

# MODELLING AND CONTROL OF A THREE-PHASE ELECTRIC ARC FURNACE

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**Abstract:** This paper investigates the modelling and control of the electric energy input of a three-phase electric arc furnace (EAF) using electrode position control as the main control strategy. Two methods to keep the electrical energy input constant at a known set point value are to control the arc-current or the arc-impedance. These variables are controlled by moving the electrodes up or down using an electrode position controller. Plant data are used to do system identification on an industrial EAF to model the close loop electrode system. *Copyright © 2003 IFAC*

**Keywords:** Electrodes; Arc resistance; Modelling; Ward Leonard drive; Control; Step inputs

## 1. INTRODUCTION

EAFs produce steel by melting scrap using a three-phase electrical supply as the electrical energy input. The fundamental problem in the EAF industry is the production of steel at a specified quality at the lowest cost possible. This three-phase electrical input serves as the main energy input in the electric arc furnace. The electrical energy input needs to be controlled with the aim of achieving the lowest possible production cost.

Each phase of the three-phase electrical input supplies power to one of the three electrodes that is mounted above the furnace bath through the roof. The furnace roof is closed when power is supplied to the system. The furnace operation is based on heat transfer into the bath from arcs drawn between the tips of the electrodes to the metallic charge. Thus, electrical energy is converted into heat which is transmitted to the charge through the electrodes (Billings and Nicholson, 1975). Constant melting causes the arc length to change and results in a change in the electric energy input if control is not supplied to the system. Two variables are mainly used to control the electrical energy input, i.e. arc impedance and arc current. Both these variables are

controlled via an electrode position controller which moves the electrodes in a vertical position to adjust the arc current or arc impedance according to specified reference values (Billings *et. al.*, 1979).

Some electrode position controllers used in the 1970's made use of a well known Ward Leonard drive to position the electrodes (Billings and Nicholson, 1977). In this paper a mathematical model of an electric arc furnace with a Ward-Leonard drive in the controller is used to investigate the control of a three-phase electric arc furnace.

Most modern EAFs use newly designed hydraulic systems to move the electrodes and although Ward Leonard drives are not commonly used anymore it can still be used to compare different control strategies.

Section 2 of this paper introduces the model for the electrical energy input of an electric arc furnace. The model used for arc impedance control is different from the model that is used for arc current control. Both these models are described in section 2. Section 3 describes the modelling and simulation of an electrode position controller that makes use of a

Ward-Leonard drive to change the position of the electrodes. In section 4 industrial data are used to do system identification of the EAF. The data that were used for the system identification comes from a plant that uses more modern techniques than that of the current model discussed.

## 2. MODEL DESCRIPTION

The EAF electrical input is supplied from a furnace transformer which in our case is three-phase. Each of the three phases serves as a power input to one of the three electrodes.

Depending on the control strategy, an electrode position controller might be necessary for each of the electrodes. Single phase modelling is thus needed for realistic simulation. A single phase representation for the electrical energy input to the EAF is shown in figure 1 (Billings and Nicholson, 1975).

The furnace transmission system from the power generation system to the arcs is described by the following equations (Billings and Nicholson, 1975):

$$I_1 = \frac{E(h'Z_{t2} - Z_{t3})}{\left(\sum_{k,l=1}^3 Z_{tk}Z_{tl}\right)} \quad (1)$$

$$I_2 = \frac{E(Z_{t3} - h'^2Z_{t1})}{\left(\sum_{k,l=1}^3 Z_{tk}Z_{tl}\right)} \quad (2)$$

$$I_3 = \frac{E(h'^2Z_{t1} - h'Z_{t2})}{\left(\sum_{k,l=1}^3 Z_{tk}Z_{tl}\right)}, \quad k \neq 1 \quad (3)$$

$$z_{tk} = (R_{ak} + R_{ck}) + jX_k = R_{tk} + jX_k \quad (k = 1,2,3) \quad (4)$$

where  $Z_{tk}$  is the total phase impedance referred to the transformer secondary windings,  $R_{ak}$  represent arc resistance,  $R_{ck}$  is the system line resistance,  $X_k$  is the line reactance,  $E$  is the line voltage and  $h'$  is a complex three-phase operator.

