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Optimal energy management of a building cooling system with thermal storage: A convex formulation ⋆

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Abstract: This paper addresses the optimal energy management of a cooling system, which comprises a building composed of a number of thermally conditioned zones, a chiller plant that converts the electrical energy in cooling energy, and a thermal storage unit. The electrical energy price is time-varying, and the goal is to minimize the electrical energy cost along some look-ahead time horizon while guaranteeing an appropriate level of comfort in the building. A key feature of the approach is that the temperatures in the zones are treated as control inputs together with the cooling energy exchange with the storage. This simplifies the enforcement of comfort, which can be directly imposed through appropriate constraints on the control inputs. Furthermore, a model that is easily scalable in the number of zones and convex as a function of the control inputs is derived based on energy balance equations. A convex constrained optimization program is then formulated to address the optimal energy management with reference to the forecasted operating conditions of the building. Simulation results show the efficacy of the proposed approach.

Keywords: Optimal energy management; building cooling system; thermal storage; constrained control; convex optimization.

1. INTRODUCTION

Almost 40% of the US overall electricity consumption can be attributed to buildings, almost a half of this fraction being used by cooling, heating, and air conditioning systems (D&R International (2012)). In the perspective of the smart grid challenge of integrating renewable energy production and distributed energy generation, buildings can be viewed as consumers that can actively contribute to the electrical energy demand/generation balance. Effective building energy management strategies should be implemented to increase efficiency and eventually track some energy consumption profile, according to the demand/response strategy. The introduction of thermal storage systems can be particularly useful in this respect since they can be used for the twofold purpose of i) making the chiller plant work closer to its highest efficiency condition, thus reducing the electrical energy consumption, and ii) shifting in time the electrical energy request from the grid, thus avoiding peaks in demand, see e.g. Deng et al. (2013); Powell et al. (2013); Ma et al. (2009). The "building thermal mass" can be beneficially exploited as a passive thermal storage to add further flexibility, Balaras (1996); Kinter-Meyer and Emery (1994); Ma et al. (2012).

Here, we consider the optimal energy management problem of a building cooling system with thermal storage. More precisely, we suppose that the electrical energy price is time-varying, and minimize the electrical energy cost along some look-ahead time horizon while guaranteeing an appropriate level of comfort in the building. In most of the related works in the literature this is achieved by acting directly on the cooling system, e.g., by regulating flows and temperatures of the chiller water circuit. The resulting behavior of the temperature in the building depends on the control variables through a possibly complex nonlinear model, which prompts the need of introducing a secondary controller (usually a PID) to counteract disturbances and modeling errors. In Ma et al. (2012) it is argued that this scheme leads to unpredictable and hardly quantifiable behaviors in the presence of disturbances. Moreover, the model complexity makes the problem hard to be tackled via stochastic optimal control methods like those based on dynamic programming, Borghesan et al. (2013).

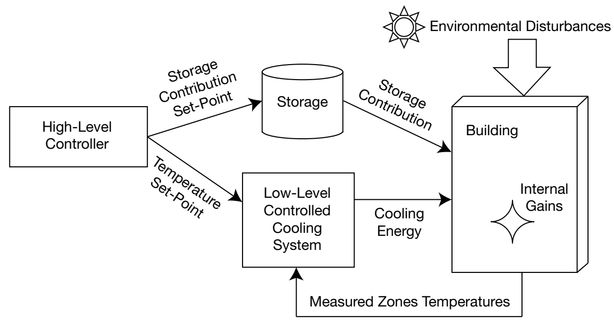

Fig. 1. Proposed energy management scheme

In this paper, we propose a different scheme: we set as control inputs the building temperature set-point together with the cooling energy exchange with a storage. We then compute the cooling energy needed for the building temperature to track its set-point (Figure 1), with the understanding that a lower control layer is present to this purpose. Issues related to the nonlinear characteristics of the system are then relegated at this lower level, Ceriani et al. (2013); Borghesan et al. (2013). Suitable constraints

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are imposed on the control input with the twofold objective of making the tracking feasible and guaranteeing comfort conditions.
The electrical energy cost is determined based on the thermal energy balance within the building, and readily accounts for thermal effects related to the building structure and thermal phenomena related, for example, to occupancy and radiation through glazed surfaces. The thermal model of the building is derived following Kim and Braun (2012) and is simple yet accurate, Kim et al. (2013). The building is decomposed in zones, each one characterized by its own temperature set-point.
The expression derived for the electrical energy cost is convex as a function of the control variables, which leads to an easily solvable constrained convex optimization problem when reference is made to the system operating in nominal conditions (certainty equivalence solution). Numerical results are presented for different variants of the energy management scheme, including a single-zone and a multi-zone example.

