EXPLICIT DAMPING FACTOR SPECIFICATION IN SYMMETRICAL OPTIMUM TUNING OF PI CONTROLLERS

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Abstract:

The Symmetrical Optimum tuning proposed by Kessler (1958) and further modified by Voda and Landau (1995) ensures that maximum phase margin is achieved for the resulting closed loop system. The equations for Symmetrical Optimum tuning as defined by Astrom and Hagglund (1995) have recently been improved by Preitl and Precup (1999). In this paper the Preitl and Precup equations for Symmetrical Optimum tuning are further refined to allow explicit specification of the closed loop damping factor. The resulting tuned controller values are applied to position control of a dc servomotor.

Keywords: Damping factor; PI controller; Symmetrical Optimum; Phase margin

1. INTRODUCTION

The adjustable control parameters of PI and PID controllers for a given process can be tuned using a variety of methods such as Ziegler - Nichols, Cohen and Coon, 3C and Symmetrical Optimum. (Pollard, 1971)

The Ziegler – Nichols method sets the controller parameters required for reasonably good performance based on the step response of the open loop system. The response is an exponential curve of a multi-capacitance process and can be characterized by two parameters measured from the response curve.

These are the delay time, L and the maximum slope, N, as a function of the total change in the variable per unit time. The total change in the measured variable K and the maximum slope N are both proportional to the magnitude of the change in the input variable M. (Pollard, 1971)

Cohen and Coon further extended the Ziegler – Nichols method. They used the following transfer function $\frac{K_p e^{-Ls}}{Ts+1}$ to determine the theoretical values of the controller parameters to give reasonable and acceptable responses. (Pollard, 1971)

The Symmetrical Optimum (S.O) tuning method proposed by Kessler (1958) and further modified by Voda and Landau (1995) ensures that the tuned controller produces maximum phase margin for the resulting closed loop system.

The equations for Symmetrical Optimum tuning as defined by Astrom and Hagglund (1995) have recently been improved by Preitl and Precup (1999) to included variable damping. In this paper the Preitl and Precup equations for Symmetrical Optimum tuning are further refined to allow explicit specification of the closed loop damping factor, or in other words this paper proposes a pole placement interpretation of the Symmetrical Optimum method in which the damping factor is explicitly defined as part of the tuning procedure.

The paper defines Symmetrical Optimum tuning in section 2, followed by explicit damping factor specification in section 3. Section 4 deals with the application of the damping factor approach to the dc servomotor and results are given and discussed in section 5. Section 6 is the conclusion. The derivations of equations and tables are given in the appendix.

2. SYMMETRICAL OPTIMUM TUNING

The Symmetrical Optimum controller tuning method is designed to ensure maximum phase margin. As expressed by Astrom and Hagglund (1995) the optimization conditions are as follows

$$2a_0a_2 = a_1^2 \text{ and } 2a_1a_3 = a_2^2 \qquad (1)$$

Preitl and Precup (1999) generalized the equation above by using the parameter β hence

$$\beta^{\frac{1}{2}} a_0a_2 = a_1^2 \text{ and } \beta^{\frac{1}{2}} a_1a_3 = a_2^2 \qquad (2)$$

The additional tuning parameter β that is introduced into the basic Symmetrical Optimum equations effectively sets the damping factor of the closed loop system, as shown in Fig 2.1.

Their research indicated that the parameter β should be chosen to fall in the range 4 to 16, and that three different situations occur:

(1) If $\beta < 9$ two of the three poles produced by the characteristic equation are complex conjugated.
(2) If $\beta = 9$ then all poles are real and equal.
(3) If $\beta > 9$ all poles are real and distinct.

Preitl and Precup state that if $\beta < 4$ the phase margin is very small, being less than 36°, while if $\beta > 16$ the phase margin is greater than 60°. Therefore the domain for β is chosen so as to find the best trade off between performance and the minimum value of the desired phase margin hence the domain [4,16]. Thus the design engineer can change the damping factor by varing the β value.

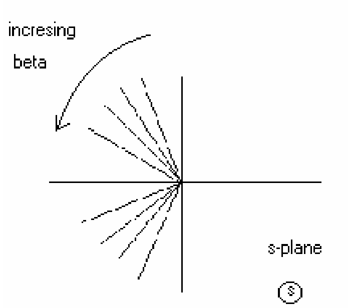

Fig 2.1 Effect of varying beta β on pole position.

