbf{E}^r_{nk}\right) \\ &-\left(\mathbf{T}_m - \mathbf{E}^r_{nk}\right)\Delta + \mathbf{E}^r_{ni}\end{aligned} \tag{9}$$

In practical designs generally, the $\mathbf{P}$-uncertainty dominates the $\mathbf{H}$- and $\mathbf{G}$-uncertainties — $\mathbf{L}_i\mathbf{L}_k^{-1} \approx \mathbf{P}_i\mathbf{P}_k^{-1}$ — and small errors are specified within $\mathbf{T}_m$-bandwidth — $[\mathbf{E}^r_{nk}]_{jj} << [\mathbf{T}_m]_{jj}$. The situation with the off-diagonal elements of $\mathbf{E}^r_{nk}$ is not necessarily clear, especially when $\mathbf{T}_m = \mathbf{I}$. Assuming nevertheless that they too are negligible (? as in Boje, 2002), eq. (9) simplifies to

$$\mathbf{E}^r_k \approx \mathbf{S}_i\left(\mathbf{P}_i\mathbf{P}_k^{-1} - \mathbf{I}\right)\mathbf{T}_m - \mathbf{T}_m\Delta + \mathbf{E}^r_{ni} \tag{10}$$

For $\mathbf{T}_m = \mathbf{I}$, we have (as in Eitelberg, 2000b)

$$\mathbf{E}^r_k \approx \mathbf{S}_i\left(\mathbf{P}_i\mathbf{P}_k^{-1} - \mathbf{I}\right) - \Delta + \mathbf{E}^r_{ni} \tag{11}$$

After the design of $\mathbf{L}_i$, the nominal $\mathbf{E}^r_{ni}$ can be eliminated with

$$\mathbf{F}_n = \mathbf{H}_n\left(\mathbf{I} + \mathbf{L}_i^{-1}\right)\mathbf{T}_m \tag{12}$$

There remain two contributions to the error transfer matrix $\mathbf{E}^r_k$ that are independent of the pre-filter $\mathbf{F}_n$ — the nominal system variability $\mathbf{S}_i\left(\mathbf{P}_i\mathbf{P}_k^{-1} - \mathbf{I}\right) = \mathbf{S}_i\left(\mathbf{P}_i - \mathbf{P}_k\right)\mathbf{P}_k^{-1}$ at $\Delta = \mathbf{0}$ (which was considered by Boje, 2002) and the relative instrumentation uncertainty Δ (which was not considered by Boje, 2002). It is a relief that the instrumentation uncertainty is additive in the simplified error equation (11). However, some unresolved doubts remain about the universal correctness of the simplification of eq. (9) — there was no such doubt in the SISO case in Eitelberg (2000b).

Both of these contributions to overall relative error are independent, generally. Hence, of a total error budget for the relative tracking error, some portion *should* be allocated to the imperfect elimination of the nominal error $\mathbf{E}^r_{ni}$ (Boje, 2002, referred to this as over-design of the sensitivity) and some *must* be allocated to the instrumentation $\mathbf{H}^{-1}\mathbf{F}$ (Boje, 2002, did not do so). It is quite possible that nothing remains of the error budget and the problem cannot be solved with given instrumentation. This is the reason why the present writer disagrees with the general attitude in control literature where sensors and command transfer are accepted (or assumed) as given before the feedback system design. In reality, they may be either inadequate or too expensive for what is in the end required of them.

Rather, a reasonably good feedback design should be carried out first, the achieved system variability should be deducted from the overall error budget and the remaining $\Delta \neq \mathbf{0}$ must then serve as the specification for the selection of adequate instrumentation and command algorithms. If no such sensors and command transfer exist then the feedback design must be made more radical or specifications must be modified — and occasionally the project must be buried to limit the losses.

3. Distillation column benchmark.

The 'benchmark'.

Let us look again at the problem as described and solved in Boje (2002) — who refers to Skogestad, Morari and Doyle (1988); Yaniv and Barlev (1990); Limebeer (1991); Horowitz (1993); Limebeer, Kasenally and Perkins (1993); Lundström, Skogestad and Doyle (1999). The plant is given as

$$\mathbf{P} = \frac{1}{1+75s}\begin{bmatrix}0.878 & -0.864\\ 1.082 & -1.096\end{bmatrix}\begin{bmatrix}k_1 e^{-sT_1} & 0\\ 0 & k_2 e^{-sT_2}\end{bmatrix}$$
$$k_1, k_2 \in [0.8,\ 1.2]; \quad T_1, T_2 \in [0,\ 1]\text{min} \tag{13}$$

The step response specifications are given as

- Response ≥ 90% of final value in less than 30 min.
- On-channel overshoot ≤ 10%.
- Steady state error (on- and off-channel) ≤ 1%.
- Off-channel response ≤ 50%.