The current magnitudes can then be linearised with a first order Taylor series expansion with

$$I_k = I_k^o + i_k \quad (5)$$

and

$$R_{ai} = R_{ai}^o + r_{ai} \quad (6)$$

Assuming zero interaction between the three arc resistances and an infinitely stiff supply voltage

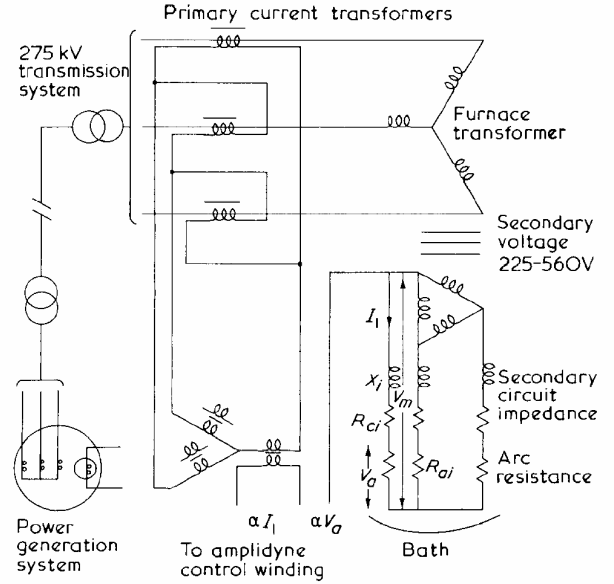


Fig. 1. Electrical power supply system (single phase) (Billings and Nicholson, 1977)

gives the arc current, arc resistance relationships as follow with  $k \neq l$  (Billings and Nicholson, 1975):

$$\frac{i_1}{r_{a1}} = \frac{(-I_1^o)^3 [(R_{t2} + R_{t3}) (\sum_{k,j=1}^3 (R_{tk}R_{tj} - X_kX_j)) + (X_3 + X_2) (\sum_{k,j=1}^3 R_{tk}X_j)]}{E^2 [(-R_{t3} - R_{t2}/2 - \sqrt{3}X_2/2)^2 + (\sqrt{3}R_{t2}/2 - X_2/2 - X_3)^2]} \quad (7)$$

$$\frac{i_2}{r_{a2}} = \frac{(-I_2^o)^3 [(R_{t2} + R_{t3}) (\sum_{k,j=1}^3 (R_{tk}R_{tj} - X_kX_j)) + (X_1 + X_3) (\sum_{k,j=1}^3 R_{tk}X_j)]}{E^2 [(R_{t3} - R_{t1}/2 - \sqrt{3}X_1/2)^2 + (X_3 - X_1/2 + \sqrt{3}R_{t1}/2)^2]} \quad (8)$$

$$\frac{i_3}{r_{a3}} = \frac{(-I_3^o)^3 [(R_{t2} + R_{t3}) (\sum_{k,j=1}^3 (R_{tk}R_{tj} - X_kX_j)) + (X_1 + X_2) (\sum_{k,j=1}^3 R_{tk}X_j)]}{E^2 [4(-R_{t1} + \sqrt{3}X_1 + R_{t2} + \sqrt{3}X_2)^2 + (-X_1 - \sqrt{3}R_{t1} + X_2 - \sqrt{3}R_{t2})^2]} \quad (9)$$

or

$$i_k = -F_k r_{ak} \quad (10)$$

The arc discharge model for the system can be found by using Nottingham's equation that relates arc voltage and arc length.

$$V_{ak} = A_k + D_k H_k + \frac{C_k + B_k H_k}{I_k^n} \quad (11)$$

$H_k$  is the effective arc length and  $D_k$  is the arc-discharge coefficient. The arc-discharge coefficient is a function of the ambient arc temperature.

A linearised version of the voltage measured at the transformer secondary terminal provides one of the components for the error current used as controller input and can be expressed as follows:

$$v_{mk} = i_k |Z_{tk}| + r_{ak} I_k^o (R_{ak} + R_{ck}) |Z_{tk}|^{-1} \quad (12)$$

where

$$|Z_{tk}| = [(R_{ak} + R_{ck})^2 + X_k^2]^{1/2} \quad (13)$$

$$I_k = I_k^o + i_k \quad (14)$$

$$R_{ak} = R_{ak}^o + r_{ak} \quad (15)$$

Using equation (10) gives

$$i_k = \frac{F_k D_k h_k}{F_k R_{ak}^o - I_k^o} \quad (16)$$

or

$$i_k = -(WD)_k h_k \quad (17)$$

where  $WD$  is constant and are usually called the arc gain. After substituting equation (17) into equation (12), the relationship between the change in the measured voltage  $v_{mk}$  and the arc length  $h_k$  is defined by:

$$v_{mk} = D_k h_k [(1 - W_k R_{ck})(R_{ak}^o + R_{ck}) - W_k X_k^2] [Z_{tk}]^{-1} \quad (18)$$

or

$$v_{mk} = D'_k h_k \quad k = 1,2,3 \quad (19)$$

From here models have to be derive separately depending on the type of control method used.

