A PID automatic tuning method for distributed-lag processes ⋆

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Abstract: In this paper we present an automatic tuning methodology for PID controllers for distributed-lag processes. The technique is based on the evaluation of a closed-loop set-point or load disturbance step response and it can be therefore employed with process routine operating data. Further, a performance assessment index is also proposed in order to establish when the performance of a PID controller can be improved by retuning it according to the proposed method. Simulation results show the effectiveness of the approach.

1. INTRODUCTION

Distributed-lag processes are frequently encountered in the process industry. For example, transmission lines, heat exchangers, stirred tanks and distillation columns might have a dynamic characteristic so that they can be modelled as an infinite series of infinitesimally small interacting lags and therefore as a distributed lag (Shinskey, 1994). Despite this fact, this kind of processes are rarely considered in the academic literature (Shinskey, 2002), with the notable exception of the works of Shinskey (see, for example, (Shinskey, 2001)). Therein a tuning tule for Proportional-Integral-Derivative (PID) controllers has been proposed based on process parameters that are obtained by evaluating an open-loop step response.
Indeed, many tuning rules have been developed for PID controllers (O'Dwyer, 2006) and the great majority of them are based on a first-order-plus-dead-time (FOPDT) or second-order-plus-dead-time (SOPDT) model of the process that can be obtained typically by evaluating an open-loop step response. However, this experiment can be time-consuming and, above all, it can imply that the normal process operations are stopped, which is obviously not desirable. For this reason, automatic tuning methodologies have been developed also based on closed-loop experiments, usually by considering a relay-feedback experiment (Yu, 1999).

In this paper we present a methodology for the automatic tuning of PID controllers for distributed-lag processes which is based on the evaluation of a *closed-loop* set-point or load disturbance step response. In particular, we assume that a (possibly badly tuned) PID controller is operating and the evaluation of the step response is employed to retune the PID controller if the achieved performance is not satisfactory, as in (Veronesi and Visioli, 2009). In order to assess the performance of the controller, a performance index is proposed, so that the methodology can be applied both for tuning-on-demand (namely, the controller is tuned after an explicit request of the operator) and for self-tuning (namely, the controller itself determines that the control performance is not satisfactory and a new tuning is provided). It is worth stressing that the tuning rule applied is devoted to the load disturbance rejection task which is usually of main concern in the above mentioned processes.

The paper is organised as follows. A model for distributed-lag processes is given in Section 2. The autotuning method is presented in Section 3, where we explain how the relevant process parameters can be obtained and how the PID parameters can be selected. Finally, the practical implementation of the method is addressed. Simulation results are given in Section 4, and conclusions are drawn in Section 5.

2. MODELLING

A distributed-lag process can be described by the following transfer function (Shinskey, 1994)

$$P(s) = \frac{2\mu}{e^{\tau s} + e^{-\tau s}} = \frac{\mu}{\cosh\sqrt{\tau s}} \tag{1}$$

where μ is the process gain. The hyperbolic cosine can be expanded into an infinite-product series, so that we obtain

$$P(s) = \frac{\mu}{[1+(2/\pi)^2\tau s][1+(2/3\pi)^2\tau s][1+(2/5\pi)^2\tau s]\cdots} \tag{2}$$

It is worth noting that the sum of all time constants, denoted as T_0, is equal to 0.5τ. If a unit step is applied to the process input, the sum of all time constants can be estimated easily as the time the process variable takes to attain the 63.2% of its steady-state value (see Figure 1). Then, the process gain can be estimated easily by considering the steady-state value of the process output and the amplitude of the step input (Visioli, 2006a). However, the open-loop experiment can be time-consuming and, in order to perform it, it can be necessary to stop the routine process operations. Thus, we propose a method to estimate the value of T_0 and of the process gain μ with a closed-loop experiment, namely by employing a PID controller with any values of the parameters (provided that the closed-

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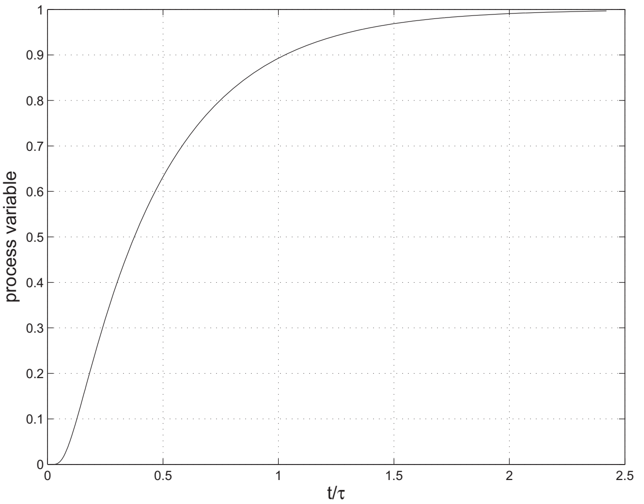

Fig. 1. Open-loop step response of a distributed-lag process. The process variable attains the 63.2% of its steady-state value at time $t = T_0 := 0.5\tau$.

