Quasi Steady State Modelling of an Evaporator

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Abstract: *Even though the thermal capacity of the evaporator tubes and of the water within these tubes are not very different, their combined dynamic model is stiff. Application of the conventional singular perturbation simplification to the full model yields an inaccurate result. Here, a modified quasi steady state technique is shown to yield a good low frequency approximation.*

Key-words: quasi steady state; singular perturbation; model reduction; stiff systems; strong coupling.

1. Introduction.

Most real dynamical processes are composed of components that proceed at very different speeds, or in very different frequency ranges. This is a grave problem for numerical simulation and is known as system stiffness — the eigenvalues of a stiff system's Jacobian have very different magnitudes. In the context of feedback control, the fast components of the system are an unnecessary complication, if their dynamic behaviour falls significantly beyond the loop bandwidth. In many situations, it is advantageous to eliminate the fast process components (the large-magnitude eigenvalues of the Jacobian) from a complex model without impairing the model's accuracy in the lower frequency range.

It is not necessarily easy to identify the 'fast components' in a complex model. One of the difficulties is that the so-called slow and fast physical variables generally contain both slow and fast transients. However if they are identified, then the corresponding (non-linear) state differential equations can be written as

$$\begin{aligned} \dot{\boldsymbol{x}}_1 &= \boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}) \\ \dot{\boldsymbol{x}}_2 &= \boldsymbol{f}_2(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}) \end{aligned} \tag{1}$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are the 'slow' and 'fast' state vectors respectively and $\mathbf{u}$ is the system input vector. Readers used to block diagrams may appreciate the equivalent Figure 1.

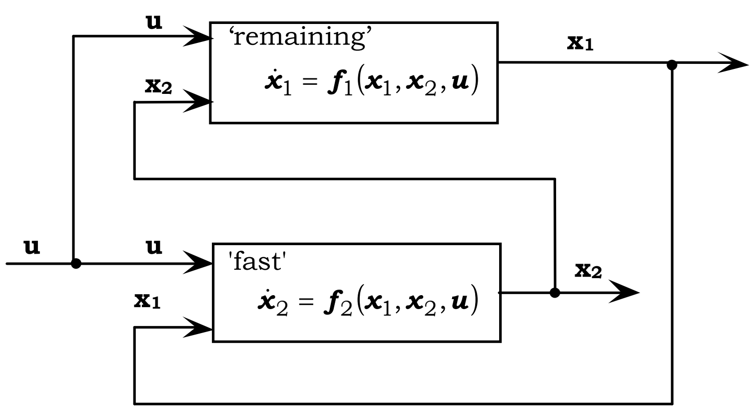

Figure 1: A model with an identified fast component.

The two subsystems are generally in a (multivariable) feedback loop. The slow subsystem inputs are $\mathbf{x}_2$ and $\mathbf{u}$ and the fast subsystem inputs are $\mathbf{x}_1$ and $\mathbf{u}$. The corresponding 'slow' and 'fast' outputs are $\mathbf{x}_1$ and $\mathbf{x}_2$ respectively. Now it should not be difficult to see what 'slow' and 'fast' may mean generally. In particular, the fast subsystem output $\mathbf{x}_2(t)$ reacts to any changes in its inputs, $\mathbf{x}_1(t)$ and $\mathbf{u}(t)$, much faster than the state, $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$, of the combined system. Equivalently, the fast subsystem output $\mathbf{x}_2(t)$ reacts to sufficiently slow changes in its inputs, $\mathbf{x}_1(t)$ and $\mathbf{u}(t)$, with negligible dynamic delays — $\mathbf{x}_2(t)$ is *strongly coupled* to $\mathbf{x}_1(t)$ (Sandell *et al.*, 1978). This relationship of system's state to the inputs is called *quasi steady state* (QSS) and is defined by

$$\mathbf{f}_2(\mathbf{x}_1(t),\mathbf{x}_2(t),\mathbf{u}(t)) = \mathbf{0} \tag{2}$$