2. OPTIMAL ENERGY MANAGEMENT PROBLEM

Consider a building composed of n_z zones, each one characterized by its own (average) temperature $T_{z,j}$, $j = 1, \ldots, n_z$. Let $\boldsymbol{T_z} = [T_{z,1} \cdots T_{z,n_z}]^\top$. Our objective is to determine an optimal profile for $\boldsymbol{T_z}$ along some finite time-horizon $[t_0, t_f]$, so as to minimize the energy cost for cooling the building while maintaining an appropriate level of comfort for the occupants of the building. To this purpose, the time horizon $[t_0, t_f]$ is discretized into M time slots of the same duration dt, and the energy cost is computed as $J = \sum_{k=1}^{M} \psi(k) E_\ell(k)$, where $E_\ell(k)$ and $\psi(k)$ are, respectively, the electrical energy consumption and its unitary cost within the k^{th} time slot. The energy contribution $E_\ell(k)$ can be expressed as a function of the cooling energy $E_{ch}(k)$ requested to the cooling system within the k^{th} time slot. In the case when the cooling system is composed by a single chiller, we can adopt the static nonlinear Ng-Gordon model Gordon et al. (1997). Dropping the dependance from k for ease of notation, E_ℓ can be expressed as follows as a function of E_{ch}

$$E_\ell = \frac{a_1 T_o T_{cw} dt + a_2 (T_o - T_{cw}) dt + a_4 T_o E_{ch}}{T_{cw} - a_3 E_{ch}/dt} - E_{ch}, \quad (1)$$

where E_{ch}/dt represents the cooling power approximated by its average value over each time slot, T_o is the outdoor temperature, T_{cw} is the temperature of the water in the chilled water circuit, and $a_1, \ldots, a_4$ are suitable coefficients that characterize the chiller performance. While T_o is a disturbance, T_{cw} is a controlled variable that is set to some appropriate constant value by a low level controller. We can then adopt a convex approximation of (1) by expressing E_ℓ as a biquadratic function of E_{ch}:

$$E_\ell = c_1(T_o) E_{ch}^4 + c_2(T_o) E_{ch}^2 + c_3(T_o), \quad (2)$$

where $c_1(T_o)$, $c_2(T_o)$, and $c_3(T_o)$ are derived according to the least squares criterion. Figure 2 shows the Coefficient Of Performance (COP) given by E_{ch}/E_ℓ as a function of E_{ch} in some admissible range of values for E_{ch}, for sensible values of the parameters and temperatures involved.

In our setup, a thermal storage is present. We describe the thermal storage as a first order system $S(k+1) = aS(k) - s(k)$, where $S(k)$ is the amount of cooling energy stored in it and $s(k)$ is the cooling energy exchanged, in the k^{th} time slot, while $a \in (0, 1)$ is a coefficient introduced to model energy losses. We can also reformulate the thermal storage dynamics in vector form as $\boldsymbol{S} = \Xi_0 S(0) + \Xi_1 \boldsymbol{s}$ where $\boldsymbol{S} = [S(1) \cdots S(M)]^\top$, $\boldsymbol{s} = [s(0) \cdots s(M-1)]^\top$, and Ξ_0 and Ξ_1 are suitable matrices, $\boldsymbol{s}$ being a control input. If $s(k) > 0$, the storage supplies part of the cooling energy, thus reducing the amount of cooling energy requested to the chiller, whereas if $s(k) < 0$, then the chiller has to produce the additional cooling energy that is stored.

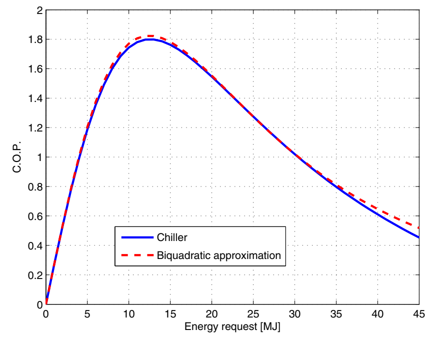

Fig. 2. Chiller COP

The cooling energy requested to the chiller is given by

$$E_{ch} = E_c - s, \quad (3)$$

where E_c is obtained as the sum of the cooling energies $E_{c,j}$ requested by the zones, each one given by the sum of four terms, namely

$$E_c = \sum_{j=1}^{n_z} E_{c,j} = \sum_{j=1}^{n_z} (E_{w,j} + E_{p,j} + E_{\text{int},j} + E_{z,j}). \quad (4)$$

$E_{w,j}$ in (4) is the amount of energy exchanged between the walls and zone j, $E_{p,j}$ and $E_{\text{int},j}$ is the heat produced respectively by people and by other sources of heat inside zone j, and $E_{z,j}$ is the energy contribution of the thermal inertia of zone j.

By selecting s in (3), we can shift the electrical energy requested to the grid, to an extent that depends on the capacity of the storage and on the rate at which it can be charged and discharged. Moreover, given that E_{ch} in (3) is linear as a function of E_c and s, then, E_ℓ in (2) is convex in E_c and s.

We next show that the energy contributions in (4) are affine as a function of the temperatures $\boldsymbol{T_z}$ of the zones, which are taken as control variables. This ensures that (2) is convex in all the control variables, i.e., $\boldsymbol{T_z}$ and $\boldsymbol{s}$.

2.1 *Wall-zone energy exchange E_w*

In order to adequately take into account the amount of heat exchanged between walls and zones, we need to derive a model for the building that consists of walls and zones. As for the walls we employ a one-dimensional finite volumes model. Each wall is divided into vertical layers ('slices') that differ in width and material composition. The area of each slice coincides with the wall area and each slice is assumed to have a uniform density and a uniform temperature. The one-dimensional discretization has been chosen exploiting the fact that the heat flow is perpendicular to the surface it is passing through. Each internal slice exchanges heat only with nearby slices through conduction, whilst boundary slices also exchange heat via convection and thermal radiation through surfaces that are exposed towards either a zone or the outside of the building. External surfaces are assumed to be gray and opaque, with equal absorbance and emissivity and with zero transmittance. Absorbance and emissivity are wavelength-dependent quantities, and here we shall consider two different values for shortwave and longwave radiation.

