ROBUST QFT-BASED POSITION CONTROL OF ELECTROHYDRAULIC SERVO SYSTEM

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Abstract: This paper presents the design of a robust controller using the Quantitative Feedback Theory technique for an asymmetric hydraulic cylinder electro-hydraulic servo system based upon a linear, parametrically uncertain model in which some of the uncertainties reflect the variation of the parameters, and taking the external disturbance into account. After the derivation of a realistic nonlinear differential equations model, the linearized plant transfer function model is developed. The effects of parametric uncertainty are accounted for. In this paper, the tracking performance index and disturbance attenuation performance index are transformed into the constraints of the parametrically uncertain sensitivity functions respectively using the sensitivity-based QFT technique. From this point, the QFT design procedure is carried out to design a feasible robust controller that satisfies performance specifications for tracking and disturbance rejection. A nonlinear closed-loop system response is simulated using the designed controller. The results show that the robust stability against system uncertainties is achieved and the robust performances are also satisfied.

Keywords: asymmetric electro-hydraulic servo system, uncertain dynamics system, quantitative feedback theory, robust control, position control

1. INTRODUCTION

The single-rod cylinders have been widely used in the electro-hydraulic control systems, due to their some advantages, such as small room occupied, simplicity of structure, and low cost. Because of the complexity of hydraulic system and its corresponding operation environment, the systems are highly nonlinear and subject to parameter uncertainty in large scale. Model parameters change with time as a result of variations in operating conditions and uncertain environment. For example, the supply pressure is subject to fluctuation, which may be caused by the operation of other actuators in a multi-user environment. The flow and pressure coefficients, characterizing fluid flow into and out of the valve, are functions of load and supply pressure and can vary under different operating conditions. The effective bulk modulus in hydraulic systems can significantly change under various load conditions, oil temperature, and air content in the oil (Yu, *et al.*, 1994). So it is necessary to account for these uncertainties in control systems design of the hydraulic servo systems. This paper presents the application of QFT to the design of a robust position controller for the asymmetric electro-hydraulic servo systems.

QFT is a robust controller design theory aimed at plants with parametric and unstructured uncertainties. The theory was first put forward by Horowitz (Horowitz, 1972; Horowitz, 1973; Horowitz, 1991). This method has been applied to many engineering fields, especially in the robust flight control systems (Houpis, *et al.,* 1994; Phillips, *et al.,* 1997). Additionally, Chait et al settled the controller design for a compact disc player using QFT (Chait, *et al.*, 1994). Ismail introduced the application of the QFT for the TBT control of MSF desalination plants (Ismail, 2001). Regarding the application of QFT to the hydraulic systems, Thompson and Kremer developed a QFT controller for a variable-displacement hydraulic vane pump (Thompson, and Kremer, 1997). The simulation results were reasonable and satisfactory. The objective of this paper is to use QFT to settle the controller design for the position control of the electro-hydraulic servo system with the parametric uncertainties and disturbances.

2. ASYMMETRIC ELECTRO-HYDRAULIC SERVO SYSTEM MODELING

A schematic diagram of the asymmetric electro-hydraulic servo system controlled by servo-valve is shown in Fig.1. In this section, we derive the nonlinear differential equations model of the asymmetric electro-hydraulic servo system and further the linearized plant transfer function, which is fit for QFT design with the parametric uncertainties.

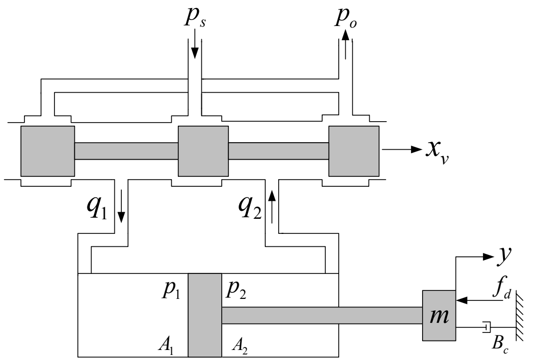

Fig.1 Schematic diagram of the asymmetric electro-hydraulic servo system controlled by servo-valve

2.1 Nonlinear dynamic equations

The governing nonlinear equations describing the fluid flows through the valve orifices are written as (Merritt, 1967):

