MODELLING AND OPTIMAL CONTROL OF A TELECOMMUNICATIONS MARKET OPERATOR

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Abstract: Model Predictive Control is used to design a feedback control system for a GSM telecommunication market modelled using system dynamics with three incumbent operators. The controller calculates the optimal tariffs for the products of one of the operators to be used as decision support by the marketing management of the operator when setting prices. The controller is able to optimise the Average Revenue Per User for the operator in the closed-loop non-linear simulation, as well as increase total revenue substantially.

Keywords: Model Based Control, Predictive Control, Optimal Control, Decision Support Systems, Telecommunication, Forecasts.

1. INTRODUCTION

A GSM telecommunications market consisting of two incumbents and an entering third player is modelled utilising a multivariable non-linear, state space, system dynamics approach (Forrester, 1995; Lommerud et al., 2002). Past researchers in this area have mostly used an econometric approach (Fildes et al., 2002) which does not capture dynamic or real-time information (Markidakis et al., 1983) and which does not lend itself to dynamic optimisation (Cox et al., 2002, McBurney et al., 2002).

The aim of this paper is to contribute to the scarcely researched area of dynamic real-time optimisation of marketing strategy (pricing) of telecommunication services. In the field of dynamic optimisation, work has been published focussing on market models that use historical data, and also models of organisations in the start-up phase (Parker, 1997). Little has been published in the field of telecommunications related to real-time optimisation and decision support systems (Jonason et al., 2002; Singh, 1990) for stable industries, especially in the South African context as addressed in this paper.

The forecasting model calculates subscriber choice from a calculated utility based on the multinomial logit (Ben-Akiva et al., 1985) and conjoint analysis (Attenborough, 1998; Wittink et al., 2001). The utility, which determines demand, is based on the fact that subscribers strive to maximise their satisfaction (Jun et al., 2002) and minimize their expense (Kang et al., 1996). The utility is used to obtain a probability that is fed into a Bass diffusion type (Bass, 1969) aggregate growth (Meade et al., 1995; Parker, 1994; Venkatesan et al., 2002) differential equation. This equation relates the different states in the model to their time derivatives, and also incorporates the industry trend that subscribers tend to be loyal to operators (Lommerud et al., 2002). The probability depends on the price, the amount of new subscribers joining the industry, the cost of connecting to an operator, network quality, marketing effort, connection incentive and monthly subscription fee (Williamson et al., 1997). The actual network usage for the particular service option chosen, which is modelled as a function of price (Fildes et al., 2002), determines the Average Revenue Per User (ARPU), and hence the revenue that the operator will be earning.

The model encapsulates all the prominent post-paid price plans on the market, as well as five different demographic market segments (Fildes et al., 2002). This approach enables a sharper focus on the determinants of price elasticity (Parker, 1997).

Relevant industry data, which are often difficult to obtain (Fildes et al., 2002; Islam et al., 2002), was supplied by Icon Corporation (Pty) Ltd, and used to validate the model.

A Model Predictive Controller (Goodwin et al., 2001; Seborg et al., 1989) is designed for the telecommunication operator market model. It uses the observed market state to optimally determine a price time-series for one of the operators' products (this operator will be referred to as the optimising operator) that will maximise ARPU over the simulation time interval. Except for ARPU, the controller is also able to increase total revenue by 3.6% and minimise churn (the movement of customers from one operator to another) over the simulated interval for the optimising operator. It therefore provides valuable decision support to the marketing management of such an operator (Braun et al., 2002; Bui et al., 1996; Singh, 1990).

2. MARKET MODEL DEVELOPMENT

This paper focuses on the control part of the research done, hence the model will be discussed very briefly. The basic structure of the model is that of a state space model, with the generic non-linear representation

$$\begin{aligned} \dot{x}[k] &= f\left(x[k], u[k]\right) \qquad x[0] = x_0, \\ y[k] &= g\left(x[k], u[k]\right) \end{aligned} \tag{1}$$

and the generic linear representation

$$\begin{aligned} \delta x[k] &= A\,x[k] + B\,u[k] \qquad x[0] = x_0, \\ y[k] &= C\,x[k] + D\,u[k] \end{aligned} \tag{2}$$

Where $x \in R^n$, $u \in R^m$, $y \in R^p$, $\mathbf{A} \in R^{n\,x\,n}$, $\mathbf{B} \in R^{p\,x\,n}$, $\mathbf{C} \in R^{p\,x\,m}$, $\mathbf{D} \in R^{p\,x\,m}$ and k is the discrete time variable representing months. In (2) the definition of the dimension n is the number of states, m is the number of inputs and p is the number of outputs.

In essence the model simulates the movement of subscribers between the different products on offer in the market based on the choice of the subscriber.