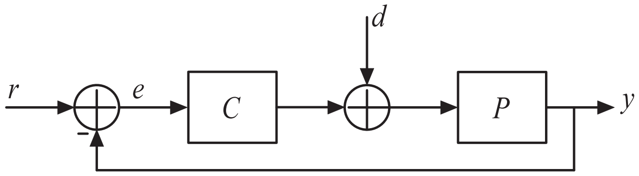

Fig. 2. The considered control scheme.

loop system is asimptotically stable).

Note that, for the purpose of simulation, transfer function (2) can be written as

$$P(s) = \frac{\mu}{\prod_{i=0}^{n-1}\left[1 + \left(\frac{2}{(2i+1)\pi}\right)^2 \tau s\right]} \tag{3}$$

with n at least equal to 20, because the dynamics of the process does not change significantly for $n > 20$ (Shinskey, 2001).

3. AUTOMATIC TUNING

3.1 Estimation of the process parameters from set-point step response

We consider the unity-feedback control system of Figure 2 where the process P is controlled by a PID controller whose transfer function is in series ("interacting") form:

$$C(s) = K_p\left(\frac{T_i s + 1}{T_i s}\right)(T_d s + 1). \tag{4}$$

The series form has been chosen for the sake of simplicity, however, the use of other forms is straightforward by suitably applying translation formulae to determine the values of the parameters (Visioli, 2006a). Note also that the use of a first-order filter that makes the controller transfer function proper has been neglected for the sake of clarity but it can be easily selected so that it does not influence the PID controller dynamics significantly.

We assume that the PID controller has been (roughly) tuned and a step signal of amplitude A_s is applied to the set-point. The process gain μ can be determined by considering the following trivial relations which involve the final steady-state value of the control variable u and of the control error e:

$$\lim_{t\to+\infty} u(t) = \frac{K_p}{T_i}\int_0^\infty e(t)dt = \frac{A_s}{\mu} \tag{5}$$

and therefore we have

$$\mu = A_s \frac{T_i}{K_p \int_0^\infty e(t)dt}. \tag{6}$$

The determination of the sum of the time constants T_0 of the process can be performed by considering the following variable:

$$e_u(t) = \mu u(t) - y(t). \tag{7}$$

By applying the Laplace transform to (7) and by expressing u and y in terms of the reference signal r we have

$$E_u(s) = \mu U(s) - Y(s) = \frac{C(s)(\mu - P(s))}{1 + C(s)P(s)}R(s). \tag{8}$$

At this point, for the sake of clarity, it is convenient to write the controller and process transfer functions respectively as

$$C(s) = \frac{K_p}{T_i s}\tilde{C}(s) \tag{9}$$

where

$$\tilde{C}(s) := (T_i s + 1)(T_d s + 1) \tag{10}$$

and (see (3))

$$P(s) = \frac{\mu}{q(s)} \tag{11}$$

where

$$q(s) = \prod_i(\tau_i s + 1) = \prod_i \tau_i s^n + \cdots + \sum_i \tau_i s + 1 \tag{12}$$

with

$$\tau_i := \left[\frac{2}{(2i+1)\pi}\right]^2 \tau, \qquad i = 0, 1, \ldots. \tag{13}$$

Then, expression (8) can be rewritten as

$$E_u(s) = \frac{\mu K_p \tilde{C}(s)}{T_i s q(s) + \mu K_p \tilde{C}(s)}(q(s) - 1)R(s). \tag{14}$$

By applying the final value theorem to the integral of e_u when a step is applied to the set-point signal we finally obtain (see (12))

$$\begin{aligned}\lim_{t\to+\infty}\int_0^t e_u(v)dv &= \lim_{s\to 0} s\frac{A_s}{s}\frac{\mu K_p \tilde{C}(s)}{T_i s q(s) + \mu K_p \tilde{C}(s)}\frac{q(s)-1}{s}\\ &= A_s \lim_{s\to 0}\frac{q(s)-1}{s}\\ &= A_s \sum_i \tau_i\\ &= A_s T_0.\end{aligned} \tag{15}$$

Thus, the sum of the time constants of the process can be obtained by evaluating the integral of $e_u(t)$ at the steady-state when a step signal is applied to the set-point and by dividing it by the amplitude A_s of the step.