The same modelling approach has been characterised as *singular perturbation* (Tihonov, 1952; Hoppenstead, 1971 and 1974; Kokotovic *et al.*, 1976; Eitelberg, 1983 and 1985), whereby a perturbation parameter ε is introduced as follows:

$$\begin{aligned} \dot{\boldsymbol{x}}_1 &= \boldsymbol{f}_1(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{u}) \\ \varepsilon\dot{\boldsymbol{x}}_2 &= \boldsymbol{f}_2(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{u}) \end{aligned} \tag{3}$$

Nominally ε = 1, but letting ε go to zero will, under certain conditions, lead to the quasi steady state condition in eq. (2). The singularly perturbed reduced-order model is then defined by

$$\begin{aligned} \dot{\boldsymbol{x}}_1 &= \boldsymbol{f}_1(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{u}) \\ 0 &= \boldsymbol{f}_2(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{u}) \end{aligned} \tag{4}$$

Eitelberg (2003) has shown why this singular perturbation of stiff models cannot be expected to yield good low frequency approximations and has instead proposed a modified quasi steady state technique as follows.

First, the output of the identified fast subsystem is defined exactly as

$$\boldsymbol{x}_2(t) = \boldsymbol{g}_2(\boldsymbol{x}_1(t),\boldsymbol{u}(t),\dot{\boldsymbol{x}}_2(t)) \tag{5}$$

Substitution into the 'slow' subsystem yields without approximation

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{f}_1(\boldsymbol{x}_1,\boldsymbol{g}_2(\boldsymbol{x}_1,\boldsymbol{u},\dot{\boldsymbol{x}}_2),\boldsymbol{u}) \tag{6}$$

Then the *quasi steady state derivative* of the 'fast' state is evaluated, either from the implicit equation (2), or equivalently from the explicit equation (5):

$$\dot{\boldsymbol{x}}_2(t) = \frac{d\boldsymbol{g}_2(\boldsymbol{x}_1(t),\boldsymbol{u}(t),\boldsymbol{0})}{dt} = \frac{\partial \boldsymbol{g}_2}{\partial \boldsymbol{x}_1}\dot{\boldsymbol{x}}_1 + \frac{\partial \boldsymbol{g}_2}{\partial \boldsymbol{u}}\dot{\boldsymbol{u}} \tag{7}$$

Substitution into eq. (6) yields the low-order approximation

$$\begin{aligned} \dot{\boldsymbol{x}}_1 &= \boldsymbol{f}_1\left(\boldsymbol{x}_1,\boldsymbol{g}_2\left(\boldsymbol{x}_1,\boldsymbol{u},\left[\frac{\partial \boldsymbol{g}_2}{\partial \boldsymbol{x}_1}\dot{\boldsymbol{x}}_1 + \frac{\partial \boldsymbol{g}_2}{\partial \boldsymbol{u}}\dot{\boldsymbol{u}}\right]\right),\boldsymbol{u}\right) \\ \boldsymbol{x}_2(t) &= \boldsymbol{g}_2(\boldsymbol{x}_1(t),\boldsymbol{u}(t),\boldsymbol{0}) \end{aligned} \tag{8}$$

It may be of some interest that

$$\begin{aligned} \frac{\partial \boldsymbol{g}_2}{\partial \boldsymbol{x}_1} &= -\left[\frac{\partial \boldsymbol{f}_2}{\partial \boldsymbol{x}_2}\right]^{-1}\frac{\partial \boldsymbol{f}_2}{\partial \boldsymbol{x}_1}; \\ \frac{\partial \boldsymbol{g}_2}{\partial \boldsymbol{u}} &= -\left[\frac{\partial \boldsymbol{f}_2}{\partial \boldsymbol{x}_2}\right]^{-1}\frac{\partial \boldsymbol{f}_2}{\partial \boldsymbol{u}} \end{aligned} \tag{9}$$

2. A simple evaporator model.

In the evaporator of a power boiler, most of the heat energy is transferred from the furnace into the steel pipes by radiation. This heat flow rate q_{rad} is essentially imposed on the evaporator and independent of the evaporator temperatures. With this assumption, the thermal state of ***any section*** of an evaporator can be characterised by the following differential equations.