The model includes 16 price plans between the 3 operators in the market. Each price plans has 6 tariffs linked to it: Three possible destinations that can be called during two different times of day. Therefore tariff inputs into the model constitute 96 different inputs.

The market is divided into 5 market segments according to income: Consumer Low, Consumer High, Medium, Business Low and Business High. For each market segment an input representing the amount of new subscribers joining the total market per time period is modelled – $\mathbf{d}_1[k]$.

In addition the following input variables are modelled per price plan offered:

1. $\mathbf{d}_2[k]$ - the cost of connecting to a price plan for the first time.
2. $\mathbf{d}_3[k]$ – soft issues not covered in the other inputs, i.e. the subscriber's value for money parameter.
3. $\mathbf{d}_4[k]$ - the connection incentive when connecting to a new contract.
4. $\mathbf{d}_5[k]$ - the monthly subscription fee.

In response to the inputs (adding the amount of input variables defined above gives m = 161), subscribers choose the product that fits their market segment the best and minimises their expense. The movement of subscribers between price plans and operators is tracked by means of 80 states in the model. In addition the model has another 30 states used for output calculation – i.e. n = 110.

The model has p = 30 outputs, which are the following 6 vectors having 5 elements each (one for each market segment):

1. The amount of subscribers that have joined the whole market during the month.
2. The net gross connections for the optimising operator.
3. The ARPU for the optimising operator.
4. The revenue from usage for the optimising operator.
5. The average amount of minutes phoned by all subscribers of the optimising operator.
6. The total amount of minutes phoned by all subscribers of the optimising operator.

3. OPEN-LOOP SIMULATION

The non-linear model in (1) was simulated and fed with relevant industry data over a period of 9 months. It tracks the actual data with sufficient accuracy for the purposes of the project.

3.1 Results

The optimising operator is referred to as OO, and the other two operators as O2 and O3. In this section some of the results of the open-loop simulation are shown and discussed.

From Fig. 1 it is clear that the majority of the subscribers belonging to the optimising operator (and to the market in general) resorts in the lower end of the market segment spectrum.

A tariff increase was affected by all operators in the model at the start of the 4th month of the simulation period. The positive impact of this tariff increase is clearly discernable in the total revenue per market segment in Fig 2.

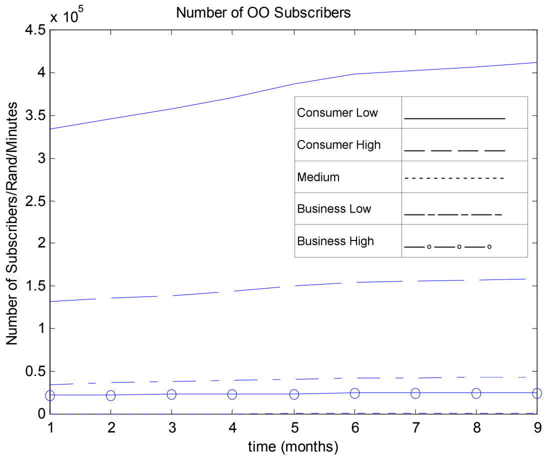

Fig. 1. Subscribers per Market Segment for OO.

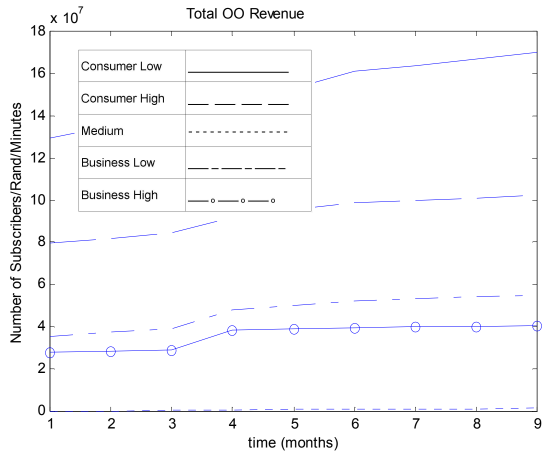

Fig. 2. Total Revenue per Market Segment for OO.

4. CONTROLLER DESIGN

4.1 Model Linearisation

The model (1) was linearised by making use of standard linear control theory (Nise, 1995) and an average operating point. Of the 161 inputs, only the tariffs of 5 of the optimising operator's price plans were chosen to be manipulated variables in the controller. The rest are measured disturbances. Thus 5 x 6 = 30 inputs make the MIMO problem square (p = 30).

4.2 Model Analysis

The linear model has the majority of its poles in the Left Half Plane (LHP), but there are five poles at zero and five at 1 in the Right Half Plane (RHP). The linear model is thus open-loop unstable.

Controllability and observability analysis gave the following results: The rank of the controllability matrix is 49 and the rank of observability matrix: 51. The linear model is therefore neither controllable nor observable. A remarkable property of MPC, however, is that stability of the resultant feedback system can be established (Goodwin et al., 2001). Furthermore, the plant is simulated as non-linear.