Remark 1. It is worth noting that both the value of the gain μ and of sum of the time constants T_0 of the process are determined by considering the integral of signals and therefore the method is inherently robust to the measurement noise.

Remark 2. Note also that the set-point step signal can be applied just for the purpose of (re)tuning the PID controller (in this case its amplitude should be as small as possible in order perturb the process as less as possible) but also a step response during routine process operations can be employed. This issue will be further discussed in subsection 3.5.

Remark 3. It is worth stressing that the value of T_0 is obtained independently on the values of the PID parameters. This is an advantage with respect to the use of other methods for the identification of the process transfer function, whose result depends on the control variable and process variable signals.

3.2 Estimation of the process parameters from load disturbance step response

The process parameters can be estimated by evaluating also a load disturbance step d of amplitude A_d. However, in this case the amplitude A_d is not known and therefore must be estimated as well. This can be determined by considering the final value of the integral of the control error. In fact, the expression of the Laplace transform of the control error is:

$$E(s) = -\frac{P(s)}{1+C(s)P(s)}D(s) = -\frac{T_i s\mu}{T_i s q(s) + K_p\tilde{C}(s)\mu}\frac{A_d}{s}, \tag{16}$$

and therefore we obtain

$$\begin{aligned}\lim_{t\to+\infty}\int_0^t e(v)dv &= \lim_{s\to 0} s\frac{1}{s}\frac{A_d}{s}\left(-\frac{T_i s\mu}{T_i s q(s) + K_p\tilde{C}(s)\mu}\right)\\ &= -\frac{A_d T_i}{K_p}.\end{aligned} \tag{17}$$

Thus, the amplitude of the step disturbance can be determined as

$$A_d = -\frac{K_p}{T_i}\int_0^\infty e(t)dt. \tag{18}$$

Once the amplitude of the step disturbance has been determined, the process gain μ can be determined by first considering the Laplace transform of the process input $i = u + d$, that is:

$$\begin{aligned}I(s) &= U(s) + D(s)\\ &= -\frac{C(s)P(s)}{1+C(s)P(s)}D(s) + D(s)\\ &= \frac{1}{1+C(s)P(s)}\frac{A_d}{s}\\ &= \frac{T_i s q(s)}{T_i s q(s) + K_p\tilde{C}(s)\mu}\frac{A_d}{s}.\end{aligned} \tag{19}$$

Thus, if we integrate $i(t)$ and we determine the limit for $t \to +\infty$ we obtain

$$\lim_{t\to+\infty}\int_0^t i(v)dv = \lim_{s\to 0} s\frac{1}{s}\frac{T_i s q(s)}{T_i s q(s) + K_p\tilde{C}(s)\mu}\frac{A_d}{s} = \frac{T_i A_d}{\mu K_p} \tag{20}$$

The process gain μ can be therefore found easily, once the value of A_d has been determined by using (18), as

$$\mu = A_d\frac{T_i}{K_p\int_0^\infty (u(t)+A_d)dt}. \tag{21}$$

Finally, the determination of the sum of the time constants of the process can be performed by initially considering the variable

Table 1. Tuning rules for distributed-lag processes.

	K_p	T_i	T_d
PI	$5/\mu$	$0.54T_0$	0
PID	$100/15/\mu$	$0.25T_0$	$0.10T_0$

$$e_i(t) := \mu(u(t) + d(t)) - y(t). \tag{22}$$

By applying the Laplace transform to (22) and by expressing u and y in terms of d we have

$$\begin{aligned}E_i(s) &= \frac{\mu - P(s)}{1+C(s)P(s)}D(s)\\ &= \frac{\mu - P(s)}{1+C(s)P(s)}\frac{A_d}{s}\\ &= \frac{\mu T_i A_d s}{T_i s q(s) + K_p\mu\tilde{c}(s)}\frac{q(s)-1}{s}.\end{aligned} \tag{23}$$