The heat energy mCT_{m} in the metal mass m (with the heat capacity C and temperature T_{m}) of the tubes is increased by the imposed radiated heat flow rate q_{rad} and decreased by the heat flow rate $q(T_{\mathrm{m}},T)$ into the water or steam with the temperature T within the pipes:

$$\frac{dmCT_{\mathrm{m}}}{dt} = q_{\mathrm{rad}} - q(T_{\mathrm{m}},T) \tag{10}$$

The heat energy ρVh of the water/steam with density ρ and specific enthalpy h in the inside volume V of the tubes is increased by the heat that is transported into the evaporator section in the water and by the heat flow rate $q(T_{\mathrm{m}},T)$ into the water. It is decreased by the heat that is transported out of the evaporator section in the steam or water:

$$\frac{d\rho Vh}{dt} = \dot{m}_{\rm in} h_{\rm in} - \dot{m}_{\rm out} h + q(T_{\rm m}, T) \qquad (11)$$

The section mass in- and out-flow rates, $\dot{m}_{\rm in}$ and $\dot{m}_{\rm out}$ respectively, are related by the mass conservation equation

$$\frac{d\rho V}{dt} = \dot{m}_{\rm in} - \dot{m}_{\rm out} \qquad (12)$$

A differential equation for the metal temperature is obtained from eq. (10) simply by dividing with the (approximately) constant mC:

$$\frac{dT_{\rm m}}{dt} = \frac{q_{\rm rad} - q(T_{\rm m}, T)}{mC} \qquad (13)$$

A differential equation for the steam/water specific enthalpy is obtained by developing eq. (11) as follows:

$$\rho V \frac{dh}{dt} = -h \frac{d\rho V}{dt} + \dot{m}_{\rm in} h_{\rm in} - \dot{m}_{\rm out} h + q(T_{\rm m}, T) \qquad (14)$$

Substitution of eq. (12) yields

$$\frac{dh}{dt} = \frac{\dot{m}_{\rm in}(h_{\rm in} - h) + q(T_{\rm m}, T)}{\rho V} \qquad (15)$$

We have found that, under some realistic conditions, the combined thermal model

$$\begin{aligned} \frac{dh}{dt} &= \frac{\dot{m}_{\rm in}(h_{\rm in} - h) + q(T_{\rm m}, T)}{\rho V} \\ \frac{dT_{\rm m}}{dt} &= \frac{q_{\rm rad} - q(T_{\rm m}, T)}{mC} \end{aligned} \qquad (16)$$

is stiff and very time-consuming to simulate. The stiffness is related to the very good thermal conductivity between the metal and fluid. The differential equation for the metal temperature $T_{\rm m}$ will be considered here as the 'fast' subsystem.

Singular perturbation.

The conventional singular perturbation technique would set the metal temperature derivative to zero in eq. (16), hence

$$q(T_{\rm m}, T) = q_{\rm rad} \qquad (17)$$

And substitution into eq. (16) would yield the simplified model

$$\frac{dh}{dt} = \frac{\dot{m}_{\rm in}(h_{\rm in} - h) + q_{\rm rad}}{\rho V} \qquad (18)$$

However, this cannot be right generally — good thermal conductivity between the metal and fluid does not mean that the heat capacity of the one or the other can be automatically ignored. The heat capacity of metal is ignored in eq. (18).

Quasi steady state.

According to the quasi steady state technique from Eitelberg (2003), we first have to eliminate $T_{\rm m}$ from the specific enthalpy differential equation. For this purpose, we can solve the metal temperature equation for the heat flow rate

$$q(T_{\rm m}, T) = q_{\rm rad} - mC \frac{dT_{\rm m}}{dt} \qquad (19)$$

Substitution of this heat flow rate into the specific enthalpy differential equation yields

$$\rho V \frac{dh}{dt} + mC \frac{dT_{\rm m}}{dt} = \dot{m}_{\rm in}(h_{\rm in} - h) + q_{\rm rad} \qquad (20)$$