3. EXPLICIT DAMPING FACTOR SPECIFICATION

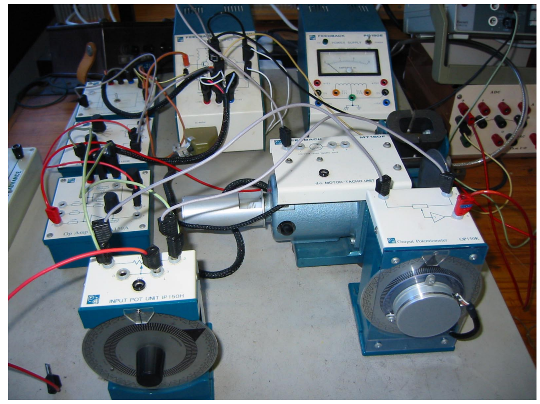

Fig 3.1: The dc servomotor picture

The dc servomotor used in this work is shown in Fig 3.1, and its dynamics can be modeled by the following transfer function

$$H_p(s) = \frac{k_p}{s(1+sT_\varepsilon)} \qquad (3)$$

where k_p is the gain and T_ε is the time constant. The PI controller is chosen for this tuning design and its transfer function is

$$H_c(s) = \frac{k_c(1+sT_c)}{s} \qquad (4)$$

where k_c is the controller gain and T_c is the controller time constant. Consider the unity feedback control loop shown in Fig 3.2.

Its open loop system is then defined by the transfer function

$$H_0(s) = H_c(s)H_p(s)$$

or

$$H_0(s) = \frac{k_ck_p(1+sT_c)}{s^2(1+sT_\varepsilon)} \qquad (5)$$

The closed loop transfer function is given by

$$H_w(s) = \frac{H_0(s)}{(1+H_0(s))}$$

or

$$H_w(s) = \frac{k_pk_c(1+sT_c)}{s^2(1+sT_\varepsilon)+k_pk_c(1+sT_c)} \qquad (6)$$

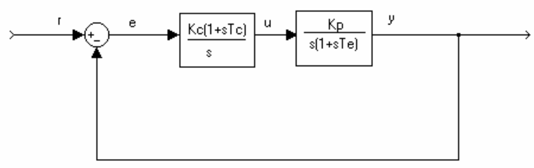

Fig 3.2: Feedback position control

By choosing a $\beta < 9$ condition, two of the three poles produced by the characteristic equation of the closed loop system are complex conjugate and the third is real as show on Fig 3.3

Let the damping factor of the complex mode be defined by ζ

$$\zeta = \cos\theta = \frac{\sigma}{\sqrt{\sigma^2 + \omega^2}} \qquad (7)$$

Rearranging the equation in terms of ω yields

$$\omega = \left(\frac{1-\zeta^2}{\zeta^2}\right)\sigma^2 \qquad (8)$$

The real pole from Fig 3.3 is defined as

$$p = \alpha\sigma \qquad (9)$$

where $\alpha > 1$ meaning that the real pole is always faster than the conjugate pair.

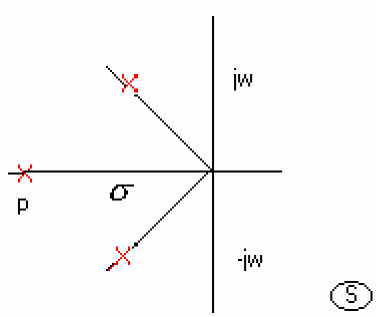

Fig 3.3: Poles position for β less than 9

By comparing the polynomial resulting from the pole position shown in Fig 3.3 and the closed loop characteristic function in equation (6) the tuning formulae can be expressed as

$$T_c = \frac{T_\varepsilon\left(2\alpha\zeta^2 + 1\right)\sigma^2}{k_p k_c \zeta^2} \qquad (10)$$

$$\sigma = \frac{1}{(\alpha + 2)T_\varepsilon} \qquad (11)$$

$$k_c = \frac{\alpha\sigma^3 T_\varepsilon}{k_p \zeta^2} \qquad (12)$$

The derivation of these equations is given in the appendix.

4. APPLICATION TO DC SERVOMOTOR

The transfer function in equation (3) represents the dc servomotor in position control, while equation (4) is the PI controller that is applied to it.

