CONTROLLING VELOCITY AND STEERING FOR BICYCLE STABILIZATION

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Abstract: Control of a naturally unstable riderless bicycle around zero equilibrium speed is investigated. A simple parametric model is derived. It predicts basic known dynamics. Jacobian linearization reveals that zero speed tilt stabilization is a MIMO non-minimum phase problem. It is shown that at certain operating conditions, the bicycle can be controlled only through velocity or steering. Combining both loops to maintain vertical balance at all speeds is the challenge. Some control structures and ideas are explored.

Keywords: multi-loop control, bicycle stabilization.

1. INTRODUCTION

The wider problem presented in this paper concerns stabilization and path tracking control of a naturally unstable nonlinear riderless bicycle, which belongs to a class of problems in single track mobile robots. The inputs are steering and speed commands and the outputs are speed, tilt and heading angle. Some issues on the physics of the problem are dealt by Jones (1970) and Fajans (2000). This paper looks at modelling and control structures for vertical balance around zero speed.

Path tracking has so far met with limited success for speeds very close to zero, a problem which is discussed in the context of what is termed rocking. Getz (1994, 1995, 1999); Yavin (1998) derive linearizing state space feedback laws for point to point path tracking for higher speed. Analysis and simulation around zero speed has shed more light on factors that limit performance and directed effort to synthesize simple controllers that meet the requirements of balance. Till now most literature is confined to bicycles moving above a certain speed and often with complicated controllers. There are however severe hardware constraints that impose bandwidth limitations in the balancing problem.

2. BICYCLE MODEL

2.1 Convention and model concept

Hand (1988) gives a historical summary of bicycle modelling. The models consider the bicycle as a system of interconnected masses, namely the rear wheel, frame / rider, steering fork and front wheel. These masses rotate in local co-ordinates which in turn can be referred to a fixed reference frame. Co-ordinate transformation is required to refer the motions to the fixed inertial co-ordinates frame.

Generally there are two methods used to develop the models, Newton's conservation of momentum and the Lagrange method. The latter is often associated with loss of direct physical interpretation. On the other hand coordinate transformation makes Newton's derivation very difficult in comparison to the Lagrange technique but it has merits of transparency. In anticipation of future work the Newton's principle of conservation of angular momentum is preferred over the Lagrange technique.

For simplicity we use the approach of Åstrøm (1977) and Klein (1989) which focuses on the important effects of steering geometry and mass placement. The result is a single mass inverted pendulum subject to kinematic constraints. Rider behaviour, frames rotational geometry and tire-road phenomena which complicate the model are ignored. Vertical tilt θ, rear wheel/frame heading or yaw angle ψ, steering angle φ and translation speed v_r or v_f as illustrated in Fig. 1. constitute the bicycle states.

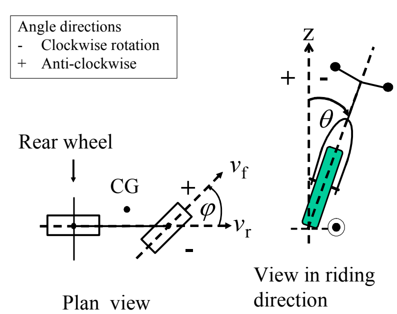

Fig.1. Projections showing model concept and sign convention of rotation angels.

The derived model agrees with the works of Shashikanth (2002), Fajans (2000), and Lowel and McKell (1982), Timoshenko and Young (1948) upon appropriate substitutions and harmonization of the sign convention used.

2.2 Definitions: Model geometry

A co-ordinate system defining the position of the bicycle at a given time is shown in Fig. 2. Generalized co-parameters required to describe the basic motions are also illustrated in Fig. 1 and 2. The definitions are;

xyz	Inertial co-ordinates on a horizontal plane.
ψ	Yaw angle, angel between the x-axis and the bicycle frame axis, degrees.
β	Direction of bicycle centre of gravity (CG) velocity $\mathbf{V}$ with respect to $\overline{\mathrm{AB}}$, degrees.
$\mathbf{V}_\mathrm{f}, \mathbf{V}_\mathrm{r}$	Bicycle front and rear wheels velocity, m/s.
a,b	Distance between the rear wheel ground contact point and CG, and $\overline{\mathrm{AB}}$, m.
M	Total mass of bicycle and rider, kg.
l	Height of bicycle CG above ground, m.
J	Bicycle moment of inertia about the z-axis, kgm^2.
r_o	Rear wheel contact point curvature radius.
r	Front wheel contact point curvature radius.