### 2.1 Model description for arc impedance control

Arc-impedance control is based on maintaining the arc-impedance at a constant preset value determined by the tap setting on the transformers secondary terminal. The advantage of this method is that there is none or little interaction between the arc-impedances of the three different phases.

Eliminating  $v_{mk}$  and  $i_k$  from equation (19) and rearranging gives the arc resistance/arc length relationship as follow:

$$r_{ak} = \frac{h_k (D'_k + (WD)_k |Z_{tk}|)}{I_k^o R_{tk} |Z_{tk}|^{-1}} \quad (20)$$

or

$$r_{ak} = B_k h_k \quad k = 1,2,3 \quad (21)$$

Each of the arc resistances is represented as a function only of its associated arc length and the arc characteristics  $D$  and  $D'$ .

In the arc-impedance-controlled model the error signal feedback can be represented by:

$$\varepsilon_k = G_5 i_k - G_4 v_{mk}, \quad k = 1,2,3 \quad (22)$$

where  $G_4$  and  $G_5$  are constants associated with the arc-impedance measuring circuit. Eliminating  $v_{mk}$  and  $i_k$  using equations (17) and (18) gives the three-phase transmission system model when using arc-impedance control and can be presented as follow:

$$\varepsilon_k = -[G_4 D'_k + G_5 (WD)_k] h_k, \quad k = 1,2,3 \quad (23)$$

### 2.2 Model description for arc current control

Current control, where the magnitude of the phase currents are controlled, produces inherent interaction between the three different currents and also between the electrode position controllers. When a disturbance occurs on one of the electrode positions all the arc currents will change and control must be applied to all three phases. In the process all the arc-resistances and the arc-lengths will change. Consequently, linearising equations (7), (8) and (9) using a first-order Taylor series expansion with

$$I_k = I_k^o + i_k \quad (24)$$

and

$$R_{ak} = R_{ak}^o + r_{ak}, \quad k = 1,2,3 \quad (25)$$

give, (for  $k \neq l$ ):

$$i_1 = \frac{E^2 [\alpha_1 r_{a1} + \beta_1 r_{a2} + \gamma_1 r_{a3}]}{I_1^o [(\sum_{k,l=1}^3 R_{tk} R_{tl} - X_k X_l)^2 + (\sum_{k,l=1}^3 R_{tk} X_l)^2]^2} \quad (26)$$

$$i_2 = \frac{E^2 [\alpha_2 r_{a1} + \beta_2 r_{a2} + \gamma_2 r_{a3}]}{I_2^o [(\sum_{k,l=1}^3 R_{tk} R_{tl} - X_k X_l)^2 + (\sum_{k,l=1}^3 R_{tk} X_l)^2]^2} \quad (27)$$

$$i_3 = \frac{E^2 [\alpha_3 r_{a1} + \beta_3 r_{a2} + \gamma_3 r_{a3}]}{I_3^o [(\sum_{k,l=1}^3 R_{tk} R_{tl} - X_k X_l)^2 + (\sum_{k,l=1}^3 R_{tk} X_l)^2]^2} \quad (28)$$

These equations relate the change in arc current to the changes in arc resistances as the latter are adjusted by the electrode position controllers. Eliminating  $r_{a1}$ ,  $r_{a2}$  and  $r_{a3}$  using equation (21) gives the current controlled model as follow:

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \frac{E^2}{\left[ \left( \sum_{k=1}^3 R_k R_{ll} - X_k X_l \right)^2 + \left( \sum_{k=1}^3 R_k X_l \right)^2 \right]} \begin{bmatrix} B_1 \alpha_1 & B_2 \beta_1 & B_3 \gamma_1 \\ I_1^o & I_1^o & I_1^o \\ B_1 \alpha_2 & B_2 \beta_2 & B_3 \gamma_2 \\ I_2^o & I_2^o & I_2^o \\ B_1 \alpha_3 & B_2 \beta_3 & B_3 \gamma_3 \\ I_3^o & I_3^o & I_3^o \end{bmatrix} \quad (29)$$

with  $k \neq l$  where  $\alpha_k$ ,  $\beta_k$  and  $\gamma_k$  are constants.