Experiments were done on the dc servomotor to find the value of the parameters k_p and T_ε. The following are the values found from these tests.

$$k_p = 80.87[v/v] \qquad T_\varepsilon = 0.55\,\text{sec} \qquad (13)$$

When used in the tuning equations 10, 11 and 12 the following controller constants were produced:

$$T_c = 3.3\,\text{sec} \qquad k_c = 0.00255[v/v] \qquad (14)$$

In this experiment the damping factor ζ and alpha α are specified, to be 0.7071 and 2 respectively.

4.1 The predicted responses

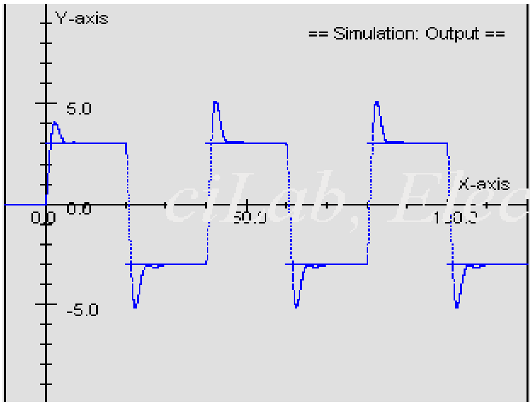

Fig 4.1: The predicted response

Figure 4.1 shows the response predicted from the closed loop transfer function using the values for the motor model and the tuned constants in equation (13) and (14). It shows the following:

The response has approximately 35% overshoot and it takes approximately 8 seconds to settle.

Figure 4.2 shows the predicted input ut, also known as the control output, from the closed loop transfer function using the model values and the tuned constants in equation (13) and (14). The input values are very small (approximately 0.05), and it takes just under 10 seconds to settle.

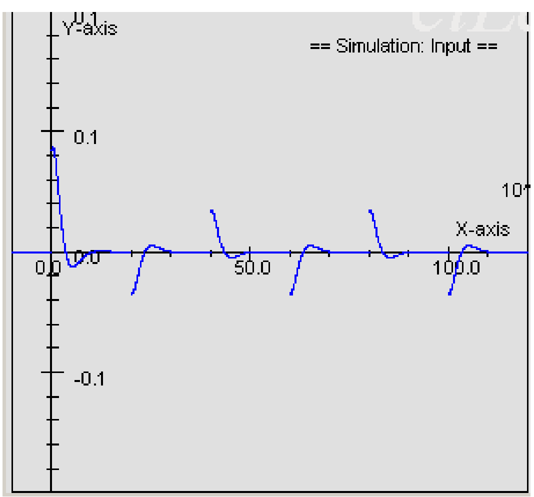

Fig 4.2: The predicted controller output u(t)

4.2 The experimental results

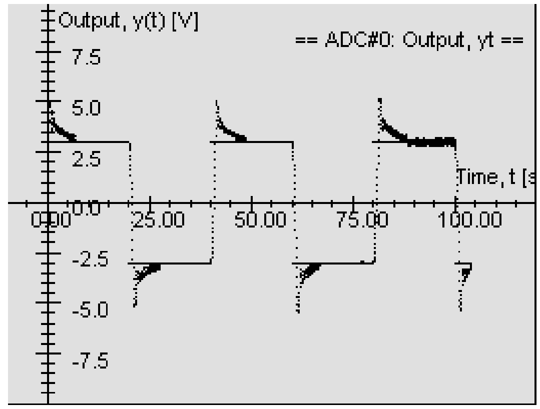

Fig 4.3: The experimental response results

Figure 4.3 shows the actual dc servomotor response to a step input, and it is as predicted. The over shoot is slightly above the predicted value of 35%, while the settling time is almost the same 8 seconds.

The discrepancy between the experimental and predicted responses is attributed to the striction effect of the servomotor system.

Figure 4.4 shows the experimental input, the settling time is faster than expected because of the striction effect of the servomotor. The value for the input is approximately 0.05 as expected.

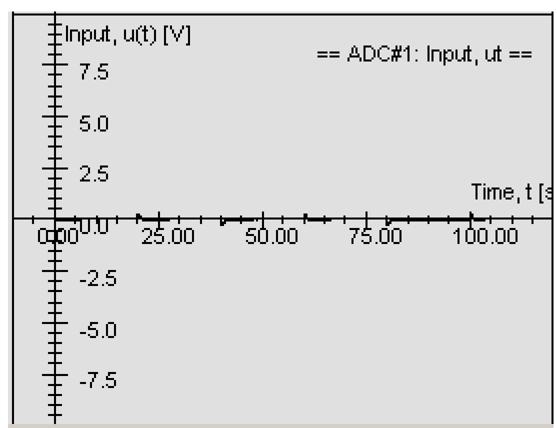

Fig 4.4: The experimental controller output u(t)

5. CONCLUSION

Pole placement interpretation of the Symmetrical Optimum method in which the damping factor is explicitly defined as part of the tuning procedure has been presented.