Some pertinent comments on the plant.

As there is no cross-channel uncertainty in this model of a distillation column with parameters known and fixed to the unrealistic three or four decimal digits, a fixed de-coupler is academically possible *for this model*. However, Boje (2002) came to the conclusion that no significant benefits could be obtained here with de-coupling, because of 'robust stability considerations'. It is suggested here that de-coupling can be outright damaging in practice — see also Shinskey (1988), Luyben (1990), Leithead and O'Reilly (1992) and Eitelberg (1999a, pp. 86–87; 1999b) in the same sense.

A diagonal feedback control structure with potentially full feed-forward is shown in Figure 2. The plant relative gain (RG) of Bristol (1966) (see McAvoy, 1983, about the frequency dependent Bristol's number) is here independent of frequency:

$$\Lambda(s) = \left[1 - \frac{P_{12}(s)P_{21}(s)}{P_{11}(s)P_{22}(s)}\right]^{-1} = 35 \tag{14}$$

(The relative gain array is Λ = $\mathbf{P}.*\mathbf{P}^{-\mathrm{T}}$.) In case of *high-gain feedback* in both loops, the Bristol number Λ is the multi-variable version of the cross sensitivity as defined in Eitelberg (1999a, p. 86). As a consequence, the plant input becomes

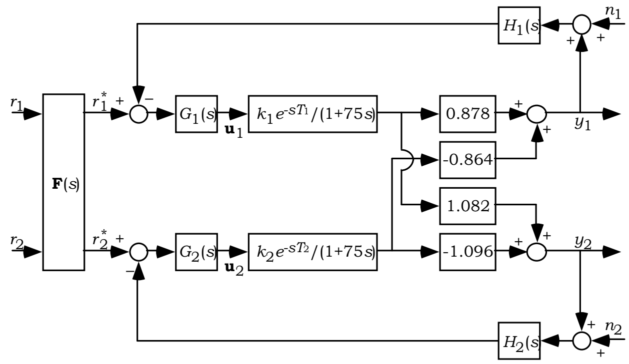

Figure 2: 2DOF feedback structure for the 'benchmark'.

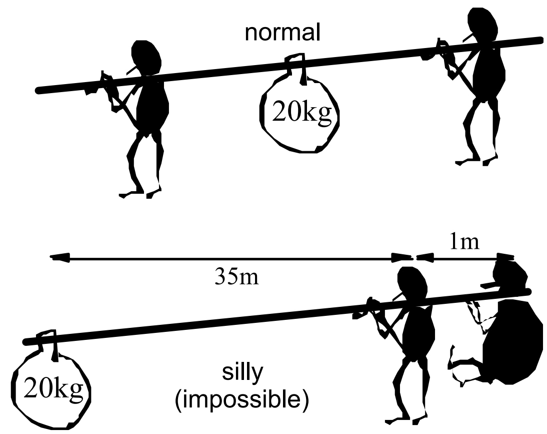

Figure 3: An example of low and high interaction. Each person caries 1/2 of the 20 kG total load 'normally'. They have to exert the *opposing* forces of about 700 kG (35×20 kG) in the 'silly' arrangement.

$$\begin{aligned} U_1 &\approx \frac{\Lambda}{P_{11}}\left[\left(R_1^* - H_1N_1\right) - \frac{P_{12}}{P_{22}}\left(R_2^* - H_2N_2\right)\right] \\ U_2 &\approx \frac{\Lambda}{P_{22}}\left[\left(R_2^* - H_2N_2\right) - \frac{P_{21}}{P_{11}}\left(R_1^* - H_1N_1\right)\right] \end{aligned} \tag{15}$$

Every change in references, or difference between sensor errors, will lead to 35 times larger effort at the plant input (and elsewhere) than in the case when an output is regulated by only one actuator in a high gain loop. Designing such a *plant* is like the impossibly silly bag carrying arrangement in Figure 3 — it can be necessary (even 'clever') for special purposes (as in animal limbs) but should be rejected as *bad design* in general.