The energy balance equation for the i^{th} slice is given by:

$$\begin{aligned}\dot{T}_i = \frac{1}{C_i}\Big[&(k_i^{i-1}+h_i^{i-1})T_{i-1}+(k_i^{i+1}+h_i^{i+1})T_{i+1}\\ &-(k_i^{i-1}+h_i^{i-1}+k_i^{i+1}+h_i^{i+1})T_i+\\ &+\alpha_i^S Q^S+\alpha_i^L Q^L-\varepsilon_i Q_r(T_i)+Q_{g,i}\Big],\end{aligned} \tag{5}$$

where T_i denotes the temperature of the slice, C_i being its thermal capacity per unit area, and k_i^j and h_i^j, with $j = i\pm 1$, representing respectively the conductive and convective heat transfer coefficients between the i^{th} and the j^{th} slice. The incoming shortwave and longwave radiation power per unit area are Q^S and Q^L, respectively, and α_i^S and α_i^L are the corresponding absorbance rates. $Q_r(T_i)$ is the emitted radiation, $\varepsilon_i < 1$ is the emissivity and $Q_{g,i}$ is the thermal power generation inside slice i. In (5), $i = 1,\dots,m$, m being the number of slices composing the wall. Superscripts/subscripts 0 and $m+1$ denote either a zone of the building or the outside of the building. Note that $k_1^0 = k_m^{m+1} = 0$ as there is no thermal conduction on walls boundary surfaces, $h_i^{i-1} = 0\ \forall i > 1$, $h_i^{i+1} = 0\ \forall i < m$ and $\alpha_i^S = \alpha_i^L = \varepsilon_i = 0\ \forall i : 1 < i < m$, since there is no thermal convection nor radiation in between slices. Since we assumed each wall as a gray body, the power $Q_r(T_i)$ radiated from each slice is governed by $Q_r(T_i) = \sigma T_i^4$, where σ is the Stefan-Boltzmann constant. This expression is approximately linear around the slice mean operating temperature $\bar{T}_i$ so that it can be replaced by

$$Q_r(T_i) = 4\sigma\bar{T}_i^3 T_i - 3\sigma\bar{T}_i^4. \tag{6}$$

If we consider a generic wall w composed of m slices, the evolution of $\boldsymbol{T_w} = [T_{w,1}\cdots T_{w,m}]^\top$, with $T_{w,i}$ denoting the temperature of the i^{th} slice of wall w, can be described in matrix form by

$$\dot{\boldsymbol{T}}_{\boldsymbol{w}} = \boldsymbol{A_w T_w} + \boldsymbol{B_w T_z} + \boldsymbol{W_w d}, \tag{7}$$

where we recall that $\boldsymbol{T_z}$ is the vector containing the temperature of the n_z zones, that are treated as control inputs. The disturbance $\boldsymbol{d} = [T_{out}\ Q^S\ Q^L\ 1]^\top$ collects the outdoor temperature T_{out}, and the incoming shortwave Q^S and longwave Q^L radiation. The constant 1 in $\boldsymbol{d}$ is introduced to account for the constant term in (6). Finally, $\boldsymbol{A_w}$, $\boldsymbol{B_w}$ and $\boldsymbol{W_w}$ are suitably defined matrices that are easily derived based on the scalar equation (5), whose coefficients depend on the wall characteristics.

Equation (7) refers to a single wall. If there are n_w walls in the building, then, we can collect all walls temperatures in vector $\boldsymbol{T} = [\boldsymbol{T_1}^\top\cdots\boldsymbol{T_{n_w}}^\top]^\top$, and write the following equation for the evolution in time of $\boldsymbol{T}$:

$$\dot{\boldsymbol{T}} = \boldsymbol{AT} + \boldsymbol{BT_z} + \boldsymbol{Wd}, \tag{8}$$

where $\boldsymbol{A}$ is a block-diagonal matrix in which the w^{th} block is $\boldsymbol{A_w}$, $\boldsymbol{B} = [\boldsymbol{B_1}^\top\cdots\boldsymbol{B_{n_w}}^\top]^\top$ and $\boldsymbol{W} = [\boldsymbol{W_1}^\top\cdots\boldsymbol{W_{n_w}}^\top]^\top$.

For our control purposes, we need to consider the amount of heat exchanged between each wall and each adjacent zone. Considering wall w with surface S_w and zone j, the thermal power transferred from the wall to the zone is given by

$$Q_{w\to j} = S_w h_{w,b}^{b'}(T_{w,b} - T_{z,j}),$$

with $w = 1,\dots,n_w$ and $j = 1,\dots,n_z$. The pair (b,b') can either be $(1,0)$ or $(m,m+1)$ according to the notation introduced for (5). The total amount of thermal power transferred from the building walls to zone j can be expressed as $Q_{b,j} = \sum_{w\in\mathcal{W}} Q_{w\to j}$, where $\mathcal{W}$ is the set of walls w adjacent to zone j. Defining $\boldsymbol{Q} = [Q_{b,1}\cdots Q_{b,n_z}]^\top$, we obtain

$$\boldsymbol{Q} = \boldsymbol{CT} + \boldsymbol{DT_z}, \tag{9}$$

where $\boldsymbol{C}$ and $\boldsymbol{D}$ are suitable matrices. From (8) and (9), we finally get

$$\begin{cases}\dot{\boldsymbol{T}} = \boldsymbol{AT} + \boldsymbol{BT_z} + \boldsymbol{Wd}\\ \boldsymbol{Q} = \boldsymbol{CT} + \boldsymbol{DT_z}\end{cases} \tag{10}$$

