Robust Nonlinear Controls for Two Problems of Rejecting Disturbances

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***Abstract*— In this paper we present the design of tow robust nonlinear controllers for two mechanical problems of rejecting disturbances such as eccentricity and friction compensation. In the eccentricity compensation problem, the considered disturbances are here assumed to be produced by eccentricity in mechanical systems and drives, the frequency and amplitude of this disturbance is unknown. The objective of this paper is the rejecting disturbances, this disturbances are bounded but the bound are unknown. These methods ensure asymptotic tracking of eccentricity and friction compensation which are problems of rejecting disturbances in both cases and the global asymptotic tracking is ensured.**

Keyword : Eccentricity compensation, friction compensation, sliding mode control, Nonlinear PI control.

I. Introduction

In the last years the eccentricity and friction compensation problems were largely studied in the literature. Several references can be founded in [7] [8] [9]. Recently Canudas de Wit and Praly [6] have proposed an adaptive controller to eccentricity problem. They assumed that disturbance depends on position. This scheme is based on an internal model which describes the perturbation dynamic. This dynamic is estimed by using an adaptive observer.

In the other hand, Canudas de Wit and Linschinsky [8] presented an adaptive controller to friction compensation problem. This work is also based of the estimation of the disturbance by an adaptive observer.

In this paper we present two new solutions to the above problems. The first one is based on a new approach developed by Astolfi and Ortega [13] and called nonlinear PI (Proportional Integrator). This controller are a generalization of the PI linear to nonlinear systems. The major advantages of this method is the global asymptotic stability and tracking of some nonlinear parameterized systems. The second solution is based on sliding mode theory. The originality of this approach is the use of an adaptive law in order to estimate the disturbance bounds which are assumed unknown. In both cases the asymptotic stability is proved by using Lyapunov theory.

The paper is organized as follow. Section 2 we describe the eccentricity and friction problems. In the 3^{rd} section we present the nonlinear PI approach and applied it the to our systems. Section 4 present the sliding mode controllers applied it the two our systems and in the section 5 we show some numerical simulations and in the last section we conclude by some comments.

II. Uncertain mechanical systems

In this section we present two examples of uncertain mechanical systems;

A. Eccentricity compensation Problem

Let us consider the two dimensional rotational system described by the following equations [7]

$$\left\{ \begin{array}{lcl} \dot{y} & = & v \\ J\dot{v} & = & u + d(y) \end{array} \right. \quad (1)$$

Where y is the system angular position, J is the inertia, u is the control input and $d(y)$ is the position-dependent oscillatory disturbance defined as

$$d(y) = a\cos(by + c) \quad (2)$$

where the angular position y and velocity $\dot{y}$ are mesurable. We suppose that a,b,c are unknown positive parameters. The control objective here is to compensate the effect of the disturbance on the system described below. This disturbance results from eccentricity in many mechanical systems where the center of rotation does not corresponds with its geometric center. The dependency of $d(y)$ on position is caused by the fact that the friction forces depends on the normal force acting between two surfaces. In accuracies in the geometric position of the rotating axis of a rolling mill (eccentricity), will produce position dependant disturbance. In gear boxes, friction will vary as a function of the effective surface in contact with the gearś teeth. The two dimensional rolling and spinning frictions causes in ball bearings the frictional torque to be dependent on both position and velocity. Fig.1 shows some of these examples.

Many of the existing works consider d not as a position function, but as a time-dependent exogenous signal, of the form [2]

$$d(t) = acos(bt + c) \quad (3)$$

In the previous mentioned systems this hypothesis is only valid if we assume that the system is operating and regulated, at constant velocity v_d so that x(t) becomes proportional $t.v_d$. Disturbances of the forme (3) have been considered in problems such as active noise and vibration control. the noise $d(t)$ is thus assumed to be generated by the rotating machinery and transmitted through the sensor path The problem is the design of controller which ensures $\dot{y}$ will asymptotically track the bounded reference trajectory $\dot{y}_r(t)$ with known bounded first derivative.