Now we have to evaluate the quasi steady state derivative of the metal temperature (similarly to eq. (7)). This can be found by differentiating the implicit equation (17) (the quasi steady state eq. (19)) with respect to time,

$$\frac{\partial q}{\partial T_{\rm m}} \frac{dT_{\rm m}}{dt} + \frac{\partial q}{\partial T} \frac{dT}{dt} = \frac{dq_{\rm rad}}{dt} \qquad (21)$$

and then solving for

$$\frac{dT_{\rm m}}{dt} = \left[\frac{\partial q}{\partial T_{\rm m}}\right]^{-1} \left[\frac{dq_{\rm rad}}{dt} - \frac{\partial q}{\partial T} \frac{dT}{dt}\right] \qquad (22)$$

Mostly, but not always, the heat transfer seems to be symmetrical in the sense that

$$\frac{\partial q}{\partial T_{\rm m}} = -\frac{\partial q}{\partial T} \qquad (23)$$

This would not be (strictly) correct, for example, in the case of radiated heat transfer. However, radiation is negligible between the tube-wall and H_2O in conventional evaporators. Therefore

$$\frac{dT_{\rm m}}{dt} = \left[\frac{\partial q}{\partial T_{\rm m}}\right]^{-1} \frac{dq_{\rm rad}}{dt} + \frac{dT}{dt} \qquad (24)$$

The steam temperature derivative is related to the steam specific enthalpy h and pressure p via the thermodynamic state relationships (steam tables):

$$\frac{dT}{dt} = \frac{\partial T}{\partial h}\frac{dh}{dt} + \frac{\partial T}{\partial p}\frac{dp}{dt} \tag{25}$$

Substituting eq. (25) into eq. (24) yields

$$\frac{dT_{\mathrm{m}}}{dt} = \left[\frac{\partial q}{\partial T_{\mathrm{m}}}\right]^{-1}\frac{dq_{\mathrm{rad}}}{dt} + \frac{\partial T}{\partial h}\frac{dh}{dt} + \frac{\partial T}{\partial p}\frac{dp}{dt} \tag{26}$$

Substituting eq. (26) into eq. (20) yields finally

$$\frac{dh}{dt} = \frac{\dot{m}_{\mathrm{in}}(h_{\mathrm{in}} - h) + q_{\mathrm{rad}}}{\rho V + mC\dfrac{\partial T}{\partial h}} - \frac{mC\left(\left[\dfrac{\partial q}{\partial T_{\mathrm{m}}}\right]^{-1}\dfrac{dq_{\mathrm{rad}}}{dt} + \dfrac{\partial T}{\partial p}\dfrac{dp}{dt}\right)}{\rho V + mC\dfrac{\partial T}{\partial h}} \tag{27}$$

In systems that we have investigated recently, ρV and $mC\partial T/\partial h$ have similar magnitudes when water is below saturation temperature — hence, neither heat capacity can be ignored. In *two-phase flow*, however, T is a function of pressure alone and

$$\frac{dh}{dt} = \frac{\dot{m}_{\mathrm{in}}(h_{\mathrm{in}} - h) + q_{\mathrm{rad}}}{\rho V} - \frac{mC\left(\left[\dfrac{\partial q}{\partial T_{\mathrm{m}}}\right]^{-1}\dfrac{dq_{\mathrm{rad}}}{dt} + \dfrac{\partial T}{\partial p}\dfrac{dp}{dt}\right)}{\rho V} \tag{28}$$

The singularly perturbed model in eq. (18) describes the thermal dynamics identically only under the condition of two-phase flow at constant pressure p and with constant heat uptake q_{rad}.

3. Simulation.

Figure 2 compares the differences of the full order, singularly perturbed and quasi-steady state models developed above applied to the model of a large once through (Benson®) boiler during start-up. The model divides the economiser and evaporator of the boiler into five and ten spatial sections respectively. As shown in Figure 2a, the start-up has first one and then two mills brought into service to move the load demand from 10% to 30% and then to 50% at a rate of around 20 MW/min. In order to excite a faster transient behaviour, a (temporary) mill trip is simulated, starting at 45 minutes. All simulations have constant feedwater pressure (6 MPa), economiser inlet massflow (280 kg/s) and feedwater temperature (180 °C).