2.3 Yaw motion dynamics of the bicycle

The following assumptions are made (i) the bicycle's wheels are rigid and subject to small tilt small steering angles, (ii) wheels do not slide and (iii). the only forces that are applied to the bicycle are the 'body' forces due to gravity and ground constraint forces preventing slipping. Trail torque which results from internal feedback at the front fork is set to zero.

From Fig.2. velocities can be resolved leading to kinematic constraint equations in inertial co-ordinates. The rear/front wheel ground contact points and centre of gravity travel in concentric arcs when the bicycle is turning. This implies the three radii from these points are coincident at the centre of curvature O. These radii are also orthogonal to the velocities $\mathbf{V}_\mathrm{r}, \mathbf{V}$ and $\mathbf{V}_\mathrm{f}$ at respective points of tangency. Expressions of these velocities in the fixed inertial frame with basis vectors $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ are given in Eqs. (1).

$$\begin{aligned}
&\mathbf{V}_\mathrm{f} = \mathbf{i} v_f \cos(\psi+\varphi) + \mathbf{j} v_f \sin(\psi+\varphi), \quad v_f = |\mathbf{V}_\mathrm{f}| \\
&\mathbf{V} = \mathbf{i} v \cos(\psi+\beta) + \mathbf{j} v \sin(\psi+\beta), \quad v = |\mathbf{V}| \\
&\mathbf{V}_\mathrm{r} = \mathbf{i} v_r \cos\psi + \mathbf{j} v_r \sin\psi, \quad v_r = |\mathbf{V}_\mathrm{r}| \qquad (1)
\end{aligned}$$

With no wheels side slip at the rear and front contact points A (x_r, y_r) and B (x_f, y_f) respectively then,

$$\dot{x}_r \sin\psi - \dot{y}_r \cos\psi = 0 \qquad (2)$$

$$\dot{x}_f \sin(\psi+\varphi) - \dot{y}_f \cos(\psi+\varphi) = 0 \qquad (3)$$

Eqs. (2) and (3) are the kinematic constraints. From geometric expressions for $\dot{y}_f$ and $\dot{x}_f$ in Eq.(1) and manipulation of the constraint equations the first expression in Eqs. (4) is obtained.

$$\dot{\psi} = \frac{v_r}{b}\tan\varphi \qquad \dot{\psi} = \frac{v_r}{a}\tan\beta \qquad (4)$$

Similarly defining co-ordinates of the centre of gravity and the non-slippage constraints the second expression in Eq. (4) is obtained. These equations constitute the yaw model for the bicycle.

2.4 Tilt Dynamics

The tilt model describes how steering φ and bicycle velocity influence frame tilt. Consider the inverted pendulum idealization, Fig. 3.
When the bicycle travels in an arbitrary path it is tilted by the centripetal component normal to the frame $Mv^2 r^{-1}\cos\beta$, the component of the translation force $M\dot{v}_\perp$ ($v_\perp$ is the component of $\mathbf{V}$ orthogonal to the bicycle frame, $\overline{\mathrm{AB}}$) and the gravitational component $gM\sin\theta$. From Figs.2. and (3) if r^{-1} is the curvature at CG then $v = \dot{\psi} r$ and

$$v_\perp = v\sin\beta \quad \text{or} \quad v_\perp = \frac{a}{b} v_\mathrm{r} \tan\varphi \qquad (5)$$

$$\frac{Mv^2}{r}\cos\beta = M\frac{v_\mathrm{r}^2}{b}\tan\varphi \qquad (6)$$

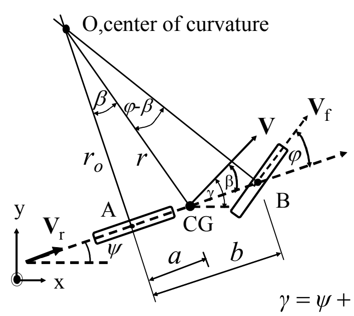

Fig.2. Kinematic plan projection in inertial frame.