The figure (1.a) shows an exemple of a systems where the shear force acting on the surface of contact may vary as a function of the joint angle positions, due to eccentricity on the axis of rotation if $r_2 \neq r_1$. In the fig.1b the geometric axis of the cylinder does note coincide with the axis of rotation due to unbalanced masses distribution. in the fig.1c the variation on the effective area of contact is due to the relative position of the gears teeth and fig1.d shows the spinning and rolling resistances induce a variation of a effective area, this variation as a function of the inner race position.

B. Friction compensation problem

Frictions is a nonlinear phenomenon that causes the performance of servomechanisms to deteriorate, such as in the case of robotics and machine tools. Typical errors caused by friction are steady- state errors in position regulation and tracking lags. Model- oriented friction compensation techniques are based on the knowledge of a suitable friction model that predicts the real friction and commands an opposed control action to compensate it.
The dynamic model of friction turns out to be suitable for the design of model-based friction compensation schemes. The successful application of these ideas relies on the quality of the estimated friction parameters. These parameters are difficult to estimate since they appear in the model in a nonlinear fashion [8].
Consider a DC motor with friction given by

$$\left\{\begin{array}{rcl} J\ddot{y} & = & -F(\dot{y}(t),z)+u \\ F(\dot{y}(t),z) & = & \sigma_0 z+\sigma_1 z+\sigma_2\dot{y} \\ \dot{z} & = & \dot{y}-\frac{\sigma_0}{g(\dot{y})}z|\dot{y}| \\ g(\dot{y}) & = & a_0+a_1 e^{-(\frac{\dot{y}}{a_2})^2} \end{array}\right. \tag{4}$$

Where $J(kgm^{-2})$ is the total motor and load inertia, $y(rad)$ is the motor shaft angular position, $u(N/m)$ is the DC motor torque and $F(Nm)$ is the friction torque.
F in (4) is the friction torque given by the dynamics of the friction internal state z, which describes the average deflection of the contact surfaces during the stiction phases. This state is note measurable. The function $g(\dot{y})$ $(\infty > a_0+a_1 \geq g(\dot{y}) \geq a_0 > 0)$ describes part of the 'steady state' characteristics of the model for constant velocity motions, including Stribeck velocity $a_2(rad/sec)$, static friction $a_0+a_1(Nm)$ and coulomb friction a_0.

The position y and speed $\dot{y}$ is measurable, and the friction force F is an unknown continuous function satisfying, for all $t \geq 0$, the bound

$$|F(\dot{y}(t),z)| \leq M(1+|\dot{y}(t)|) \tag{5}$$

with M a known positive parameter.

III. Nonlinear PI Controller

In this section, we will present the approach of nonlinear PI controller.
Given the following simplest scalar system :

$$\dot{y} = \phi(y)+u, \tag{6}$$

where $\phi(y)$ is an unknown continuous function that ranges in the interval $0 < \phi(y) < 1$
First, we select some desired dynamics that we would like to impose to our system, in this case we choose $\dot{y} = -\lambda y$, with $\lambda > 0$. Defining the signal

$$z = \beta_P(y)+\beta_I$$

The nonlinear PI controller has the following form

$$\left\{\begin{array}{rcl} u & = & \beta(\beta_P(y,y_r)+\beta_I,y,y_r) \\ \dot{\beta}_I & = & \omega_I(y,y_r) \end{array}\right. \tag{7}$$

The main result of this approach is summarized in the following propositions

Proposition 1 [13]

Consider the n-dimensional single-input linear time-invariant system with matched uncertainty

$$\dot{x} = Ax+B[u+\phi(x)]$$

with the following assumptions.

- *There exists a linear output map $y = C^T|x|$ such that $C^TB \neq 0$ and we know C^TB and a bound on $|C^TA|$.Moreover $|y(t)|$ bounded implies $x(t)$ bounded .*
- *The additive uncertainty $\phi(x)$ satisfies, uniformly in x, the bound $|\phi(x)| \leq \phi_M$.*