Figure 2b shows the water outflow rate of the boiler (it is not yet in Benson mode). Figure 2c shows the specific enthalpy of the water or two-phase fluid at the evaporator outlet. As can be seen in Figures 2b and 2c, the full order and quasi-steady state model give almost exactly the same response. In contrast, direct application of the singular perturbation method results in a model that does not include the effect of energy absorbed by the boiler tubes during transients and is therefore too fast.

Figure 3 shows the effect of a step change in the feedwater temperature from 180 °C to 220 °C on the water outlet massflow rate. For this simulation, the following are held constant: feedwater pressure (6 MPa); economiser inlet massflow (280 kg/s); and 50% boiler demand. The simulation highlights that the quasi-steady state method is somewhat limited in that the transient heat exchange between the hotter fluid and boiler tubes is not captured correctly. The singularly perturbed model is however much worse.

4. Conclusion.

This paper has shown that while direct application of the singular perturbation method yields a model with incorrect transient behaviour, the quasi steady state technique achieves model order reduction with good low frequency approximation. This technique in the particular application of a power plant boiler also makes technical sense as it correctly captures the thermal capacity of both the evaporator tubes and of the water within them.

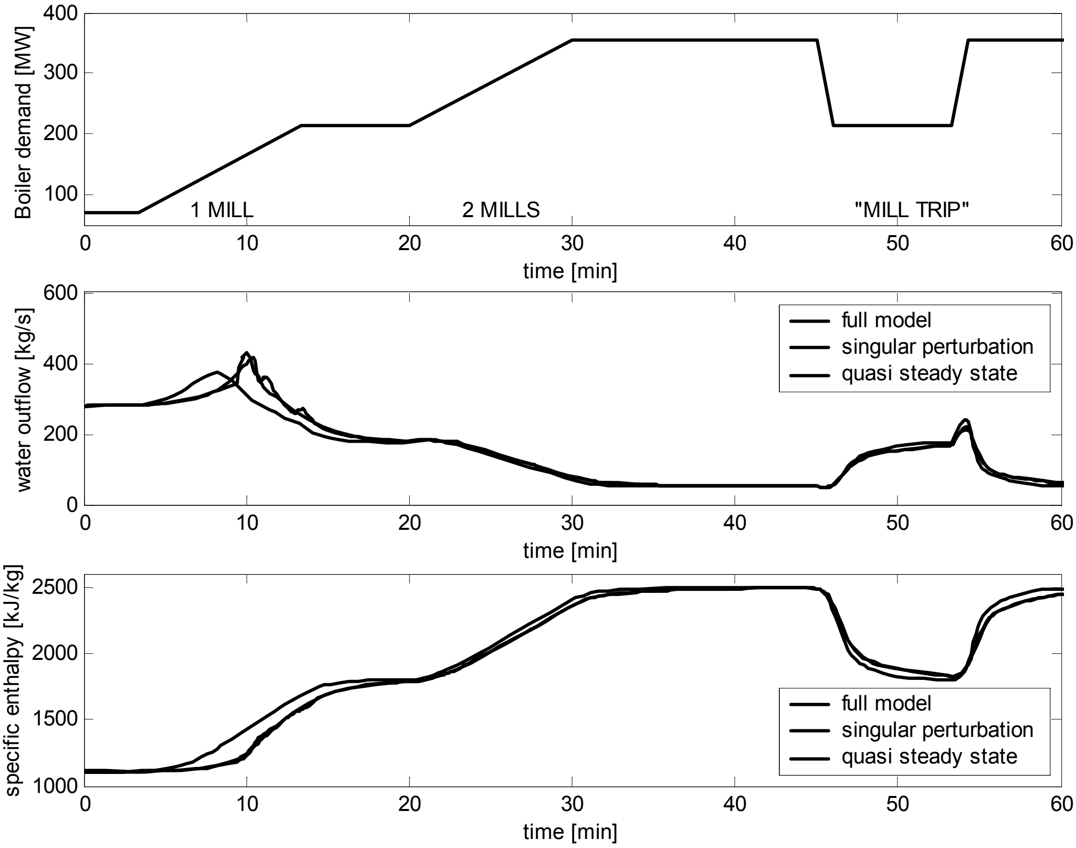

Figure 2: Simulation of start-up of once-through boiler using quasi-steady state model order reduction.