Remark 1. Note that the obtained model, though linear, can be quite large. However, following Kim and Braun (2012) its order can be greatly reduced by applying the model reduction algorithm based on Hankel's Single Value Decomposition (HSVD). □

To solve the discrete-time optimal energy management problem, we need to consider a discretized version of (10). Given the linearity of (10), it holds that

$$\begin{aligned}\boldsymbol{T}((k+1)dt) &= e^{\boldsymbol{A}dt}\boldsymbol{T}(kdt)+\\ &+\int_{kdt}^{(k+1)dt} e^{\boldsymbol{A}((k+1)dt-\tau)}(\boldsymbol{BT_z}(\tau)+\boldsymbol{Wd}(\tau))d\tau.\end{aligned} \tag{11}$$

If we assume that $\boldsymbol{T_z}$ (i.e. our control variables) and $\boldsymbol{d}$ are linearly varying within each time slot, then the integral in (11) can be computed analytically. Formally, given

$$\boldsymbol{T_z}(\tau) = \frac{\boldsymbol{T_{z,k+1}} - \boldsymbol{T_{z,k}}}{dt}(\tau - kdt) + \boldsymbol{T_{z,k}}, \tag{12}$$

where $\boldsymbol{T_{z,k}} = \boldsymbol{T_z}(kdt)$, and

$$\boldsymbol{d}(\tau) = \frac{\boldsymbol{d_{k+1}} - \boldsymbol{d_k}}{dt}(\tau - kdt) + \boldsymbol{d_k},$$

where $\boldsymbol{d_k} = \boldsymbol{d}(kdt)$, $\forall\tau : \ kdt \le \tau < (k+1)dt$ and $k = 1,\dots,M$, if we set $\boldsymbol{T_k} = \boldsymbol{T}(kdt)$ and $\boldsymbol{Q_k} = \boldsymbol{Q}(kdt)$, $k = 1,\dots,M$, then the dicretized system can be expressed as follows

$$\begin{cases}\boldsymbol{T_{k+1}} = \boldsymbol{\Gamma_x T_k} + \boldsymbol{\Gamma_{u,1}T_{z,k+1}} + (\boldsymbol{\Gamma_{u,0}} - \boldsymbol{\Gamma_{u,1}})\boldsymbol{T_{z,k}}+\\ \qquad\quad + \boldsymbol{\Gamma_{w,1}d_{k+1}} + (\boldsymbol{\Gamma_{w,0}} - \boldsymbol{\Gamma_{w,1}})\boldsymbol{d_k}\\ \boldsymbol{Q}_k = \boldsymbol{CT_k} + \boldsymbol{DT_{z,k}}\end{cases}$$

where $\boldsymbol{\Gamma_x} = e^{\boldsymbol{A}dt}$, $\boldsymbol{\Gamma_{u,0}} = (\int_0^{dt} e^{\boldsymbol{A}s}ds)\boldsymbol{B}$, $\boldsymbol{\Gamma_{u,1}} = (\int_0^{dt} e^{\boldsymbol{A}s}(dt-s)ds)\boldsymbol{B}/dt$, $\boldsymbol{\Gamma_{w,0}} = (\int_0^{dt} e^{\boldsymbol{A}s}ds)\boldsymbol{W}$, and $\boldsymbol{\Gamma_{w,1}} = (\int_0^{dt} e^{\boldsymbol{A}s}(dt-s)ds)\boldsymbol{W}/dt$.

Applying the transformation $\boldsymbol{\xi_k} = \boldsymbol{T_k} - \boldsymbol{\Gamma_{u,1}T_{z,k}} - \boldsymbol{\Gamma_{w,1}d_k}$ we obtain

$$\begin{cases}\boldsymbol{\xi_{k+1}} = \boldsymbol{\Gamma_x\xi_k} + ((\boldsymbol{\Gamma_x} - \boldsymbol{I})\boldsymbol{\Gamma_{u,1}} + \boldsymbol{\Gamma_{u,0}})\boldsymbol{T_{z,k}}+\\ \qquad\quad + ((\boldsymbol{\Gamma_x} - \boldsymbol{I})\boldsymbol{\Gamma_{w,1}} + \boldsymbol{\Gamma_{w,0}})\boldsymbol{d_k}\\ \boldsymbol{Q_k} = \boldsymbol{C\xi_k} + (\boldsymbol{C\Gamma_{u,1}} + \boldsymbol{D})\boldsymbol{T_{z,k}} + \boldsymbol{C\Gamma_{w,1}d_k}\end{cases} \tag{13}$$

Dropping the bold notation for vectors and matrices, (13) can be rewritten as the following discrete-time system

$$\begin{cases}x(k+1) = \tilde{A}x(k) + \tilde{B}u(k) + \tilde{W}w(k)\\ y(k) = \tilde{C}x(k) + \tilde{D}u(k) + \tilde{V}w(k)\end{cases}, \tag{14}$$

where $x(k) = \boldsymbol{\xi_k}$, $u(k) = \boldsymbol{T_{z,k}}$, $w(k) = \boldsymbol{d_k}$, $y(k) = \boldsymbol{Q_k}$, $\tilde{A} = \boldsymbol{\Gamma_x}$, $\tilde{B} = (\boldsymbol{\Gamma_x} - \boldsymbol{I})\boldsymbol{\Gamma_{u,1}} + \boldsymbol{\Gamma_{u,0}}$, $\tilde{W} = (\boldsymbol{\Gamma_x} - \boldsymbol{I})\boldsymbol{\Gamma_{w,1}} + \boldsymbol{\Gamma_{w,0}}$, $\tilde{C} = \boldsymbol{C}$, $\tilde{D} = \boldsymbol{C\Gamma_{u,1}} + \boldsymbol{D}$, and $\tilde{V} = \boldsymbol{C\Gamma_{w,1}}$.