By twice integrating e_i and by applying the final value theorem we obtain (see (15))

$$\begin{aligned}&\lim_{t\to+\infty}\int_0^t\int_0^{v_2} e_i(v_1)dv_1dv_2\\ &= \lim_{s\to 0} s\frac{1}{s^2}\frac{\mu T_i A_d s}{T_i s q(s) + \mu K_p\tilde{C}(s)}\frac{q(s)-1}{s}\\ &= \frac{T_i A_d}{K_p}T_0.\end{aligned} \tag{24}$$

Thus, T_0 can be obtained as

$$T_0 = \frac{K_p}{T_i A_d}\int_0^\infty\int_0^t e_i(v)dvdt. \tag{25}$$

Remark 4. Note that also in this case the estimation of the process parameters is based on the integral of signals and therefore the method is inherently robust to the measurement noise. Further, the process parameters are obtained independently on the values of the PID parameters, because the estimation is based on steady-state values of the variables. Finally, as for the set-point step response, the step disturbance signal can be applied just for the purpose of (re)tuning the PID controller (in this case its amplitude should be as small as possible in order to perturb the process as less as possible) but also a step response during routine process operations can be employed.

Remark 5. In the proposed method, the occurrence of an abrupt (namely, step-like) load disturbance has been assumed. Indeed, this is the most relevant case for the control system, as the disturbance excites significantly the dynamics of the control system itself. Thus, the performance assessment technique has to be implemented together with a procedure for the detection of abrupt load disturbances. Methods for this purpose have been proposed in (Hägglund and Åström, 2000; Veronesi and Visioli, 2008).

3.3 Tuning of the controller

Once the sum of the time constants has been estimated by evaluating the set-point or the load disturbance step response, the PID controller can be tuned properly by considering the load disturbance rejection task, which is usually of main concern in practical applications. We propose to use the tuning rules devised by Shinskey and explained in (Shinskey, 1994, 2001). They are reported in Table 1, for the sake of clarity, for both PI and PID controller.

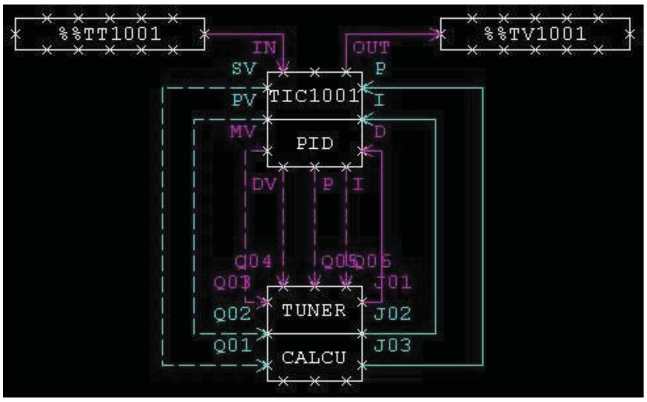

Fig. 3. The implementation of the proposed technique by means of the Yokogawa Centum VP Distributed Control System (courtesy of Yokogawa Italia).

3.4 Practical implementation

If a set-point step response is employed for estimating the process parameters, the proposed methodology can be easily implemented in a DCS with a suitable software development environment, as shown in Figure 3. The PID block (TIC1001) executes the standard PID control: its input and output are indicated respectively as TT1001 and TV1001. The calculation block TUNER determines the value of T_0 by computing the integral of the process variable (PV) and of the control output (MV) it receives from the PID block. Further, it computes the process gain. For this reason it needs also the setpoint (SV) and the PI parameters, namely, the proportional gain (or, equivalently, the proportional band) and the integral time constant. Finally the block TUNER computes the new values of the PID parameters by implementing the tuning rules shown in Table 1 and send them back to the PID block. Note that Q01..08 and J01..03 are the conventional name of the ports that the calculation block uses for exchanging data with the other function blocks.
If a load disturbance response is employed, the estimation procedure has to be estimate first the step amplitude A_d and then its value has to be employed to determine the μ and T_0 as indicated in (21) and (25).