In the current-controlled model the error feedback is defined as follow:

$$\varepsilon_k = -A_k i_k, \quad k = 1, 2, 3 \quad (30)$$

where the scalar  $A_k$  is chosen such that the error current is initially equal for both current- and impedance controlled regulators.

### 3. THE ELECTRODE POSITION CONTROLLER

Throughout the period of a melt the arc length varies erratically due to scrap movement within the furnace and some form of control is required to maintain the desired power input level. The function of an arc furnace electrode position controller is to maintain a preset arc current or arc impedance by lowering or raising an electrode. Electrode position controllers use the current-voltage reference feedback from the furnace power system to position the electrodes.

The regulator discussed in this paper employ a Ward Leonard drive to position the electrodes. However, regulating systems have successfully been introduced. These systems use modern technology and solid state electronic components but will not be discussed in this paper due to the lack of open literature. Most references on this topic date back to the 1970's.

#### 3.1 Modelling the electrode position controller

An electric arc furnace process consists of three different electrode position controllers, one for each of the three electrodes. All three regulators work on the same basis.

A circuit diagram of an electrode position controller using a Ward-Leonard-drive together with an amplidyne amplifier is shown in fig. 2. Modelling of the electrode position controller is based on modelling each component in the system individually. Fig. 3 shows the block diagram representing the different transfer functions for each component.

In the controller under discussion the error signal acts as the input to an amplidyne rotating amplifier and the output of the amplidyne provides the input to the Ward Leonard drive and the winch system moves the electrode up or down. The arc-impedance measuring circuit compares currents proportional to arc voltage and arc current with a reference value

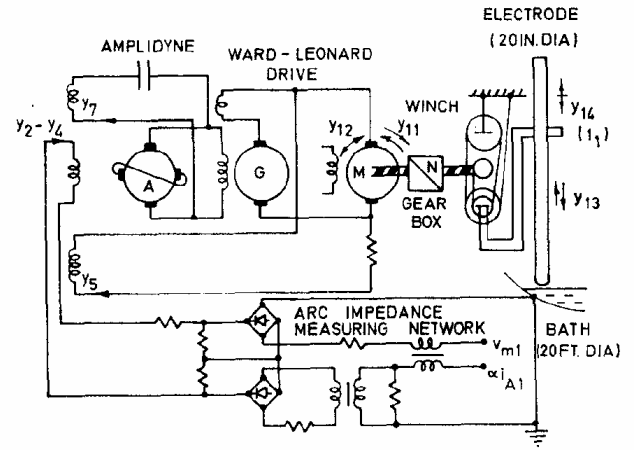


Fig. 2. A single phase electrode position controller (Billings and Nicholson, 1975)

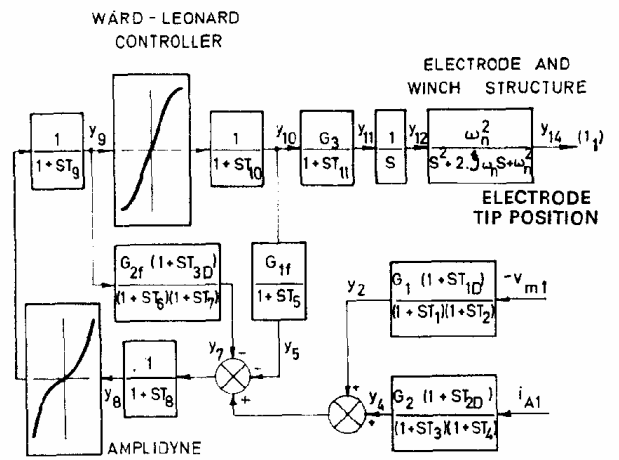


Fig. 3. Block diagram of the electrode position controller (Nicholson and Roebuck, 1972)

and produces an error when they are unequal. The following transfer function, that relates the error current in the amplidyne control winding to the output mast position, is determined from fig.3:

$$g_{kk}(z^{-1}) = \frac{z^{-2T} (0.2498z^{-1} + 0.3079z^{-2} + 0.095z^{-3})}{1 - 3.547z^{-1} + 4.826z^{-2} - 2.9967z^{-3} + 0.7177z^{-4}} \quad (31)$$

The step time for this digital transfer function is usually taken as 1/24 seconds to assure an accurate model for the arc furnace controller.