Then, the system in closed loop with a nonlinear PI controller with parameters

$$\left\{\begin{array}{rcl} \beta & = & \frac{1}{CB}(1+||x||)(-y+M-\frac{2M}{1+e^{-z/y}}) \\ \beta_P & = & \frac{1}{2}y^2 \\ \omega_I & = & y^2(1+||x||) \end{array}\right. \tag{8}$$

where $M \geq |C^TA|+|C^TB|\phi_M$ is such that, for all initial conditions $(x(0),\beta_I(0))$,

$$lim_{t\to\infty}y(t) = 0$$

with all signals bounded

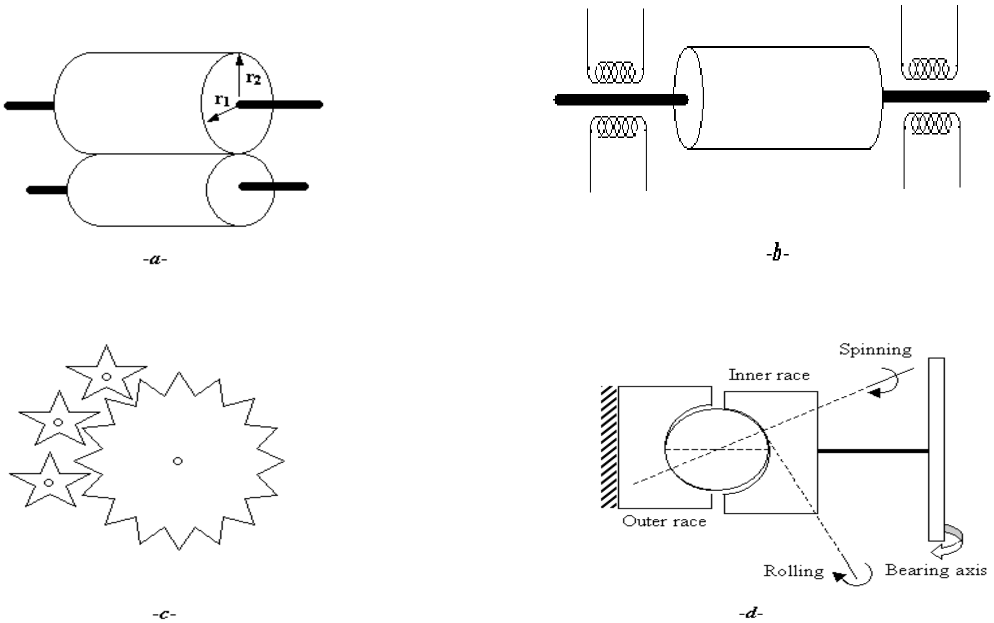

Fig. 1. Examples of systems where disturbances are position-dependent and may produce eccentricity. (a): Asymmetries on the rotation axes, (b): magnetic bearing system, (c): Different position of gears teeth, (d): Cross section of a ball bearing assembly

Proposition 2 [13]

Consider the scalar system (6), where $\phi(y)$ verifies $|\phi(y)| \leq M(1+|y|+|y|^n)$, in closed loop with a nonlinear PI controller with parameters

$$\left\{ \begin{array}{ccc} \beta & = & M(1+|y|+|y|^N)(-y+1-\frac{2}{1+e^{-z/y}}) \\ \beta_P & = & \frac{1}{2}y^2 \\ \omega_I & = & -y^2(1+|y|+|y|^N) \end{array} \right. \quad (9)$$

Then, for all initial conditions $(y(0), \beta_I(0))$

$$lim_{t \longrightarrow \infty} y(t) = 0$$

with all signals bounded.

A. Application to Eccentricity compensation problem

Let us consider the system (1), (2). The idea is to find β such that

$$\ddot{y} = \ddot{y}_r(t) - \lambda(\dot{y} - \dot{y}_r).$$

Let us set

$$e = \dot{y} - \dot{y}_r(t)$$

and

$$u = \ddot{y}_r(t) + v$$

Then, the system will written as follows

$$\dot{e} = v + a\cos(by + c).$$

Defining a signal z such that

$$z = \beta_P(e) + \beta_I$$

then

$$\dot{z} = \dot{\beta}_I + \frac{\partial \beta_P}{\partial e}(v + a\cos(by + c))$$

and considering that

$$\left\{ \begin{array}{ccc} v & = & \beta(e,z) \\ \dot{\beta}_I & = & \omega_I \end{array} \right.$$