$x_v \geqslant 0$ (extension)

$$q_1 = c_d w x_v \sqrt{\tfrac{2}{\rho}\left(p_s - p_1\right)} \tag{1}$$

$$q_2 = c_d w x_v \sqrt{\tfrac{2}{\rho}\left(p_2 - p_0\right)} \tag{2}$$

$x_v < 0$ (retraction)

$$q_1 = c_d w x_v \sqrt{\tfrac{2}{\rho}\left(p_1 - p_0\right)} \tag{3}$$

$$q_2 = c_d w x_v \sqrt{\tfrac{2}{\rho}\left(p_s - p_2\right)} \tag{4}$$

where q_1 and q_2 denote fluid flows into and out of the servo-valve, respectively. c_d represents the orifice coefficient of discharge. w represents the area gradient that relates the spool displacement x_v to the orifice area. ρ represents the mass density of the fluid. p_s , p_0 , p_1 , p_2 represent supply pressure, return line pressure, head side pressure and rod side pressure of hydraulic cylinder, respectively.

Continuity equations for oil flow through the cylinder, neglecting the leakage flow across the piston, are

$$q_1 = A_1 \dot{y} + \frac{V_1}{\beta_e} \dot{p}_1 \tag{5}$$

$$q_2 = A_2 \dot{y} - \frac{V_2}{\beta_e} \dot{p}_2 \tag{6}$$

where A_1 and A_2 are the piston effective areas. y is the piston displacement. β_e is the effective bulk modulus of the hydraulic fluid, while V_1 and V_2 are the volumes of the fluid trapped at the sides of the piston. The relationship between them and the piston displacement can be described as

$$V_1 = V_{01} + A_1 y \tag{7}$$

$$V_2 = V_{02} - A_2 y \tag{8}$$

where V_{01} and V_{02} are the initial volumes trapped in the head and rod sides chamber.

Applying Newton's second law to the forces on the piston, neglecting the nonlinear friction forces and the mass of oil, the force equation is

$$A_1 p_1 - A_2 p_2 = m\ddot{y} + B_c \dot{y} + f_d \tag{9}$$

where m denotes the total mass of the piston and payload. B_c is the viscous damping coefficient of piston and load. f_d is arbitrary external load force acted on the piston.

As for the servo-valve, it can be considered as first order system

$$u = \frac{1}{k_v}\left(\tau \dot{x}_v + x_v\right) \tag{10}$$

where u , k_v and τ denotes the input voltage, gain and time constant of the valve respectively.

Up to now, Equations (1)~(10) compose the nonlinear dynamic model of the hydraulic system which we study.

2.2 Linearized transfer function model

In the previous section, the nonlinear dynamic equations are derived. Now the linearized model with the variation of operating point dependent parameters described as uncertainties, which is fit for QFT design, can be obtained based on an operating point.

As for the fluid flow equations of the servo-valve, the linearized equations are

$$q_1 = k_{q1} x_v - k_{c1} p_1 \tag{11}$$

$$q_2 = k_{q2} x_v + k_{c2} p_2 \tag{12}$$

here k_{q1} and k_{q2} , k_{c1} and k_{c2} denote the flow and pressure gains, respectively. Their representations are

$$k_{q1} = c_d w \sqrt{\tfrac{2}{\rho}\left(p_s - p_1\right)} \quad k_{q2} = c_d w \sqrt{\tfrac{2}{\rho}\left(p_2 - p_0\right)}$$

$$k_{c1} = \frac{c_d w x_v}{\sqrt{2\rho\left(p_s - p_1\right)}} \quad k_{c2} = \frac{c_d w x_v}{\sqrt{2\rho\left(p_2 - p_0\right)}} \quad x_v \geq 0$$

and

$$k_{q1} = c_d w\sqrt{\tfrac{2}{\rho}(p_1 - p_0)} \quad k_{q2} = c_d w\sqrt{\tfrac{2}{\rho}(p_s - p_2)}$$

$$k_{c1} = \frac{-c_d w x_v}{\sqrt{2\rho(p_1 - p_0)}} \quad k_{c2} = \frac{-c_d w x_v}{\sqrt{2\rho(p_s - p_2)}} \quad x_v < 0$$

Additionally, within the vicinity of the mid-stroke, the assumption can be made

$$\frac{V_1(y)}{\beta_e} \approx \frac{V_2(y)}{\beta_e} \approx \frac{1}{\beta_e}\left(\frac{V_{01}+V_{02}}{2}\right) = \gamma$$