Two separate arrangements are needed for arc impedance control and arc current control. Figure 1 and figure 2 can simply be combined when the control strategy is based on maintaining the arc impedance at a constant value. Note that this will represent a single phase arrangement as the three different arc impedances do not show any interaction. Figure 4 shows another arrangement where current control is used to maintain the input power at a constant preset value. The three phases, when looking at arc currents, have a fair amount of interaction between them which calls for a combined control strategy.

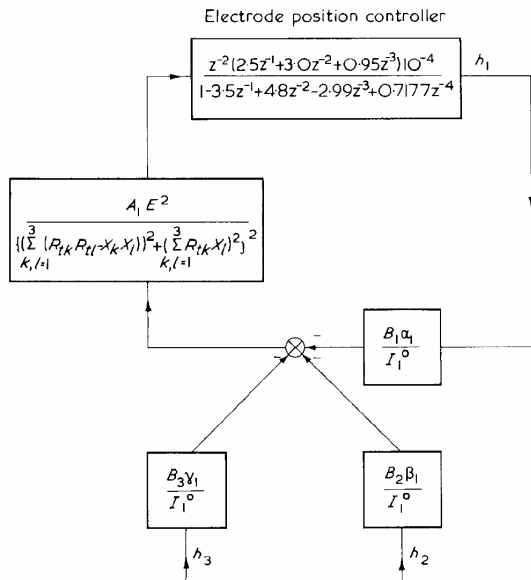


Fig. 4. Block diagram of the current controlled arc furnace (Billings, *et al.*, 1979)

To implement current control for the arc furnace one can combine the model obtained in equation (29) with the electrode position controller model in equation (31). The current- and impedance-controlled models have been formulated assuming equal arc characteristics and electrode-position controller dynamics.

When a disturbance occurs in the arc furnace the electrode position controllers operate in response to an error between the controlled variable and its referenced value to adjust the electrode position and re-establish the desired power input. A performance measure of the electrode-position controller can therefore be based on its ability to re-establish the desired input power while maintaining necessary control constraints.

### 3.2 Simulation results

The models were simulated assuming equal arc characteristics ( $D' = 3940V/m$ ) and electrode-position controller dynamics for each phase. Time steps of  $1/24sec$  were used throughout the simulations.

The responses of the current- and impedance controlled models with a disturbance of 1.25 cm on one of the arc lengths are shown in Fig. 5.

From fig. 5 (Nicholson and Roebuck, 1972) we can clearly see that the current control strategy results in a larger accumulated power discrepancy compared with impedance control. The reason for this is because of the direct interaction between the phase currents. This means that although only one phase is triggered with a disturbance all three electrode controllers act to establish the preset input power.

The ability of the current control strategy to reduce the arc-current deviations in a shorter time than arc

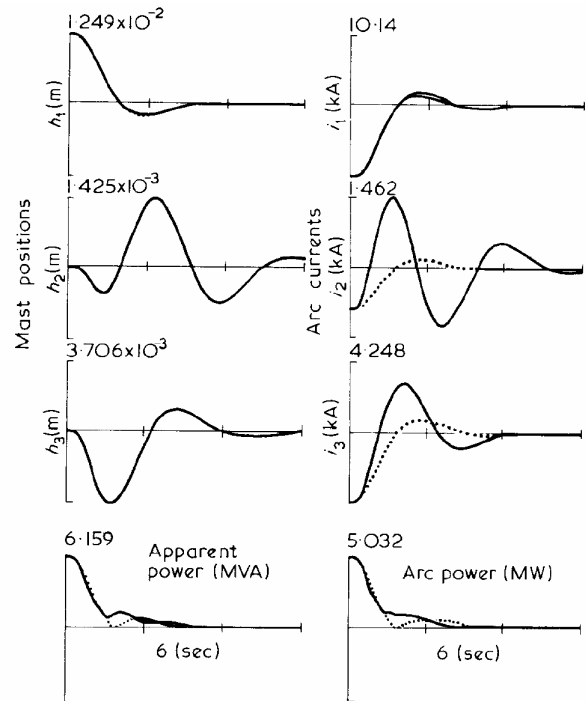


Fig. 5. Simulation results. The solid lines are current control and the dashed lines are impedance control (Nicholson and Roebuck, 1972)

impedance control may be advantageous under short circuit conditions. This results in the possibility to use these two control strategies in a dual method where impedance control is used when the system is operating under normal melting conditions and current control when a short circuit on one of the phases occur.