The uncertain term is bounded, and the system is second order, hence Proposition 1 applies. Then if we choose

$$\left\{ \begin{array}{ccc} \beta(e,z) & = & \ddot{y}_r(t) - \gamma(e + zcosz) \\ \beta_P & = & e \\ \omega_I & = & \gamma e \end{array} \right.$$

we have

$$\dot{z} = \frac{\partial \beta_p}{\partial e}[\gamma e + \cos(by + c) + \beta(e,z)],$$

from this, we can easily see that z is bounded. In order to prove the asymptotic convergence of e, let us consider the following function

$$V = \frac{1}{2}e^2 - z$$

then

$$\dot{V} = -\lambda e^2$$

which means that V is also bounded and

$$\int_0^\infty e^2(s)ds \leq V_0 - V_\infty.$$

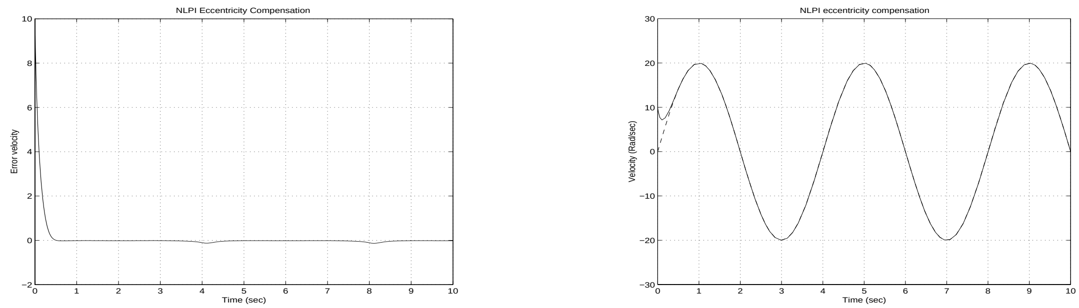

Fig. 2. NLPI control of eccentricity compensation problem

Therefore the error e goes asymptotically to zero as is shown in figure 2.

B. Application to friction compensation problem

This problem is conceptually similar to the Eccentricity compensation problem, the only difference is that the nonlinearity, representing the friction force, is linearly bounded. Hence, a combination of the results in proposition (1) and (2) has to be used.

If we choose

$$\left\{ \begin{array}{lcc} \beta & = & \ddot{y}_\star - (1+||x||)(\dot{\tilde{y}} + zcosz) \\ \beta_P & = & \dot{\tilde{y}} \\ w_I & = & \dot{\tilde{y}}(1+||x||) \end{array} \right. \quad (10)$$

In order to proof the asymptotic convergence of $\dot{y}$, we consider this function: $V = \frac{1}{2}\dot{\tilde{y}}^2 - z$.
The results of simulation of this controller are represented on the following figures

IV. Robust sliding mode controller

In this section , we will presents the technique of Sliding mode control.
Consider the following simplest scalar system

$$\dot{y} = \phi(y) + u \quad (11)$$

Where $\phi(y)$ is unknown continuous function.

Proposition 3

There exists an **unknown** positive constant μ such that $\phi(y) \leq \mu$. Then, the system in closed loop with following controller

$$\left\{ \begin{array}{lcc} u & = & -\alpha y - \hat{\mu}\text{sign}(y) \\ \dot{\hat{\mu}} & = & |y| \end{array} \right. \quad (12)$$

converges asymptotically to zero for all initial conditions.

Proof

Let us consider the following Lyapunov function

$$V = \frac{1}{2}y^2 + \frac{1}{2}(\hat{\mu} - \mu)^2$$

then

$$\dot{V} = -\alpha y^2 + y\phi(y) - \hat{\mu}|y| + \dot{\hat{\mu}}(\hat{\mu} - \mu)$$

or

$$\dot{V} \leq -\alpha y^2 + |y|\mu - \hat{\mu}|y| + \dot{\hat{\mu}}(\hat{\mu} - \mu)$$

by considering the value of $\hat{\mu}$, we can write

$$\dot{V} \leq -\alpha y^2 + |y|\mu - \hat{\mu}|y| + \hat{\mu}|y| - |y|\mu$$

which means that

$$\dot{V} \leq -\alpha y^2$$

then $y(t)$ converges asymptotically to zero.