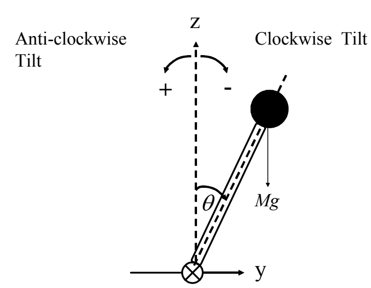

Fig. 3. Inverted pendulum model of a bicycle.

Summing torques by the principle of conservation of angular momentum about the tilt z-axis the model simplifies to Eq.(7).

$$\ddot{\theta} = \frac{Mgl}{J}\mathrm{s}_\theta + \frac{Ml}{J}\left(\frac{v_\mathrm{r}^2}{b}\mathrm{t}_\varphi + \frac{a}{b}\frac{d(v_\mathrm{r}\,\mathrm{t}_\varphi)}{dt}\right)\mathrm{c}_\theta \qquad (7)$$

The trigonometric expressions are shortened to $\sin\theta = \mathrm{s}_\theta, \tan\theta = \mathrm{t}_\theta$ and $\cos\theta = \mathrm{c}_\theta$. For nearly rectilinear motion $\mathrm{s}_\theta \approx \theta \quad \mathrm{c}_\theta \approx 1$ and $\mathrm{t}_\varphi \approx \varphi$. The constant speed model subject to these approximations is analysed by Åstrøm (1977).

3. TILT DYNAMICS AROUND ZERO SPEED

Skilled cyclists can 'rock' a bicycle around zero equilibrium speed without toppling. By rocking is implied the act of pedalling forward and backward by incremental amounts while keeping the handlebars at some fixed steering angle. In this state the bicycle has two inputs, speed command and steering, and two outputs, tilt angle and speed.

The possibility of emulating zero speed tilt stabilization i.e. rocking is investigated through linearization and a nonlinear control approach.

3.1 Rocking equilibrium conditions

The starting point is the basic inverted pendulum model of Eq.(7). The inputs steering angle φ and pedalling speed v_r are provided by first order motor models $P_\varphi(s)$ and $P_v(s)$ respectively in Eq. (8).

$$\mathrm{P}_\varphi(s): \varphi = \frac{c_1 v_{\mathrm{m}1}}{(s\tau_1 + 1)}, \quad \mathrm{P}_v(s): v_\mathrm{r} = \frac{c_2 v_{\mathrm{m}2}}{s\tau_2 + 1} \qquad (8)$$

$v_{\mathrm{m}1}, v_{\mathrm{m}2}$ are inputs, τ_1 and τ_2 time constants, and c_1, c_2 gains of the motors. Defining the state $\mathbf{x}$ and input $\mathbf{u}$ vectors, Eq. (9a) the model may be expressed as $\dot{\mathbf{x}} = \mathbf{f}(x_1,\ldots,x_4,u_1,u_2)$ expanded in Eq.(9b).

$$\mathbf{x} = (\theta,\dot{\theta},\varphi,v_\mathrm{r}),\ \mathbf{u} = (v_{\mathrm{m}1}, v_{\mathrm{m}2})^\mathrm{T} \qquad (9a)$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \omega_\mathrm{o}^2\,\mathrm{s}_{x_1} + \mu\omega_\mathrm{o}^2\left\{\frac{x_4^2}{a}\mathrm{t}_{x_3} + \frac{x_4}{c_{x_3}^2}(\dot{x}_3) + \mathrm{t}_{x_3}(\dot{x}_4)\right\}c_{x_1}$$