From (14) one can derive the expression of $x(k)$ and $y(k)$ as a function of the initial state and the inputs up to k.

Defining $\boldsymbol{u} = [u^\top(0) \cdots u^\top(M)]^\top$, $\boldsymbol{w} = [w^\top(0) \cdots w^\top(M)]^\top$ and $\boldsymbol{y} = [y^\top(0) \cdots y^\top(M)]^\top$, we finally have that

$$\boldsymbol{y} = Fx(0) + G\boldsymbol{u} + H\boldsymbol{w} \tag{15}$$

where F, G and H are suitably defined matrices.

Recalling that $y(k) = \boldsymbol{Q_k}$ and assuming that each component of $\boldsymbol{Q}(t)$ varies linearly within each time slot, the vector $E_w(k) = [E_{w,1}(k) \cdots E_{w,n_z}(k)]^\top$ of the overall thermal energy transferred from the walls to each zone can be computed as

$$E_w(k) = \frac{dt}{2}(\boldsymbol{Q_{k-1}} + \boldsymbol{Q_k}) = \frac{dt}{2}(y(k-1) + y(k)), \tag{16}$$

$k = 1, \ldots, M$. Finally, defining $\boldsymbol{E_w} = [E_w^\top(1) \cdots E_w^\top(M)]^\top$, from (15) and (16), we can derive $\boldsymbol{E_w} = \tilde{F}x(0) + \tilde{G}\boldsymbol{u} + \tilde{H}\boldsymbol{w}$.

2.2 People energy contribution E_p

Occupancy implies heat production and, in crowded places (e.g. offices), it can be a critical issue. According to an empirical model documented in Butcher (2006), if we consider the temperature $T_{z,j}(t)$ of the j^{th} zone, then the heat rate $Q_{p,j}(t)$ produced by a number of occupants $n_p(t)$ inside zone j is given by

$$Q_{p,j}(t) = n_p(t)(p_2 T_{z,j}^2(t) + p_1 T_{z,j}(t) + p_0), \tag{17}$$

where $p_2 = -0.2199$, $p_1 = 125.125$ and $p_0 = -1.7685 \cdot 10^4$. Although expression (17) is not convex as a function of $T_{z,j}$, it is almost linear in a sensible operating temperature range and can thus be linearized around some comfort temperature $\bar{T}_{z,j}$:

$$Q_{p,j}(t) = n_p(t)(\tilde{p}_1 T_{z,j}(t) + \tilde{p}_0). \tag{18}$$

Recalling that $T_{z,j}(t)$ varies linearly within each time slot (see from (12)), if we take $n_p(t)$ as a linear function of time as suggested in Borghesan et al. (2013), then equation (18) can be analytically integrated from $(k-1)dt$ to kdt to obtain the energy transferred to zone j

$$\begin{aligned} E_{p,j}(k) = {} & q_{2,k}(n_p) T_{z,j}(kdt) + \\ & + q_{1,k}(n_p) T_{z,j}((k-1)dt) + q_{0,k}(n_p) \end{aligned}$$

where $q_{2,k}(n_p) = \tilde{p}_1(n_{p,k}/3 + n_{p,k-1}/6)dt$, $q_{1,k}(n_p) = \tilde{p}_1(n_{p,k}/6 + n_{p,k-1}/3)dt$, and $q_{0,k}(n_p) = \tilde{p}_0(n_{p,k}/2 + n_{p,k-1}/2)dt$, with $n_{p,k} = n_p(kdt)$.

The total amount of energy transferred to all zones in each time slot can be packed in a vector $E_p(k) = [E_{p,1}(k) \cdots E_{p,n_z}(k)]^\top$ and then, defining $\boldsymbol{E_p} = [E_p^\top(1) \cdots E_p^\top(M)]^\top$, one can write that $\boldsymbol{E_p} = N(n_p)\boldsymbol{u} + e(n_p)$, where $N(n_p)$ and $e(n_p)$ depend on the above coefficients.

2.3 Other internal energy contributions E_{int}

There are many other types of heat sources that may affect the internal energy of a building, e.g. internal lighting, daylight radiation through windows, electrical equipment, etc. The overall heat flow rate transferred to zone j can be expressed as the sum of three contributions, namely

$$Q_{\text{int},j}(t) = \alpha_j(t) Q^S(t) + \kappa_j I_{\mathbb{R}^+}(n_p(t)) + \lambda_j, \tag{19}$$

where $\alpha_j(t)$ is a coefficient that takes into account the mean absorbance coefficient of zone j, the transmittance coefficients of the windows and their areas, sun view and shading factors, and radiation incidence angle. $I_{\mathbb{R}^+}(\cdot)$ denote the indicator function on the positive real values. The thermal energy contribution to zone j due to internal lightening and electrical equipment is composed of two contribution: a constant term λ_j, and an additional therm κ_j that represents the change in internal lightening and electrical equipment when people are present. Note that $Q_{\text{int},j}$ do not depends on Q^L because windows are usually shielded against longwave radiation. Similarly to the previous section, (19) can be discretized and integrated in order to obtain the energy $E_{\text{int},j}(k)$ during the k^{th} slot. We can collect the thermal energy of the zones in a single vector $E_{\text{int}}(k) = [E_{\text{int},1}(k) \cdots E_{\text{int},n_z}(k)]^\top$, and finally, $E_{\text{int}}(k)$, $k = 1, \ldots, M$ can be collected in a single vector $\boldsymbol{E_{\text{int}}} = [E_{\text{int}}^\top(1) \cdots E_{\text{int}}^\top(M)]^\top$.