DOF1: design of regulation.

It is convenient to define the loop transfer functions when all other loops are open:

$$L_{Pi} = P_{ii}G_iH_i \tag{16}$$

In the QFT context (at least for the first design step) loops are defined as

$$L_{Qi} = Q_i G_i H_i \quad \text{with} \qquad Q_i = V_{ii}^{-1} \quad \text{from} \quad \mathbf{V} = \mathbf{P}^{-1} \tag{17}$$

Generally (Eitelberg, 1999a, p. 84):

$$L_{Qi}(s) = \frac{L_{Pi}(s)}{\Lambda_{ii}(s)}; \quad \Lambda = [\Lambda_{ij}] = \mathbf{P}.*\mathbf{V}^{\mathrm{T}} \qquad \text{here} \quad L_{Qi} = \frac{L_{Pi}}{35} \tag{18}$$

If one of the two loops, say no. 1, is closed then the other loop around y_2 in Figure 2 is (see also eq. (4-23) in Eitelberg, 1999a):

$$L_2 = L_{P2}\frac{1+L_{Q1}}{1+L_{P1}} = L_{P2}\frac{1+L_{P1}/35}{1+L_{P1}} \tag{19}$$

If $1+L_P$ has right half-plane zeros, then the closed L_2 (or L_1) becomes *conditionally stable* (at best) and its design may become practically impossible (because of the combination of instability with non-minimum phase-lag). This is why Horowitz (1993, pp. 420–421) designed L_{Q1} and L_{P1} simultaneously for this model. Beyond L_{P1} bandwidth L_2=L_{P2} and within the L_{Q1} bandwidth L_2=L_{Q2}=$L_{P2}/35$. This is the real reason for the apparently terribly slow tracking specification (30 minutes) in comparison with the very short dead time of no greater than 1 minute.

As the plant is very nearly symmetrical and the specifications are totally symmetrical, it is reasonable to begin by considering the achievable performance under the condition L_{P1}=L_{P2}=L_P. Instead of the popular 'quantitative design', the much simpler and often more accurate *worst case design* (Eitelberg, 1999a and 2000a), with k_1=k_2=1.2 and T_1=T_2=1 minute, suffices here. The rough design perspective is shown in Figure 4. Note the *unavoidable* zero slope between the gain cross-over frequencies of L_{Q1} and L_{P2}, which corresponds to a dramatic wiggle (very prominent in the shown nominal L_2 of Boje, 2002) to zero phase angle on a logarithmic complex plane. This indicates why the traditional QFT design with L_Q is sensible for low frequency performance specification, but it can be very misleading in respect of stability. Assuming a reasonable L_P slope of about –30dB per decade around and between ω_{Qgc} and ω_{Pgc}, leads to ω_{Qgc} < 0.1. This corresponds to a first order time constant greater than 10 minutes or to a resonance with a period longer than 63 minutes. It follows that the specification time of 30 minutes is *impractical* — a contrived yet highly informative game. Around ω_{Qgc}, avoidance of sensitivity amplification requires a flatter slope than –30dB per decade — lowering the value of ω_{Qgc} even further. If, however, low frequency sensitivity reduction is needed then a slope of –30dB per decade or steeper is needed — this makes the wiggle into a large loop around 0dB on the logarithmic complex plane and creates high resonance in the closed loop system near ω_{Qgc}. Nevertheless, as no disturbance regulation is specified, this game can be played with the high gain loop approach, because the resonance can be compensated for in the filter **F**.

With L_{P1}≠L_{P2} one can sacrifice some performance in one channel to a potentially dramatically improved performance in the other channel (see also p. 87 in Eitelberg, 1999a) — this is not followed here. With the understanding of Figure 4, it is easy to design, or *tune*, two identical (tight) loops, as shown in Figure 5, with the PI regulators:

$$G_1 = 38\left(1+\frac{1}{4s}\right); \quad G_2 = -30\left(1+\frac{1}{4s}\right) \tag{20}$$

Without violating the rules of this game, either the sensors are assumed to be ideal, or they are sufficiently well compensated with the PID lead-lag terms in the return paths.