$$\dot{x}_3 = -\tau_1^{-1}x_3 + c_1\tau_1^{-1}v_{\mathrm{m}1}$$

$$\dot{x}_4 = -\tau_2^{-1}x_4 + c_2\tau_2^{-1}v_{\mathrm{m}2}\text{, where}$$

$$\mu = a/bg \ , \ \omega_o^2 = Mgl/J \qquad (9b)$$

At equilibrium $\mathbf{x}_\mathrm{e} = (\theta_\mathrm{e},\dot{\theta}_\mathrm{e},\varphi_\mathrm{e},v_\mathrm{re})^\mathrm{T}$ with inputs $v_{\mathrm{m}1\mathrm{e}}, v_{\mathrm{m}2\mathrm{e}}$, and $\mathbf{f}(x_{1e},\ldots,x_{4e},u_{1e},u_{2e}) = \mathbf{0}$. The second equation in Eq.(9) leads to the equilibrium condition,

$$g\tan\theta_\mathrm{e} + \frac{1}{b}v_\mathrm{re}^2\tan\varphi_\mathrm{e} = 0\,. \qquad (10)$$

Does Eq. (9) have equilibrium, $\mathbf{x}_\mathrm{e}$? From the sign convention φ and θ have opposing signs that renders Eq.(10) solvable. The point $(0,\varphi_e,0)$ is a possible solution but it is an unstable equilibrium. At this point steering angle is free to assume any angle between its limits $(-\pi/4,\pi/4)$ but cannot be zero. A bicycle at the state $(0,\dot{\theta}_e,\varphi_e,0)$ is identified with the rocking mode. Small signal linearization in the next section clarifies some basic factors contributing to instability during rocking.

3.2 Rocking linearized model

Schauder's fixed-point theorem enables us to compensate a nonlinear system using an equivalent uncertain LTI model. Jacobian linearization yields a rather restrictive incremental model whereas the later can have a wider range of application. We have found both approaches complimentary as will be clear in the discussion of the two controllers beginning with the incremental linear model about $\mathbf{x}_\mathrm{e}$ as expressed in Eqs.(11) and (12).

$$\Delta\dot{\mathbf{x}} = \mathbf{J}(\mathbf{x}_\mathrm{e})\Delta\mathbf{x} + \mathbf{J}(\mathbf{u}_\mathrm{e})\Delta\mathbf{u}, \quad \Delta\mathbf{y} = [1 \quad 0 \quad 0 \quad 0]\Delta\mathbf{x}\,, \qquad (11)$$

$$\mathbf{J}(\mathbf{x}_\mathrm{e}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \\ 0 & 0 & -1/\tau_2 & 0 \\ 0 & 0 & 0 & -1/\tau_1 \end{bmatrix} \mathbf{J}(\mathbf{u}_\mathrm{e}) = \begin{bmatrix} 0 & 0 \\ \frac{\partial f_2}{\partial v_{\mathrm{m}1}} & \frac{\partial f_2}{\partial v_{\mathrm{m}2}} \\ \mathrm{c}_1/\tau_1 & 0 \\ 0 & \mathrm{c}_2/\tau_2 \end{bmatrix} \qquad (12)$$

For $f_2(x_{1\mathrm{e}},\ldots,x_{4\mathrm{e}},u_1,u_2)$ the differential coefficients redefined as $a_\mathrm{i}, \mathrm{i} = 1,\ldots,4$ for $\mathbf{J}(\mathbf{x}_\mathrm{e})$ and $b_i', \mathrm{i} = 1,2$ for $\mathbf{J}(\mathbf{u}_\mathrm{e})$ are easily evaluated from Eq. (9). The coefficients of direct relevance in the discussion are given in Eqs. (13) to (15). Other Jacobian coefficients are specified in the Appendix.