A. Application to eccentricity compensation problem

In this section we present another solution based on sliding mode theory. We also consider that the parameters a, b, and c are unknown and the bounded of a are constant and unknown.
we consider $a < a_{max}$

Let us consider the following controller [2]

$$\left\{ \begin{array}{lcc} u & = & \ddot{y}_r(t) - \alpha(\dot{y} - \dot{y}_r(t)) - \hat{a}_{max}\text{sign}(e) \\ \dot{\hat{a}}_{max} & = & |e| \end{array} \right. \quad (13)$$

In order to proof the asymptotic tracking, we consider the following Lyapunov function

$$V = \frac{1}{2}e^2 + \frac{1}{2}(a_{max} - \hat{a}_{max})^2$$

Then

$$\dot{V} \leq -\alpha e^2 + |e|a_{max} - (a_{max} - \hat{a}_{max})\dot{\hat{a}}_{max} - \hat{a}_{max}|e|$$

which means that

$$\dot{V} \leq -\alpha e^2 + |e|a_{max} - a_{max}\dot{\hat{a}}_{max} + \dot{\hat{a}}_{max}\hat{a}_{max} - \hat{a}_{max}|e|$$

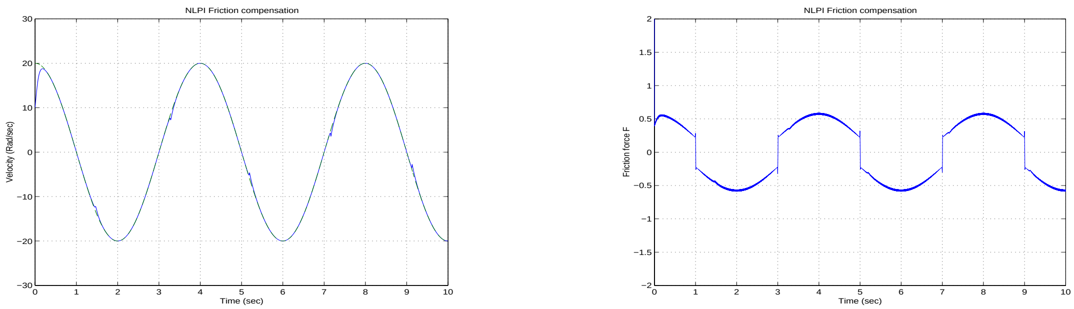

Fig. 3. NLPI control of Friction compensation problem

by replacing $\dot{\hat{a}}_{max}$, we have

$$\dot{V} \leq -\alpha e^2 + |e|a_{max} - a_{max}|e| + \hat{a}_{max}|e| - \hat{a}_{max}|e|$$

which leads to

$$\dot{V} \leq -\alpha e^2$$

from this it is obvious that e goes asymptotically to zero.

The results of simulation of this controller are represented on the figure 4.

B. Application to friction compensation problem

We consider the two dimensional system representing by equations (4), the friction force satisfying the bound [13]

$$|F(\dot{y}(t), z)| \leq M(1 + |\dot{y}(t)|) \leq M(1 + |\dot{y}(t)| + |z|)$$

$$u = J\ddot{y}_r(t) - \alpha(\dot{y} - \dot{y}_r(t)) + v \tag{14}$$

Replacing the equation (14) in (4), we obtain the following error equation

$$\begin{cases} J\dot{e} &= -\alpha e - F(\dot{y}, z) + v \\ e &= \dot{y} - \dot{y}_r \end{cases} \tag{15}$$

Let us consider the following controller

$$\begin{cases} v = -\hat{M}(1 + |\dot{y}| + |z|)sign(e) \\ \dot{\hat{M}} = \frac{|e|}{J}(1 + |\dot{y}| + |z|) \end{cases} \tag{16}$$

In order to proof the asymptotic tracking, we consider the following Lyapunov function

$$V = \frac{1}{2}e^2 + \frac{1}{2}(\hat{M} - M)^2$$

Then

$$\dot{V} = -\frac{\alpha}{J}e^2 - \frac{|e|}{J}\hat{M}(1 + |\dot{y}| + |z|) + \frac{|e|}{J}F(\dot{y}, z) + \dot{\hat{M}}(\hat{M} - M)$$

by replacing $F(\dot{y}, z)$, we have

$$\dot{V} \leq -\frac{\alpha}{J}e^2 - \frac{|e|}{J}\hat{M}(1 + |\dot{y}| + |z|) + M\frac{|e|}{J}(1 + |\dot{y}| + |z|) +$$
$$+\hat{M}\frac{|e|}{J}(1 + |\dot{y}| + |z|) - M\frac{|e|}{J}(1 + |\dot{y}| + |z|)$$

which leads to

$$\dot{V} \leq -\frac{\alpha}{J}e^2$$

The results of simulation of this controller are represented on the figure 5.