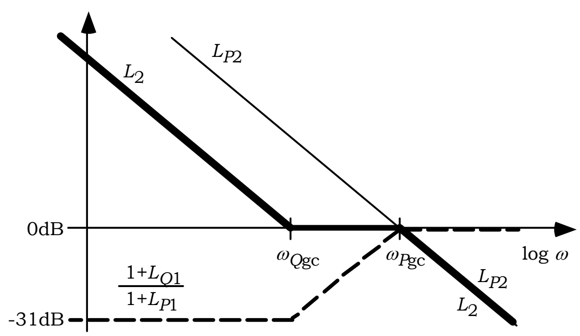

Figure 4: General loop design perspective with L_{P1}=L_{P2}. Here, ω_{Pgc} < 1 rad/min.

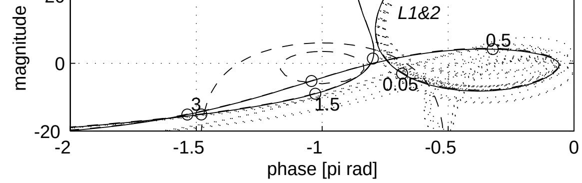

Figure 5: Loop design. Solid lines: 'worst case'. Dotted lines: a selection of L_1, L_2.

DOF2: design of command transfer.

Simulation with $\mathbf{F} = \mathbf{I}$ indicates surprisingly good behaviour, except for a brief cross channel overshoot of over 100% and (due to tight design) the expected resonance near both ω_{Qgc} and ω_{Pgc}. The fast overshoot and resonance are easy to filter out as both are beyond the specified frequency range, but the lower frequency resonance and cross coupling turned out to be more difficult to tune out heuristically. Instead, the filter $\mathbf{F}$ is calculated from eq. (12). With diagonal $\mathbf{H}$ and $\mathbf{T}_{\rm m}$, a very convenient implementation is shown in Figure 6. There is no uncertainty in $P_{12}(s)/P_{22}(s) = 0.788$ or $P_{21}(s)/P_{11}(s) = 1.232$, here, but there is some in L_{Qi}. This uncertainty is of practically no consequence within regulation bandwidth, where $\left|L_{Qi}^{-1}\right| < 1$ is negligible and it does not matter beyond the bandwidth ($<\omega_{Pgc}$) of $T_{{\rm m}i}$, where the improper (often non-causal and possibly unstable) inverse of L_{Qi} will have to be implemented with a proper stable approximation. Practically, the inverse of L_{Qi} and its uncertainty matters only near its gain cross-over frequency ω_{Qgc} where the given dead-time can be ignored. The only remaining uncertainty of relevance is due to k_i. In the filter design, there is no worst-case. As the most important operating condition is not known, the average $k_1 = k_2 = 1$ is used here.

The following simulations are carried out with the filter in Figure 6 based on:

$$L_{Q1,\rm approx}^{-1} = L_{Q2,\rm approx}^{-1} = \frac{4.2s(1+75s)}{(1+4s)(1+s)}$$

$$H_1 = 1; \quad T_{\rm m1} = \frac{1}{1+1.2\times 12s+(12s)^2} \qquad (21)$$

$$H_2 = 1; \quad T_{\rm m2} = \frac{1}{1+1.4\times 9s+(9s)^2}$$

Simulation.

Unit step responses are shown in Figures 7 and 8. It should be noted that here all uncertain plant parameters (k_1, k_2, T_1, and T_2) are varied independently and the above simple design (for clear reasons) does not quite satisfy the specifications. If, however, $T_1 = T_2$ (as in Horowitz, 1993) then the above simple PI regulation practically satisfies the specifications. All extreme traces around t = 40 min — shown with dotted lines in Figures 7 and 8 — are due to opposite extremes of the dead-times ($T_1 = 1$ with $T_2 = 0$ and $T_1 = 0$ with $T_2 = 1$). Using PID regulators would permit somewhat higher loop bandwidth and improved tracking.

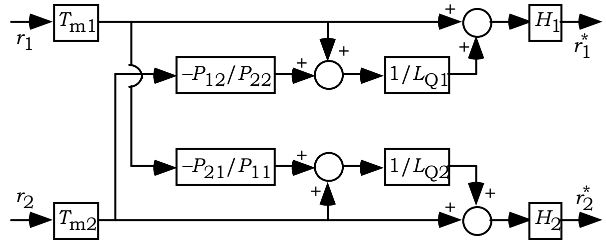

Figure 6: Filter implementation.