$$a_1 = \frac{\partial f_2}{\partial x_1}\Big|_{\mathbf{x}_\mathrm{e}} = \omega_\mathrm{o}^2\left(\mathrm{c}_{\theta_\mathrm{e}} - \mu\frac{v_\mathrm{re}^2}{a}\mathrm{t}_{\varphi_\mathrm{e}}\mathrm{s}_{\theta_\mathrm{e}}\right) \qquad (13)$$

$$b_1' = \frac{\partial f_2}{\partial u_1}\Big|_{\mathbf{x}_\mathrm{e}} = \mu\omega_\mathrm{o}^2\frac{x_{4\mathrm{e}}}{\mathrm{c}_{x_{3e}}^2}\frac{\mathrm{c}_1}{\tau_1}\mathrm{c}_{x_{1e}} \qquad (14)$$

$$b_2' = \frac{\partial f_2}{\partial u_2}\Big|_{\mathbf{x}_\mathrm{e}} = \mu\omega_\mathrm{o}^2\frac{\mathrm{c}_2}{\tau_2}\mathrm{c}_{x_{1\mathrm{e}}}\mathrm{t}_{x_{3\mathrm{e}}}\,. \qquad (15)$$

It is convenient to express Eqs.(14) and (15) as $b_2 = \tau_2\,\mathrm{c}_2^{-1}\,b_2'$, $\quad b_1 = \tau_1\,\mathrm{c}_1^{-1}\,b_1'$. The incremental transfer function corresponding to Eq. (11) then reduces to Eq. (16) and has a block diagram representation of Fig.5.

$$\Delta\theta = b_1\left(\frac{s + v_\mathrm{re}/a}{s^2 - a_1}\right)\left(\frac{c_1}{s\tau_1 + 1}\right)\Delta U_1 + b_2\left(\frac{s + 2v_\mathrm{re}/a}{s^2 - a_1}\right)\left(\frac{c_2}{s\,\tau_2 + 1}\right)\Delta U_2 \qquad (16)$$

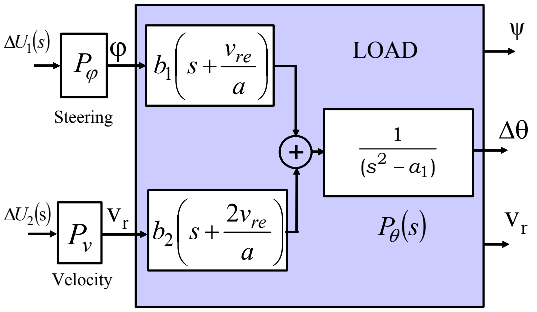

Fig. 5. Small signal bicycle linear model.

It is clear that this is a MISO or MIMO load sharing problem. Why is a stationary bicycle difficult to stabilize? Consider Eq. (16) both its zeros and gain b_1 approach zero as $v_{re} \to 0$. In this state the steering loop is not available to aid tilt stabilization. We need $b_2 \neq 0$ to control tilt via Δv_r about $v_{re} = 0$. That is why φ must be free and not zero. First turning the handle bar to an initial angle one can skilfully 'rock' a bicycle around average zero speed. Note that b_1 and b_2 are either positive or negative depending on the signs of v_{re} or φ_e.

In the next section we explore the possibility of rocking controllers. Stabilizing tilt is a means to an end. The real goal is tracking. Three control techniques were explored the standard PI, load sharing and a nonlinear scheme. Simulations and analyses are presented for the independent load sharing strategy of Eitelberg (1999) and a nonlinear to linear mapping technique suggested by Eitelberg in a personal communication. The later allows frequency domain design and works well with the original nonlinear plant.

4. CONTROL STRUCTURE FOR $v_{re} = 0$

The load sharing scheme in Fig.5. has two supply plants $P_v(s)$ and $P_\varphi(s)$ feeding a common load $P_\theta(s)$. $P_\theta(s)$ has an unstable mode and possibly other non-minimum phase characteristics. Fig.6 corresponds to $v_{re} = 0$. The steering motor-load loop forward path transfer function given in Eq. (17) is denoted $L_1(s)$ and the speed motor-load forward path transfer function in Eq. (18) is denoted $L_2(s)$. $L_1(s)$ is of no present concern as it is decoupled from the load due to b_1 being zero. This is depicted in Fig.6. by the mark '×'. $R_v(s)$, $R_\varphi(s)$ and $R_\varphi(s)$ are respective command inputs.

4.1 Tilt control using velocity

Around zero speed, tilt is controlled by velocity at a given steering. The challenge is how to tune the loop to attain vertical stability. With that done it is easier to undertake the path tracking problem. Design of the two controllers $G_{2\theta}(s)$ and $G_v(s)$ as shown in Fig. 6 is outlined in the next section.