2.4 Zones energy contributions E_z

Observe that in order to lower the temperature of a zone we need to draw energy from the zone itself. This energy contribution to the thermal balance equation (4) can be expressed as

$$E_{z,j}(k) = -C_{z,j}(T_{z,j}(kdt) - T_{z,j}((k-1)dt)), \tag{20}$$

where $C_{z,j}$ is the heat capacity of the j^{th} zone. Then, equation (20) with $k = 1, \ldots, M$ can be written in the following matrix form $\boldsymbol{E_z} = Z\boldsymbol{u}$, where $\boldsymbol{E_z} = [E_z^\top(1) \cdots E_z^\top(M)]^\top$, with $E_z(k) = [E_{z,1}(k) \cdots E_{z,n_z}(k)]^\top$ and Z is a suitably defined matrix.

2.5 Optimization problem

If we fix some nominal values for the disturbances affecting the system, then, the optimal energy management problem of minimizing the electrical energy cost while guaranteeing comfort and accounting for physical limits can be formalized as the following convex optimization problem in the control inputs $\boldsymbol{u}$ and $\boldsymbol{s}$:

$$\min_{\boldsymbol{u},\boldsymbol{s}} \quad \boldsymbol{\Psi} \cdot \boldsymbol{E_\ell}(\boldsymbol{u}, \boldsymbol{s}) \tag{21}$$

$$\begin{aligned} \text{subject to: } & \boldsymbol{u}_{min} \leq \boldsymbol{u} \leq \boldsymbol{u}_{max}, \quad |\boldsymbol{s}| \leq s_{max} \\ & 0 \leq \boldsymbol{S} \leq S_{max}, \quad \boldsymbol{E_c} \geq 0, \quad \boldsymbol{E_\ell}(\boldsymbol{u}, \boldsymbol{s}) \leq E_{max} \end{aligned}$$

where E_{max} is the maximum amount of electrical energy that the chiller can draw, S_{max} is the maximum amount of energy that the thermal storage can contain, s_{max} is the maximum amount of energy that can be stored/retrieved in/from the thermal storage in a single time slot, $\boldsymbol{u}_{min}$ and $\boldsymbol{u}_{max}$ are vectors representing temperatures lower/upper bounds for all time slots, $\boldsymbol{\Psi} = [\psi(1) \cdots \psi(M)]^\top$, according to (4) $\boldsymbol{E_c} = \boldsymbol{E_w} + \boldsymbol{E_p} + \boldsymbol{E_{\text{int}}} + \boldsymbol{E_z}$, and $\boldsymbol{E_\ell}$ can be computed from $\boldsymbol{E_c}$ using (2), (3) and (4). Obviously, $\boldsymbol{E_\ell}$ depends on $\boldsymbol{u}$ and $\boldsymbol{s}$ and this dependence is here made explicit via the notation $\boldsymbol{E_\ell}(\boldsymbol{u}, \boldsymbol{s})$.

3. NUMERICAL EXAMPLES

Consider a medium-sized office building: 20 m long, 20 m wide, and 10 m tall. The building is divided into three floors, each facade is half glazed and the roof is flat. In the following we will consider a single-zone setup, where the three floors are treated as a single zone with the same temperature set-point, and a multi-zone setup, where each floor is a zone with its own temperature set-point. The look-ahead time horizon is discretized in $dt = 10$ minutes time slots and is 48 hours long, though the strategy is then applied over a one-day time horizon. This is to avoid the depletion of the storage at the end of the day.

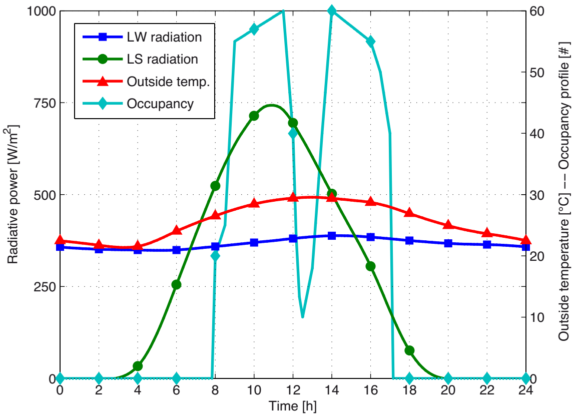

Fig. 3. External disturbances profiles.

A thermal storage with maximum capacity $S_{max} = 700$ MJ, a maximum charge/discharge rate of $s_{max} = 18$ MJ within each time slot, and a loss coefficient $a = 0.99$ is considered. As for the chiller, we set $E_{max} = 30$ MJ, and $c_1 = 1.1133 \cdot 10^{-5}$, $c_2 = 1.85 \cdot 10^{-2}$ and $c_3 = 3.6837$ in (2). To compute the policy, we consider realistic profiles for the external disturbances. Such profiles are depicted in Figure 3 for the first 24 hours. For the next 24 hours we consider the same profiles. Similarly, in Figure 4 we report the upper and lower bounds for the zones temperature (blue solid lines) and the profile of the energy price (green dashed line) during the first 24 hours. The period from 8 AM to 5 PM is referred to as "working hours". As for all other parameters, the reader is referred to Ioli (2014).