Thus, equations (5), (6) can be written as

$$q_1 = A_1\dot{y} + \gamma\dot{p}_1 \tag{13}$$

$$q_2 = A_2\dot{y} - \gamma\dot{p}_2 \tag{14}$$

From equations (9)~(14), we can obtain the linearized model in the Laplace domain

$$Y(s) = P_1(s)U(s) - P_2(s)F_d(s) \tag{15}$$

where

$$P_1(s) = \frac{k_v\left[A_1 k_{q1}(\gamma s + k_{c2}) + A_2 k_{q2}(\gamma s + k_{c1})\right]}{s(\tau s+1)\left[(ms+B_c)(\gamma s + k_{c1})(\gamma s + k_{c2}) + A_1^2(\gamma s + k_{c2}) + A_2^2(\gamma s + k_{c1})\right]} \tag{16}$$

$$P_2(s) = \frac{(\gamma s + k_{c1})(\gamma s + k_{c2})}{s\left[(ms+B_c)(\gamma s + k_{c1})(\gamma s + k_{c2}) + A_1^2(\gamma s + k_{c2}) + A_2^2(\gamma s + k_{c1})\right]} \tag{17}$$

To simplify the above transfer function, k_{q1} and k_{q2}, k_{c1} and k_{c2} are replaced by k_q, k_c, respectively. Hence transfer function (16) and (17) are reduced to (18), (19)

$$P_1(s) = \frac{k_v k_q(A_1 + A_2)}{s(\tau s+1)\left[(ms+B_c)(\gamma s + k_c) + A_1^2 + A_2^2\right]} \tag{18}$$

$$P_2(s) = \frac{(\gamma s + k_c)}{s\left[(ms+B_c)(\gamma s + k_c) + A_1^2 + A_2^2\right]} \tag{19}$$

In the above two equations, the uncertainties of k_q and k_c denote the variation of the supply pressure, the operating point and the orifice area gradient of the servo-valve. The uncertainty of γ denotes the variation of the effective bulk modulus of the hydraulic fluid and the volumes of the fluid trapped at the sides of the piston. While the uncertainty of the servo-valve dynamics can be denoted by the variation of the parameter τ.

These uncertain parameters stack in a vector, denoted as α. Then the open-loop transfer function of the system can be written as

$$Y(s,\alpha) = P_1(s,\alpha)U(s) - P_2(s,\alpha)F_d(s) \tag{20}$$

The open-loop Bode plots of the plant set $P_1(s,\alpha)$ are shown in Fig.2.

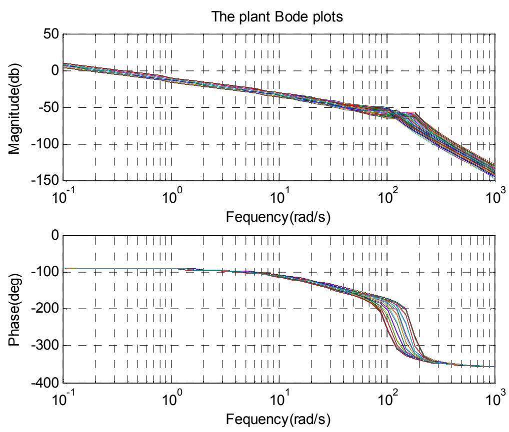

Fig.2 The open-loop Bode plots of the plant set $P_1(s,\alpha)$

3. CONTROLLER SYNTHESIS

The objective of this section is to design a robust position controller for the system that is represented by the uncertain transfer function (20). A typical two-degree-of-freedom feedback system configuration in QFT is shown in Fig.3. A proper controller, $G(s)$, and a proper prefilter, $F(s)$, are to be designed such that the following conditions are satisfied.

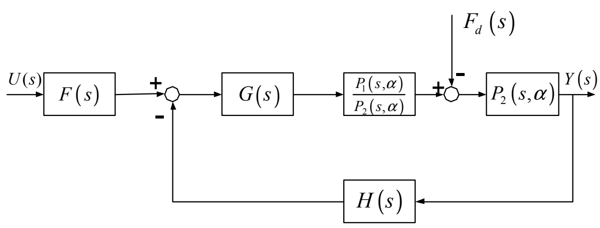

Fig.3 Two-degree-of-freedom QFT feedback control system

1) Closed-loop robust stability

$$\left|\frac{PG}{1+PG}\right| \le 1.3, \quad \forall \omega \in [0,\infty), \quad (21)$$

The above stability requirement implies an approximately 2.3dB gain margin for the closed-loop system.