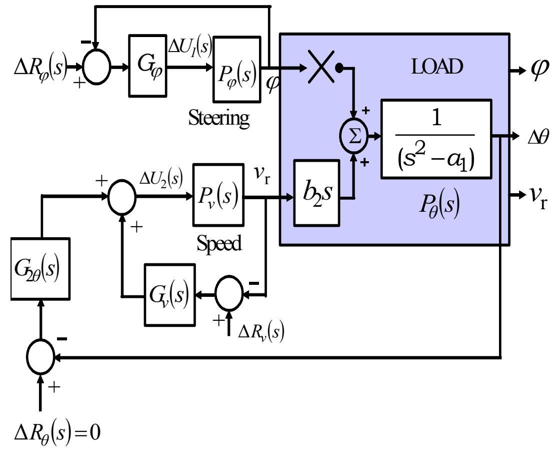

Fig. 6. Control of bicycle tilt using the velocity loop.

In Fig.6, there is an inner loop denoted $L_v(s)$ and an outer loop $L_{2\theta}(s)$. These loop transfer functions are expressed as

$$L_v(s) = G_v P_v \tag{17}$$

$$L_{2\theta}(s) = G_{2\theta} P_v P_{2\theta}, \quad P_{2\theta} = s b_2 P_\theta \tag{18}$$

The open loop transfer function of the inner loop on condition that the outer loop is closed is called the conditional open loop transfer function, $L_{sv}(s)$.

$$L_{sv}(s) = \frac{L_v(s)}{1 + L_{2\theta}(s)}. \tag{19}$$

$G_{2\theta}(s)$ and $G_v(s)$ are designed by frequency domain shaping of $L_{2\theta}(s)$ and then $L_{sv}(s)$ in that order. This is the sequential loop closure technique.

4.2 Design of tilt controller $G_{2\theta}(s)$

Nominal values for the model parameters are,

$M = 75$ kg,	$a = 0.5$ m,	$b = 1.2$m
$l = 1.0$ m,	$g = 9.81$ m/s^2,	$c_1 = 0.08$rad/V
$c_2 = 0.7$ ms^{-1}/V		

Hence $\omega_o \approx 3$ and $b_2 = \pm 0.05$. If a fast speed motor is assumed then a zero dynamics model approximation $P_v(s) = c_2$ may be used. Hence,

$$P_{2\theta}(s) = \frac{-0.05s}{(1+s/3)(1-s/3)} \tag{20}$$

With $G_{2\theta}(s) = k(1 + \omega_c / s)$ and a choice $\omega_c = 3$,

$$L_{2\theta}(s) = \frac{-0.15k\,c_2}{(1-s/3)} \tag{21}$$

$L_{2\theta}$ is closed loop stable provided $0.015k\,c_2 > 1$. If $0.15k\,c_2 = 3$ then $k = 31$ and tilt controller is,

$$G_{2\theta}(s) = 31(1 + 3/s) \tag{22}$$

4.3 *Velocity controller* $G_v(s)$

For G_v we shape L_{sv} (19). The outer loop is closed. With $G_v = k_v(1 + \omega_{cv}/s)$, $\omega_{cv} = 6$ and $k_v = -1/(3c_2)$ then,

$$L_{sv} = \frac{1 - s/3}{s} \quad \text{and} \tag{23}$$

$$G_v(s) = -(3\,c_2)^{-1}(1 + 6/s) \tag{24}$$

This technique elegantly illustrates the bandwidth limiting effect of unstable zeros and the non-minimum phase inherent in the system Åstrøm (1977) and Eitelberg (1999). Overall system gain crossover frequency is limited to 1.5 rad/s. The sensitivity plot in Fig.7 depicts the net effect of the proposed designs. By virtue of the predominant loop gain in respective frequency ranges note that velocity control dominates at low and tilt feedback at higher frequencies. This is as expected since tilt dynamics are fastest in this problem.