The main advantages of using this method are that

The PI controller parameters can be tuned by specifying the damping factor ζ , that has more explicit physical meaning than the variable β in equation (2).

The proposed method was applied to a dc servomotor and the results indicate that the specification imposed on the tuning equation were observed in practice when applied to the motor.

6. REFERENCES

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APPENDIX

Derivation of the tuning equations

Preitl and Precup (1999) investigated of the closed loop characteristic function of the third degree

$$a_0 + a_1 s + a_2 s^2 + a_3 s^3 \qquad (14)$$

The optimization conditions according to the SO method are expressed as:

$$2a_0 a_2 = a_1^2 \text{ and } 2a_1 a_3 = a_2^2 \qquad (16)$$

or

$$\beta^{\frac{1}{2}} a_0 a_2 = a_1^2 \text{ and } \beta^{\frac{1}{2}} a_1 a_3 = a_2^2 \qquad (17)$$

Substituting equation 17 into 14 and dividing by the co-efficient of s^3 the following monic polynomial function results

$$s^3 + \beta^{\frac{1}{2}} A_1 s^2 + \beta^{\frac{1}{2}} A_1^2 s + A_1^3 \qquad (18)$$

where $A_1 = \dfrac{a_1}{a_2}$

Choosing a value of β<9 results in the following pole position, one real pole and two conjugate pair. Let the poles positions be represented by the following variables

$$(s+p)(s+\sigma+j\omega)(s+\sigma-j\omega) \qquad (19)$$

$$s^3 + (p+2\sigma)s^2 + (2p\sigma+\sigma^2+\omega^2)s + p(\sigma^2+\omega^2)$$

The real pole can be further defined as

$$p = \alpha\sigma \text{ and } \alpha > 1 \qquad (20)$$

If the damping factor is defined as

$$\zeta = \frac{\sigma}{\sqrt{\sigma^2+\omega^2}} \qquad (21)$$

the equation can be rewritten in terms of ω^2 as shown below

$$\omega^2 = \left(\frac{1-\zeta^2}{\zeta^2}\right)\sigma^2 \qquad (22)$$

Substituting equation 22 and 20 into 19 the monic characteristic equation becomes

$$s^3 + (\alpha+2)\sigma s^2 + \left(2\alpha+1+\frac{1-\zeta^2}{\zeta^2}\right)\sigma^2 s + \left(1+\frac{1-\zeta^2}{\zeta^2}\right)\sigma^3 \qquad (23)$$

The characteristic equation of the closed loop given in equation 6 can be written as

$$\phi_c(s) = s^3 + \frac{1}{T_\varepsilon}s^2 + \frac{k_p k_c T_c}{T_\varepsilon}s + \frac{k_p k_c}{T_\varepsilon} \qquad (24)$$

Comparing the above characteristic equation (23) and (24) yields the tuning formulae

$$T_c = \frac{T_\varepsilon \left(2\alpha\zeta^2 + 1\right)\sigma^2}{k_p k_c \zeta^2}$$

$$\sigma = \frac{1}{(\alpha + 2)T_\varepsilon}$$

$$k_c = \frac{\alpha\sigma^3 T_\varepsilon}{k_p \zeta^2}$$

Table 7.1 shows all the parameters in the tuning of the PI controller using damping factor specification in Symmetrical Optimum. The parameters in row (4) are the ones used in this paper, by varying the damping factor only two parameters are affected namely the controller time constant, T_c and k_c .

Table 7.1: Simulations results for experiment 1: This table shows the tuning of parameters when the damping factor is varied

	k_p	T_ε	ζ	σ	α	T_c	k_c
(1)	80.87	0.55	0.866	0.4545	2	4.400	0.00170
(2)	80.87	0.55	0.819	0.4545	2	4.050	0.00190
(3)	80.87	0.55	0.766	0.4545	2	3.682	0.00217
(4)	80.87	0.55	0.707	0.4545	2	3.300	0.00255
(5)	80.87	0.55	0.643	0.4545	2	2.918	0.00310
(6)	80.87	0.55	0.574	0.4545	2	2.548	0.00390
(7)	80.87	0.55	0.500	0.4545	2	2.200	0.00510