V. Simulation Results

In this section, we present some simulations. We consider the following values for system parameters

$$J = 0.002,\ a = 0.1,\ b = 0.2 \text{ and } c = 3$$

$$k_v = J.40, k_0 = 1, k_1 = 5, \gamma = 5$$

We suppose the following reference trajectories

$$v_d(t) = 20cos(\frac{\pi}{2}t).$$

$$a_0 = 22,\ a_1 = 1,\ a_2 = 0.1,\ \sigma_0 = 280,\ \sigma_1 = 1,\ \sigma_2 = 0.017.$$

The figure 2 represent respectively the tracking error, velocity and reference in the case of nonlinear PI controller applied to eccentricity compensation problem. The figure 3 shows the performance of the nonlinear PI in the case of friction problem. In the figures 4 and 5 we present the tracking error, velocity and reference with sliding mode approach respectively to eccentricity and friction problems. All simulations show that asymptotic tracking is ensured and controller are bounded. We can also see that the disturbances have several frequencies.

VI. Conclusion

In this paper we presented two solutions to eccentricity and friction compensation problem. In both cases we have seen that our controller are robuste and asymptotic tracking is ensured. We also proved that it is not necessary to known the bound of disturbances. These approaches will be extended to another classes of nonlinear systems in future work.

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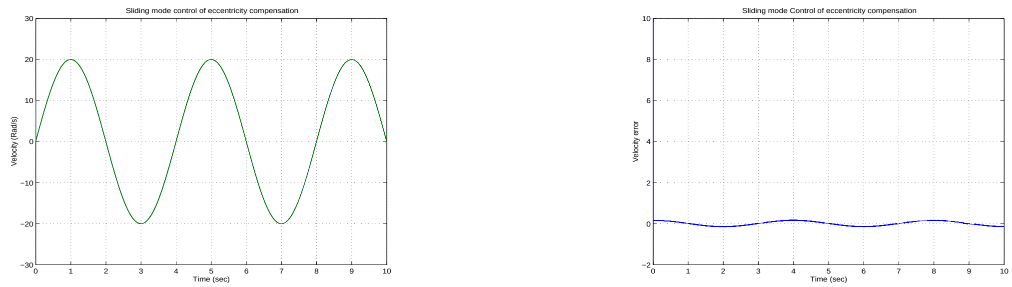

Fig. 4. Sliding mode control of eccentricity compensation problem

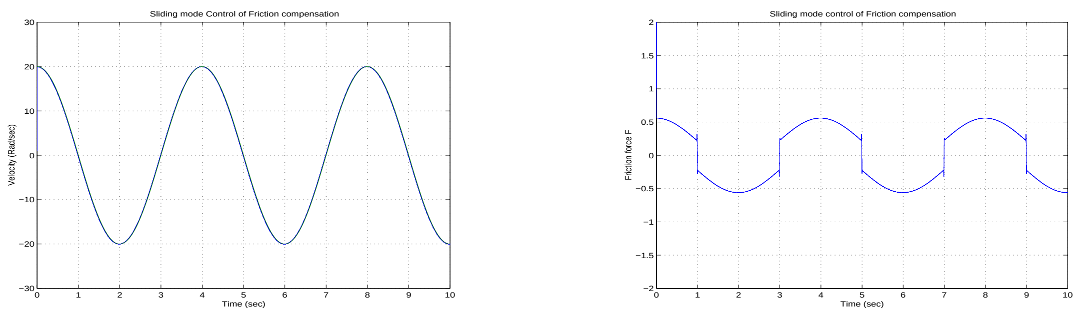

Fig. 5. Sliding mode control of friction compensation problem

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