Simulation results in Fig. 8 indicate that tilt can be stabilized when $v_{\mathrm{re}} = 0$ and $|\Delta v_{\mathrm{r}}| < 0.1$. The controllers developed for the incremental model fail when $|\Delta v_{\mathrm{r}}| \ge 0.4$ as stability is lost. The nonlinear strategy in the next section overcomes this problem but only partly for positive Δv_{r}. A possible reason for the limitation is explained in Section 5.

5. NON LINEAR TILT CONTROLLER FOR $v_{\mathrm{re}} > 0$

The basic model in Eq.(7) may be expressed as,

$$\ddot{\theta} = \frac{g}{l}\theta + \frac{1}{bl}u(t), \text{ where} \tag{25}$$

$$u(t) = v_{\mathrm{r}}u_3 + \mathrm{a}\frac{\mathrm{d}u_3}{\mathrm{d}t}, u_3 = v_{\mathrm{r}}\ \mathrm{t}_{\varphi}$$

Eq. (25) is linear with respect to $u(t)$ for any $v_{\mathrm{r}}(t)$ and linear with respect to $u_3(t)$ for any constant speed. Earlier parameter values result in transfer function,

$$\frac{\theta(s)}{u_3(s)} = \frac{v_{\mathrm{r}}}{9}\frac{(1 + s\,a/v_{\mathrm{r}})}{(1 + s/3)(1 - s/3)} \tag{26}$$

By a similar frequency domain loop shaping technique we arrive at a controller in Eq. (27). Fig.9. illustrates the nonlinear control scheme as defined above. Results of simulation are in Fig. 10. The system is stable for $v_{\mathrm{r}} > 0$. This is because of the impracticality of canceling a right half plane pole-zero dipole which results when $v_{\mathrm{r}} < 0$. From Eq.(27) the pole-zero dipole is at $s = 3$ and $s = v_{\mathrm{r}e}/a$.

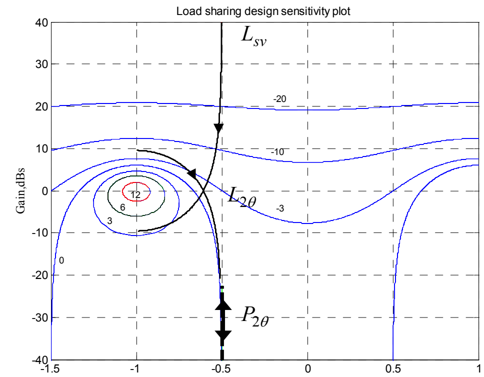

Fig.7. Loop transfer plots for $P_{2\theta}, L_{2\theta}$ and L_{sv}

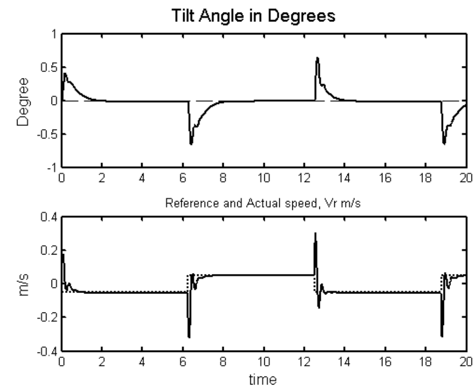

Fig. 8. Rocking simulation results for $v_{\mathrm{re}} = 0$, $\varphi_{\mathrm{re}} = \pi/4$. Speed reference is switched between +0.05 and –0.05 m/s.

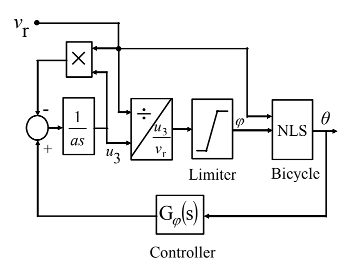

Fig. 9. Eitelberg's nonlinear tilt control scheme

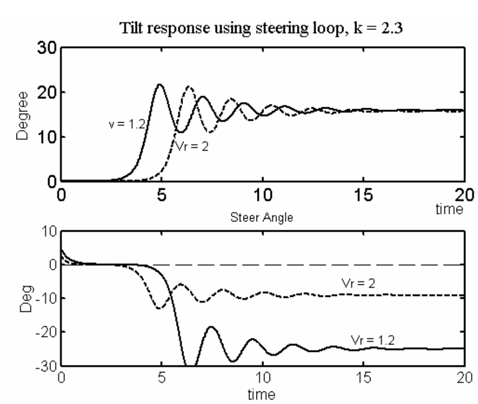

Fig. 10. Tilt response to speed step at $t = 0$ with nonlinear controller.