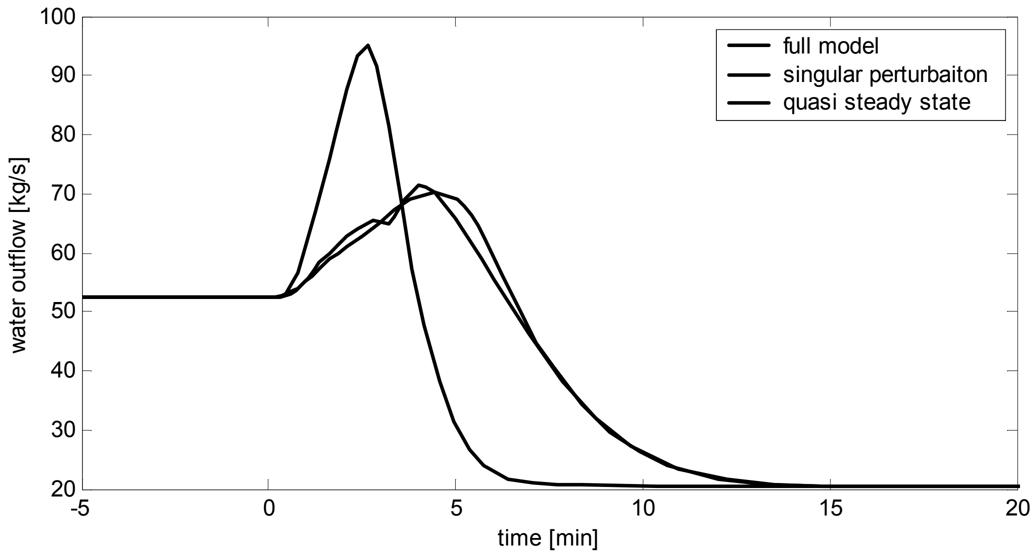

Figure 3: Simulation of once-through boiler with rate-limited step disturbance in feedwater (inlet) temperature (enthalpy) from 180° C to 220 °C

References

1. Eitelberg, Ed. (1983): *Perturbationstechniken bei der Optimierung grosser linearer Regelungssysteme*. Fortschr.-Ber. VDI-Z. VDI-Verlag, Duesseldorf.
2. Eitelberg, Ed. (1985): '*A transformation of non-linear dynamical systems with a single singular singularly perturbed differential equation*.' Int. J. Control, Vol. 41, No. 5, pp. 1301–1316.
3. Eitelberg, Ed. (2003): '*Quasi steady state modelling*.' Submitted.
4. Hoppenstead, F. (1971): '*Properties of solutions of ordinary differential equations with small parameters*.' Comm. Pure Appl. Math., Vol XXIV, pp. 807–840.
5. Hoppenstead, F. (1974): '*Asymptotic stability in singular perturbation problems*. II: *Problems having matched asymptotic expansion solutions*.' J. Differential Equations. Vol 15, pp. 510–521.
6. Kokotovic, P.V.; O'Malley, R.E.; Sannuti, P. (1976): '*Singular perturbations and order reduction in control theory – an overview*.' Automatica. Vol 12, pp. 123–132.
7. Sandell, N.R.; Varaiya, P.; Athans, M.; Safonov, M.G. (1978): '*Survey of decentralized control methods for large scale systems*.' IEEE AC-23, No. 2, pp. 108–128.
8. Tihonov, A.N. (1952): '*Systems of differential equations with small parameters at the derivatives*.' Mat. Sb. Vol. 31 (73), no. 3, pp. 575–586 (in Russian).