From the frequency response of the linear model it is apparent that the cut-off frequency and bandwidth of the system is 10^0 = 1 rad/month. This was the case for all the responses – the maximum bandwidth in the model is 1 rad/month.

4.3 Controller Design

In order to reach the objective of the research the following feedback control methodology, utilising Model Predictive Control (MPC) (Goodwin et al., 2001; Seborg et al., 1989; Van Den Boom, 1996) has been implemented:

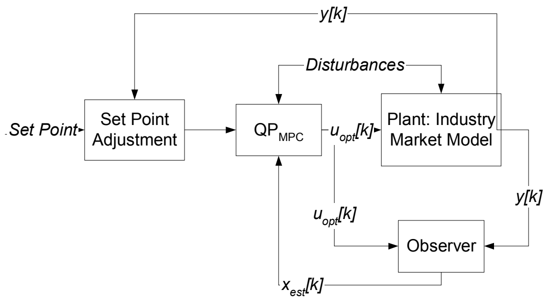

Fig. 3. The Implemented MPC Controller (Morari at al., 1998)

The observer is linear and estimates the current states from the model outputs and inputs. The constraints of the market are the maximum and minimum values of the tariffs that may be charged per minute.

It was found that if the controller focuses on all the outputs of the model with the same priority, that a meaningful solution to the problem, resulting from bounded changes in the manipulated variables (tariffs) could not be obtained. A higher priority therefore had to be assigned to some outputs. It was decided to focus the attention of the controller on the ARPU set of outputs of the model.

The set point of the ARPU outputs were made a certain percentage higher than the value of the respective ARPU outputs per month obtained from the open-loop non-linear simulation of the model. All the other outputs were given set points equal to the values of those outputs per month obtained from the open-loop simulation. If any output is lower than the desired set point for the month, the set point is kept constant and the controller would attempt to get the output to be closer to the set point (if not equal to it) for the next month by applying optimal tariff inputs. If an output is higher than the desired set point for a month, however, the controller sets this set point equal to the output for that month so as to zero the error signal for that output, to enable the controller to focus on outputs that were lower than set point (see the set point adjustment block in Fig. 3 and how it links to the other blocks in the design).

The amount of control moves that the controller plans into the future during each iteration (control horizon), and the amount of time intervals that the system output is predicted into the future in response to the control moves (prediction horizon) (Seborg et al., 1989; Van Den Boom, 1996), were empirically determined. The optimal and implemented values are a control horizon of 2, and a prediction horizon of 7.

Values for the manipulated variable and output weights were assigned empirically.

5. CLOSED-LOOP SIMULATION

5.1 Results

In this section selected key output and manipulated input graphs are shown. The closed-loop system was simulated over the last 4 months of the 9 month period simulated in the open-loop simulation. An extra month was added to make the simulation period 5 months.