$$G_\varphi(s) = -k\frac{1+s/3}{1+s/30} \tag{27}$$

It seems plausible that a combination of the schemes above can yield a system which can be steered from rest to full speed.

The controller gain is set at 2.3 in the two cases shown and the reference speed varied as shown in Fig.9. Performance degrades at very low speeds and low gains. In such cases tilt increases to large values. High speed or high gain in the steering loop is required. Though not presented here it has been shown that the conventional PI controller suffers from the critical speed limitation Shashikanth. et. al, (2002) for similar gains used.

7. CONCLUSION

Simulations done demonstrate that rocking can be used to maintain bicycle tilt balance at nearly zero forward speed. The extent to which the controllers can be applied in practice depend on adaptation to dynamics not included in the models and effects of plant uncertainty. It appears that a switched controller topology for guidance from rest to speed with tilt stability is workable. Thus tilt feedback via the pedalling loop suffice to keep the system stable at very low speeds and steering loop argumentation is required at higher speeds. Further work is being undertaken to weigh these factors against technical instrumenting challenges that limit the bandwidth called for. There is need for further investigation as the model only admits positive Δv_{re} otherwise it is unstable.

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APPENDIX

Some of the Jacobian coefficients in Eq. (12) are shown below.

$$a_3 = \frac{\partial f_2}{\partial x_3}\Big|_{\underline{x}_e} = \mu\omega_{\mathrm{o}}^2\left(\frac{x_{4\mathrm{e}}^2}{a} - \frac{1}{\tau_1}x_{4\mathrm{e}}\right)\cos x_{1\mathrm{e}}\sec^2 x_{3\mathrm{e}}$$

$$a_4 = \frac{\partial f_2}{\partial x_4}\Big|_{\underline{x}_{\mathrm{e}}} = \mu\omega_{\mathrm{o}}^2\left(\frac{2x_{4\mathrm{e}}}{a} - \frac{1}{\tau_2}\right)\tan x_{3\mathrm{e}}\cos x_{1\mathrm{e}}$$

$$\Delta\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_1 & 0 & a_3 & a_4 \\ 0 & 0 & -1/\tau_2 & 0 \\ 0 & 0 & 0 & -1/\tau_1 \end{bmatrix}\Delta\mathbf{x} + \begin{bmatrix} 0 & 0 \\ b_1 & b_2 \\ c_1/\tau_1 & 0 \\ 0 & c_2/\tau_2 \end{bmatrix}\Delta\mathbf{u},$$

$\Delta y = \mathrm{C}\cdot\Delta\mathbf{x}$, $\mathrm{C} = [1 \;\; 0 \;\; 0 \;\; 0]$ hence,

$\Delta\theta = \mathrm{C}(s\mathrm{I} - \mathbf{J}(\mathbf{x}_{\mathrm{e}}))^{-1}\mathbf{J}(\mathbf{u}_{\mathrm{e}})\Delta\mathbf{u}$ or

$$\Delta\theta(s) = b_1\left(\frac{s+z_1}{s^2-a_1}\right)\left(\frac{1}{s+1/\tau_1}\right)\Delta U_1(s) + b_2\left(\frac{s+z_2}{s^2-a_1}\right)\left(\frac{1}{s+1/\tau_2}\right)\Delta U_2(s)$$

$$z_1 = \frac{1}{\tau_1} + \frac{c_1 a_3}{\tau_1 b_1} = \frac{v_{\mathrm{re}}}{a},\quad z_2 = \frac{1}{\tau_2} + \frac{c_2 a_4}{\tau_2 b_2} = \frac{2v_{\mathrm{re}}}{a}$$