Single-zone setup: Two different strategies are compared: *Fixed*, i.e., during working hours the temperature is maintained at $24°C$, while during the rest of the time the chiller is idle; and *Optimal*, i.e., temperature profiles are determined by solving (21).

For each strategy we consider two cases: with and without thermal storage. In the fixed strategy, the thermal storage is charged during the night and discharged during working hours. In Figure 5 the zone temperature profile over one period is reported, when different strategies are applied. Almost no difference can be noticed between the fixed strategy with thermal storage (green dashed line) and the same strategy without it (blue solid line). Although both

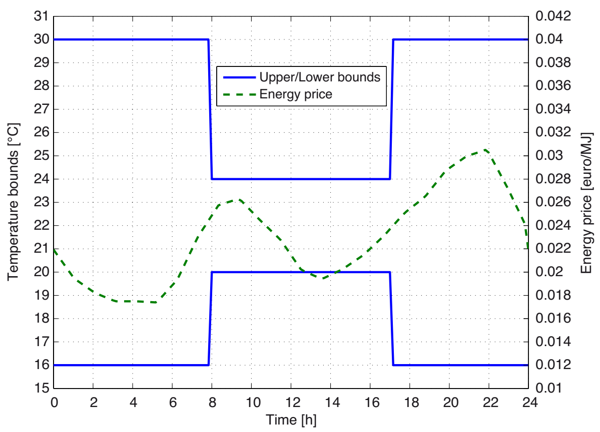

Fig. 4. Temperature bounds and energy price profile.

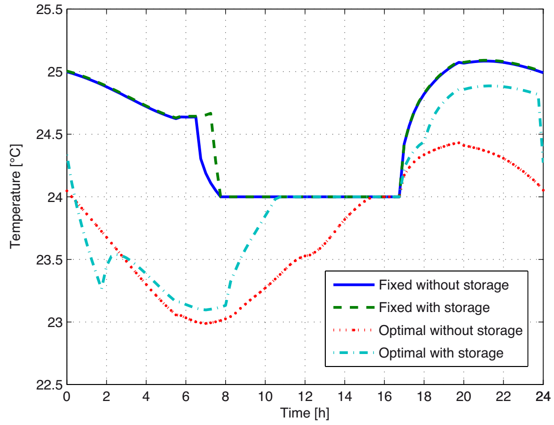

Fig. 5. Temperature profiles comparison.

optimal strategies (with and without thermal storage, cyan dot-dashed line and red dotted line respectively) provide different temperature profiles, we can identify three common phases: a pre-cooling phase, where the building temperature is lowered before working hours, a comfort phase in which the temperature is kept within the prescribed limits, and a final phase, where the temperature rises until a pre-cooling phase starts over again.

Figure 6 plots the profile of the energy requested to the chiller. A substantial difference can be seen between the fixed strategy without thermal storage and the other strategies. The proposed control strategy is able to compensate the lack of a thermal storage exhibiting an energy request similar to the fixed strategy case with thermal storage. This is achieved by exploiting the thermal inertia of the building structure as a storage, as suggested in Henze et al. (2003). In the case of the optimal control strategy with storage capabilities, the availability of both active and passive thermal storages allows the chiller to work closer to its best efficiency point. This is confirmed by the results in Table 1, where the overall energy needed for cooling, the energy requested from the chiller, the electrical energy consumption, and the total energy cost for all strategies (F = fixed without storage, F+S = fixed with storage, O = optimal without storage, O+S = optimal with storage) is reported. The optimal policy (with storage) uses only a little more than a quarter of the storage capacity. These considerations suggest the use of smaller chiller and thermal storage unit.

Strategy	F	F+S	O	O+S
E_c [MJ]	1094	1087	1288	1076
E_{ch} [MJ]	1094	1330	1288	1187
E_ℓ [MJ]	1219	742.3	750.5	694.4
Cost [euro]	29.09	16.95	16.63	14.44

Table 1. Total energy consumption and costs.

Multi-zone setup: Suppose now to adopt a different temperature profile in each floor. The optimization procedure can now rely on both thermal inertia of the building structure and thermal exchanges between zones. In the following we focus on the optimal strategy without thermal storage. The resulting temperature profiles depend on the zone and are indeed quite different (see Figure 7). In particular the ground floor (zone 1) is maintained at a tem-

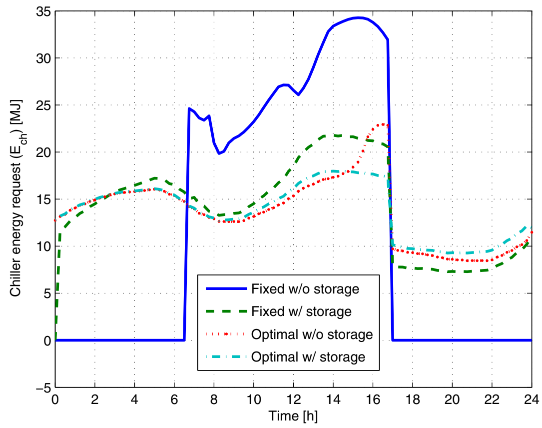

Fig. 6. Chiller energy request profiles comparison.