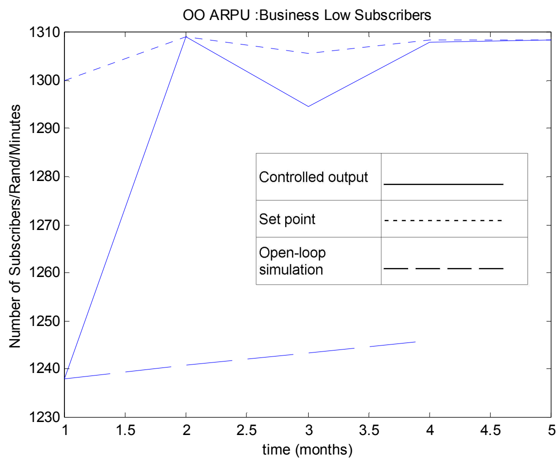

Fig. 4. OO ARPU in the Business Low Market Segment

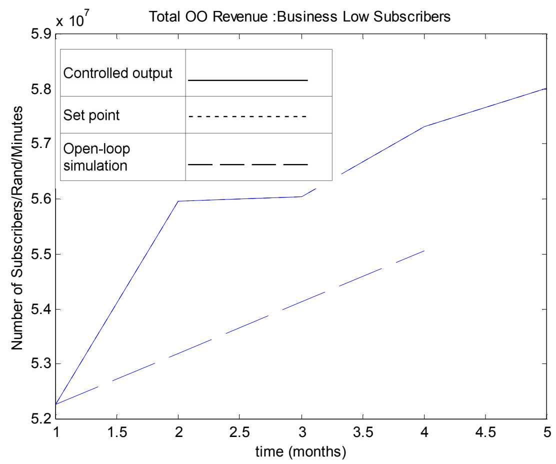

Fig. 5. Total OO Revenue in the Business Low Market Segment

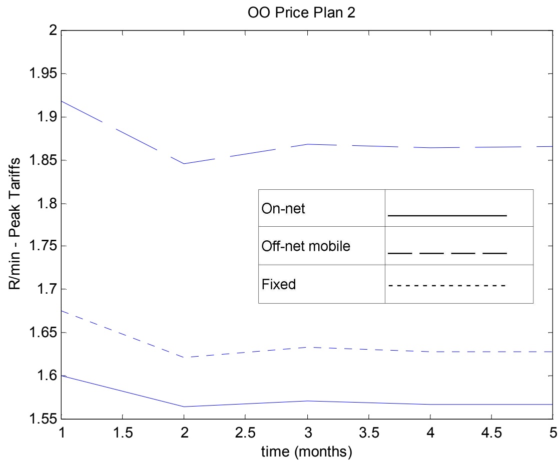

Fig. 6. Peak Tariffs for OO Price Plan 2

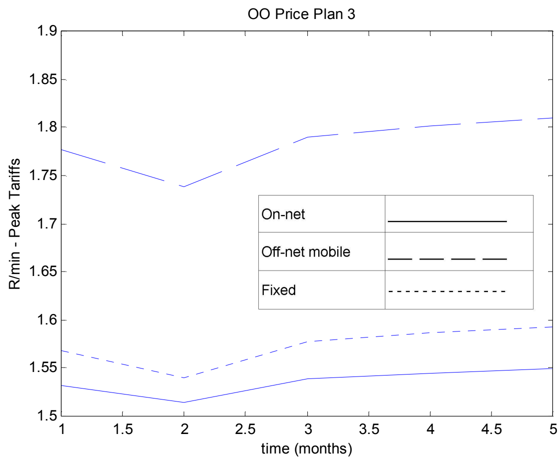

Fig. 7. Off-Peak Tariffs for OO Price Plan 3

5.2 Discussion

The controller manages to successfully maximise the ARPU signal (the ARPU signals approach the increased set points of 5% above the values recorded in the open-loop simulation) for all 5 market segments (see Fig. 4 as an example). By increasing the ARPU for each market segment the controller manages to increase the revenue per market segment above the totals recorded per month in the open-loop simulation (although this was not the primary focus on the controller) as shown in Fig. 5. This is achieved by calculating the change in the tariffs which are inputs into the model – see Fig. 6 and Fig. 7 for examples of tariff trends.

The total additional revenue achieved by increasing the ARPU by 5 %, is 3.62% over the open-loop simulation. This amounts to a very significant revenue increase in monetary terms.

The Nyquist frequency of the linear model was calculated to be $1/\pi$ per month (Haykin, 1994). Therefore the minimum sampling rate, and subsequently the minimum simulation time increment as well as controller simulation time increment is π months. Because of practical limitations – the relevant industry data provided is sampled monthly - it was decided to make the subsequent simulation time increment 1 month. The chosen sampling rate is therefore π times greater than the Nyquist frequency, but smaller than the standard control practice of sampling at not less than 10 times the Nyquist frequency. It is therefore possible that some dynamics might be lost.

Furthermore, although the linear model is open loop unstable, and neither controllable nor observable, the linear controller applied to the non-linear model in the simulation produces a stable output.

6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The following conclusions can be drawn from the research done:

1. A linear feedback, model based predictive controller implemented on the non-linear model successfully optimises the strategic ARPU output vector of the model for the optimising operator by manipulating the tariffs of the optimising operator.
2. In addition to the ARPU vector, the revenue vector is also increased by 3.62 % resulting in a large profit increase for the operator. Most of the other controlled variables are still controlled at or close to their respective set points.
3. The goal of the project as stated in section 1 has been successfully achieved, and a foundation has been built for future research.

6.2 Further Research

Further research into the following areas is needed in order to bring the research done to a state of greater maturity which might lead to commercialisation:

1. The observation that an increase in total revenue follows when ARPU is maximised will have to be verified.
2. Other outputs than the ARPU could be chosen as the primary strategic output to focus on as well. Fluctuation between different priority outputs and parameters as key performance indicators occur often in the marketing departments of operators. One particular output typically remains key for a season and then changes to a different one. The controller implemented in this research could easily be adjusted in such a scenario to focus on the new output variable.
3. Additional analysis and investigation regarding the inherent instability of the linear model should be conducted. The stability of the non-linear model need to be ascertained and formal control design techniques for open-loop unstable systems should be experimented with.
4. Other control strategies could be applied to the problem and compared with MPC.
5. Adaptive control which makes it possible to run the system on-line and real time with the internal model "learning" and adjusting to changes in the external market, would add a significant dimension of decision support to management.
6. The suggestions made by the controller for price change should be implemented in real-time on an actual case study and the market response verified with empirical data.

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