## 4. SYSTEM IDENTIFICATION ON THE EAF

Measured data from an industrial EAF can be used to verify electrode position control. However, the plant where the data were obtained from does not use a Ward-Leonard drive as part of the electrode position controller.

The industrial data obtained included two hours of recorded data for the arc currents. Live measurements together with reference values were obtained. The data can be used as input and output data for a close loop system identification on the EAF. Data were also obtained for the input power and the voltage measured at the secondary terminals of the furnace transformer. The three parameters mentioned above are directly proportional to each other. Figure 6 shows the set point data (input) and the actual measured data (output) for the first phase of an industrial electric arc furnace during an entire production phase. The data were sampled at 1 second intervals. This gives a total time of 50 minutes of data.

With three different phases to control and assuming inherent interaction between them gives a total of nine transfer functions needed. The system can be represented by the following equation:

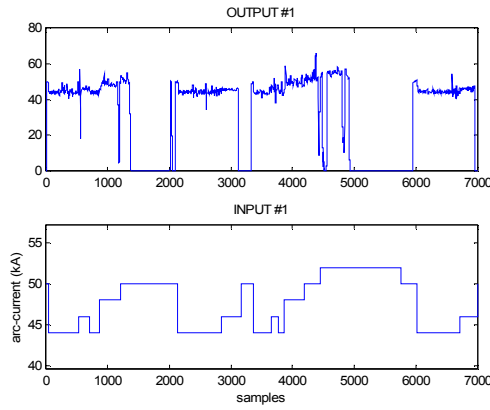


Fig. 6. Set point current and measured current

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} T_{11}(s) & T_{12}(s) & T_{13}(s) \\ T_{21}(s) & T_{22}(s) & T_{23}(s) \\ T_{31}(s) & T_{32}(s) & T_{33}(s) \end{bmatrix} \begin{bmatrix} i_{s1} \\ i_{s2} \\ i_{s3} \end{bmatrix} \quad (32)$$

In this paper however only  $T_{11}(s)$  will be given because the other diagonal transfer functions are similar. The system identification was done by using an ARX (Auto Regression with external input) model. The model for  $T_{11}(s)$  was determined in the following format:

$$T_{11}(s) = \frac{ke^{-is}}{\tau s + 1} \quad (34)$$

where  $k$  is the dc gain and should be one for perfect control,  $i$  is the time delay and  $\tau$  is the time constant.

The following result was obtained:

$$T_{11}(s) = \frac{0.883e^{-5s}}{11.127s + 1} \quad (35)$$

A step response of this model in figure 7 show that the arc current, with electrode control in place, are stable and very close to the set point. This compares favourably with a similar step response in Billings *et. al.*, 1979.

## 5. CONCLUSION

The study of electric arc furnace electrode control has illustrated that electrode arcing can be kept at a constant preset power input when control is applied to the system.

An approach to the problem using modern system identification techniques on an industrial plant for the closed loop has been presented. First order models were obtained for the three phases of the electric arc furnace electrode control system. Step responses show that stable arcing can be achieved when control strategies are applied to the system. With knowledge on the individual components of the arc furnace system one can also compute a mathematical model for the arc-impedance and arc-

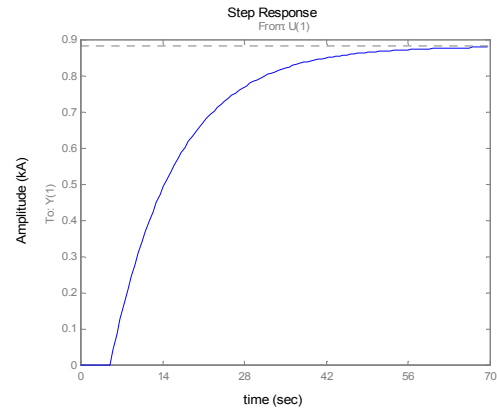


Fig. 7. Step response for  $T_{11}(s)$

current control furnace. These models can be simulated to show the response of the electrode movements when disturbances are applied to the system.

Simulation results show that arc-impedance and arc-current control can achieve effective control for the electrode tip displacement during the production of steel. The differences between these two methods show that arc-impedance control is more efficient during normal control while interacting current control can be more efficient when removing short circuits.

With accurate modelling of the system more efficient control, and hence lower production costs, can be achieved in electric arc furnace steelmaking.

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