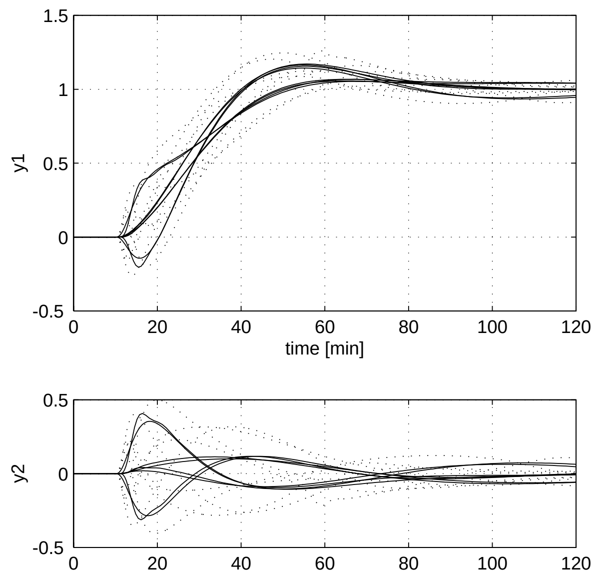

Figure 7: Response to unit step in channel 1 at t=10 min. Solid: T_1=T_2; dotted: $T_1 \neq T_2$.

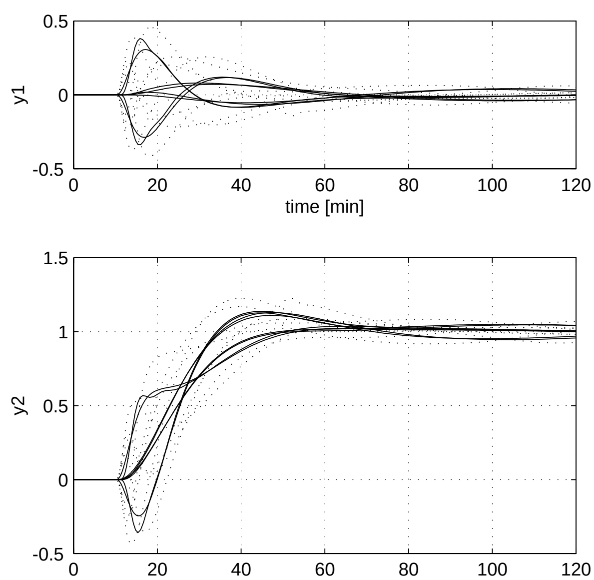

Figure 8: Response to unit step in channel 2 at t=10 min. Solid: T_1=T_2; dotted: $T_1 \neq T_2$.

Some comments on instrumentation errors.

The only benchmark specification that relates directly to the relative tracking error in equation (11), is the severe 1% steady state error. It is true that the regulator integral terms reduce the steady state sensitivity to zero. However, the personal experience of the writer indicates that the steady state entries in Δ significantly exceed the specified 1% in process plants. This accuracy and much better is possible indeed but requires regular calibration — as is the norm in military systems but not in the (South African) process industry.

This specification is illusory if 1% of the usually rather small test step is meant. If this step is, say one tenth of the plant operating range, then the total instrumentation *and installation* error must not exceed 0.1% of the sensor range!

The response time and overshoot specifications relate to instrumentation errors weakly if they are, say, less than the achievable 2 or 3% of the range, because the given model uncertainty would then dominate. Nevertheless one should check, especially at frequencies near the Nyquist point, where the sensitivity **S** cannot but the stability margins tend to be small.

4. Conclusion.

The multivariable feedback system tracking error is related to the sensitivity matrix $\mathbf{S} = [\mathbf{I} + \mathbf{L}]^{-1}$and to the uncertainty of plant and instrumentation — see eq. (11). However, some doubts remain in respect of the off-diagonal elements of $\mathbf{E}^{r}{}_{nk}$ in eq. (9) — can they be always ignored?

The 'popular' highly interacting 2×2 distillation column problem was approached in a novel manner: very simple worst-case PI regulator design followed by simple nominal command transfer design. This compares well with some previously published solutions of great complexity leading to (unpublished) very high sensitivity and resonance around the L_P gain and phase cross-over frequencies. None of the published designs have been shown to satisfy the given specifications under the condition of dynamically saturating actuators. The step response specifications will lead to actuator saturation whenever the step is not negligible in relation to the operating range.

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