2) Robust tracking performance

$$|T_l(j\omega)| \le |T_{ry}(j\omega,\alpha)| \le |T_u(j\omega)|, \quad \forall \omega \in [0,\infty) \quad (22)$$

where $T_{ry}(s,\alpha) = \dfrac{F(s)P(s,\alpha)G(s)}{1+P(s,\alpha)G(s)}$, and the upper and lower tracking bounds are defined as

$$T_u(s) = \frac{5s+150}{s^2+20s+150},$$

$$T_l(s) = \frac{1000}{(s+30)(s^2+10s+100/3)}.$$

The determination of these bounds of the closed-loop tracking frequency domain performance does not have the uniform theory. But these bounds can be defined by the time domain performances. Such as peak overshoot and settling time etc. of the system step responses. The specific procedure can be referred to the literature (D'Azzo, and Houpis, 1995). In this paper, the settling time is not more than 0.6s corresponding to $T_l(s)$. The overshoot is not more than 2% corresponding to $T_u(s)$.

3) Closed-loop disturbance attenuation

As for the disturbance attenuation at the plant output, the corresponding performance specification can be embodied by the following inequality

$$\max_{\alpha\in\Omega}\left|\frac{P_2(j\omega,\alpha)}{1+P_1(j\omega,\alpha)G(j\omega)}\right| \le w_d(\omega) \quad \forall \omega \in [0,\infty) \quad (23)$$

where

$$w_d(\omega) = 2.0(10^{-7})\left|\frac{(j\omega)^3+60(j\omega)^2+750(j\omega)+2400}{(j\omega)^2+15(j\omega)+170}\right|$$

The above design specifications can be transformed into the constraints of the loop transfer function $L_0(s) = P_{10}(s)G(s)$. These constraints shown on a Nichols chart compose the so-called QFT bounds. In the initial design, $G(s)$ can be simply evaluated by 1. Of course, it can also be obtained from other control theory, such as H_∞ method (Zhao, and Jayasuriya, 1998). In the process of loop shaping, $L_0(s)$ should satisfy these bound constraints. Then the controller can be extracted from $L_0(s)$ by dividing by the nominal plant transfer function, $P_{10}(s)$ which should be kept invariant in the design process. Design frequencies are chosen as

$$\omega = [0.1, 1, 3, 5, 10, 60, 80, 100, 130, 180, 200, 300].$$

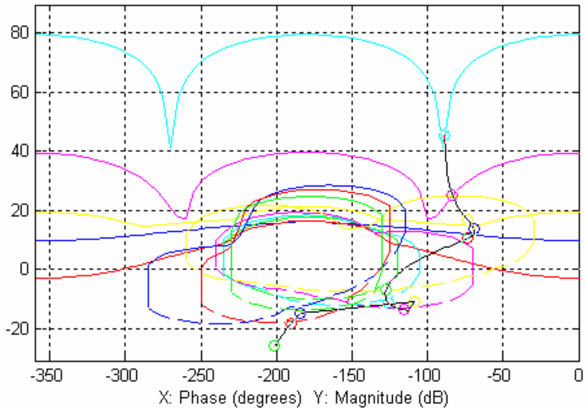

Fig.4 QFT bounds and system loop transfer function

The bounds generated by constraints (21), (22), (23) and the final loop shaping of the system are shown in Fig.4. In the process of loop shaping, $L_0(s)$ should lie on or above the bounds to satisfy the bounds at low frequencies, while for higher frequencies $L_0(s)$ should not enter the closed boundaries generated by the constraints. The controller that satisfies the specifications is

$$G(s) = \frac{73.2\left(\dfrac{s}{4.9}+1\right)\left(\dfrac{s^2}{118.6^2}+\dfrac{0.68s}{118.6}+1\right)}{\left(\dfrac{s}{17.63}+1\right)\left(\dfrac{s^2}{512.1^2}+\dfrac{0.42s}{512.1}+1\right)}$$