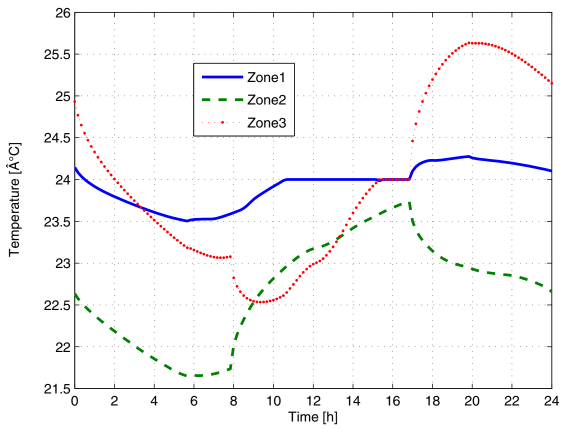

Fig. 7. Temperature profiles for each zone.

	Single-Zone	Multi-Zone	$\Delta_{\%}$
E_c [MJ]	1288	979.9	−23.9%
E_{ch} [MJ]	750.5	669.7	−14.5%
Cost [euro]	16.63	12.79	−21.1%

Table 2. Single-zone e multi-zone comparison.

perature level around the upper comfort limit of 24°C, the second floor (zone 3) follows a temperature profile similar to the single-zone case, and the first floor (zone 2) presents a strong pre-cooling phase in the morning before working hours, and right after to account for the next day thanks to the 48 hours look ahead horizon. The first floor is used as a thermal storage which drains heat from other floors through its pavement and its ceiling. Table 2 compares the results obtained with the optimal strategy without thermal storage in the single-zone and multi-zone setups, and shows that using a multi-zone setup can significantly reduce energy consumption and thus cooling costs.

4. CONCLUSIONS

We presented a novel approach to the optimal energy management of a building cooling system with thermal storage. A main distinguishing feature of our approach is that we adopt a model based on thermal energy balancing, and use the zones temperature set-points as control variables. This leads to a constrained convex optimization problem to be solved. Comparison between different strategies shows that the thermal inertia of the building structure can be effectively exploited to add flexibility to the system, and that increasing the number of controlled zones in a building can significantly improve performance.

Though in this work we refer to some forecasted operating conditions, disturbances are taken into account explicitly in the model formulation. In perspective, this allows to tackle the problem within a stochastic optimal control framework. Indeed, in Ioli (2014) preliminary promising results on a stochastic approach implementing a disturbance compensator via the "scenario approach" to constrained stochastic optimization (Campi et al. (2009)) are documented. Further work is needed in this direction.

REFERENCES

Balaras, C. (1996). The role of the thermal mass on the cooling load of buildings. an overview on computational methods. *Energy and Buildings*, 24.

Borghesan, F., Vignali, R., Piroddi, L., and M.Prandini (2013). Approximate dynamic programming-based control of a building cooling system with thermal storage. In *IEEE ISGT 2013*. Copenhagen, Denmark.

Butcher, K.J. (2006). *CIBSE Guide A: Environmental Design*. CIBSE Publications, Norwich, UK.

Campi, M., Garatti, S., and Prandini, M. (2009). The scenario approach for systems and control design. *Annual Reviews in Control*, 33(2), 149–157.

Ceriani, N., Vignali, R., Piroddi, L., and Prandini, M. (2013). An approximate dynamic programming approach to the energy management of a building cooling system. In *ECC 2013*. Zurich, Switzerland.

Deng, K., Sun, Y., Chakraborty, A., Lu, Y., Brouwer, J., and Mehta, P.G. (2013). Optimal scheduling of chiller plant with thermal energy storage using mixed integer linear programming. In *ACC 2013*. Washington, DC, USA.

D&R International, L. (2012). *2011 Buildings energy data book*. U.S. Department of Energy. Http://buildingsdatabook.eren.doe.gov/.

Gordon, J., Ng, K., and Chua, H. (1997). Optimizing chiller operation based on finite-time thermodynamics: universal modeling and experimental confirmation. *International Journal of Refrigeration*, 20(3), 191–200.

Henze, G.P., Felsmann, C., and Knabe, G. (2003). Evaluation of optimal control for active and passive building thermal storage. *International Journal of Thermal Science*, 1(1), 191–200.

Ioli, D. (2014). *Optimal energy management of a building cooling system with thermal storage: modeling and control*. Master thesis, Politecnico di Milano.

Kim, D. and Braun, J.E. (2012). Reduced-order building modeling for application to model-based predictive control. In *5th National Conference of IBPSA-USA*, 554–561. Madison, Wisconsin, USA.

Kim, D., Zuo, W., Braun, J.E., and Wetter, M. (2013). Comparisons of building system modeling approaches for control system design. In *13th Conference of International Building Performance Simulation Association*, 3267–3274. Chambery, France.

Kinter-Meyer, M. and Emery, A. (1994). Optimal control of an hvac system using cold storage and building thermal capacitance. *Energy and Buildings*, 23(1), 19–31.

Ma, Y., Borrelli, F., Hencey, B., Packard, A., and Bortoff, S. (2009). Model predictive control of thermal energy storage in building cooling systems. In *CDC/CCC 2009*, 392–397.

Ma, Y., Kelman, A., Daly, A., and Borrelli, F. (2012). Predictive control for energy efficient buildings with thermal storage: Modeling, simulation and experiments. *IEEE Control System Magasine*, 1(1), 45–66.

Powell, K.M., Cole, W.J., Ekariaka, U.F., and Edgar, T.F. (2013). Dynamic optimization of a campus cooling system with thermal storage. In *ECC 2013*, 4077–4082. Zurich, Switzerland.