To satisfy the tracking specification, a prefilter is to be designed to place the closed-loop frequency response between $T_u(s)$ and $T_l(s)$. So the suitable prefilter is designed to be

$$F(s) = \frac{13.59\left(\dfrac{s^2}{1000^2}+\dfrac{s}{1000}+1\right)}{\left(\dfrac{s}{8.54}+1\right)\left(\dfrac{s^2}{34.76^2}+\dfrac{1.41s}{34.76}+1\right)\left(\dfrac{s^2}{1000^2}+\dfrac{0.176s}{1000}+1\right)}$$

4. CLOSED-LOOP ANALYSIS

The aforementioned controller design is based on the finite frequency points. So it is necessary to verify through analysis whether it can guarantee to satisfy the specifications in total operating points of the system or not. The analysis results for the gain margin, the tracking performance and the disturbance attenuation are shown in Fig.5, Fig.6 and Fig.7, respectively. The step responses of closed-loop control system are shown in Fig.8. As can be seen, for all cases pertaining to extreme parts of the operating envelope, the specifications are satisfied.

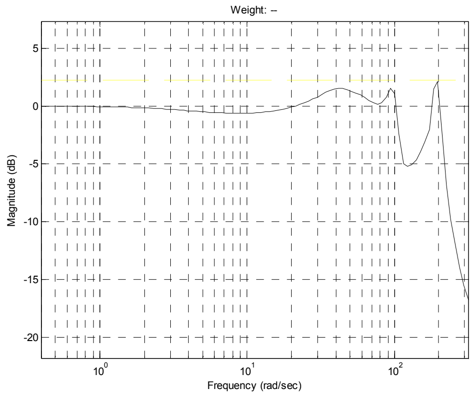

Fig.5 The analysis curves of stability

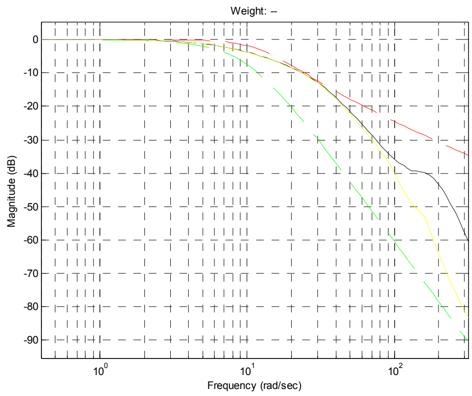

Fig.6 The analysis curves of tracking performance

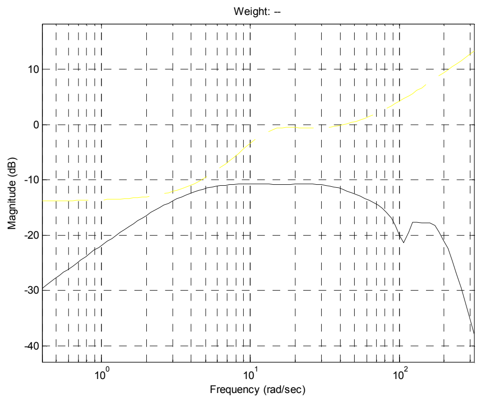

Fig.7 The analysis curves of disturbance attenuation

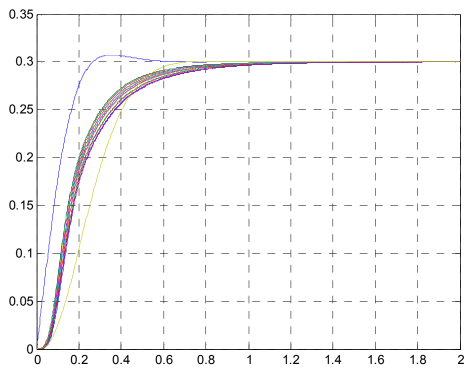

Fig.8 The step responses of closed-loop system

5. CONCLUSION

This paper has described the application of the QFT method to the development of a position controller for the electro-hydraulic servo system. A linear fourth order model with parametric uncertainties was obtained to describe the relationship between the control signal and the position of piston. A robust position controller was designed using QFT method that, along with a reasonable prefilter, maintains a satisfactory position control performance against the model parametric uncertainties and the external disturbance. The results show that the designed controller is effective and feasible to the electro-hydraulic servo system.

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