3.5 Performance assessment

In a practical context it is also useful to evaluate the performance of a (PID) controller in order to determine if it has to be retuned or not. This is especially necessary if a self-tuning procedure has to be implemented, namely, the control system itself evaluates the control performance during process routine operations and a new tuning is provided in case it is not satisfactory. In this context, a measure of the performance of a control system can be effectively based on the integrated absolute error

$$IAE = \int_0^\infty |e(t)|dt \tag{26}$$

which implicitly considers both the peak error value and the settling time. For the technique proposed in this paper it is of interest to assess the control performance when a load disturbance occurs. For this purpose, the integrated absolute error obtained by applying the tuning rules of Table 1 to distributed-lag processes (2) with different

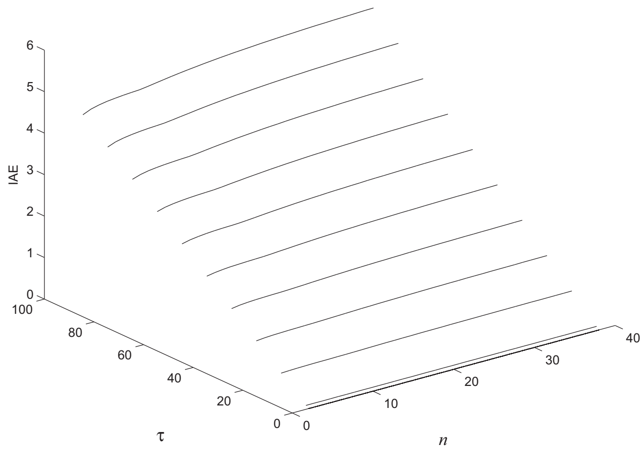

Fig. 4. Values of IAE for different values of τ and n (process order) with a PI controller tuned according to Table 1.

values of μ and τ, and different process order n has been computed. Results for $\mu = 1$ are shown in Figure 4 and 5 for PI and PID controller respectively. By interpolating these results, we obtain that the value of IAE achieved by applying the tuning rules of Table 1 are (for PI and PID controllers respectively):

$$IAE_{PI} = 0.058\tau\mu = 0.116T_0\mu \tag{27}$$

$$IAE_{PID} = 0.02\tau\mu = 0.04T_0\mu \tag{28}$$

Thus, the integrated absolute error achieved by a PI(D) controller should be ideally that expressed in (27) and (28). A performance index can be therefore defined as

$$J_{PI} = \frac{IAE_{PI}}{\int_0^\infty |e(t)|dt} \tag{29}$$

$$J_{PID} = \frac{IAE_{PID}}{\int_0^\infty |e(t)|dt} \tag{30}$$

and it can be determined, once the process parameters have been estimated by applying the technique described previously, by considering the obtained integrated absolute error.
In principle, the performance obtained by the control system is considered to be satisfactory if $J_{PI} = 1$ or $J_{PID} = 1$. From a practical point of view, however, the controller can be considered to be well-tuned if J_{PI} or J_{PID} is greater than a threshold (less than one) which can be selected by the user depending on how tight are its control specifications. In any case a sensible default value of 0.8 can be fixed.
Remark 6. It turns out from the presented results that using the derivative action allows to improve the performance significantly with respect to a PI controller.
Remark 7. It is worth noting that a performance index J greater than one can result because of the (small) interpolation error in determining (29) and (30) and because in any case the tuning formulae of Table 1 does not guarantee that the integrated absolute error is globally minimized.

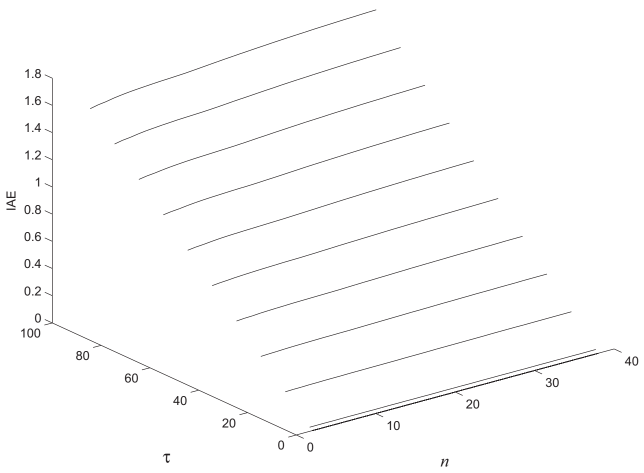

Fig. 5. Values of IAE for different values of τ and n (process order) with a PID controller tuned according to Table 1.

4. SIMULATION RESULTS

4.1 Example 1 - PID control

As a first example we consider a process with $\mu = 1$, $\tau = 10$ and $n = 30$ lags. Initially, the PID controller parameters are selected as $K_p = 3.3$, $T_i = 1.9$, $T_d = 0.25$. Then, a unit step load disturbance is applied to the process and the amplitude of the disturbance, the gain of the process and the sum of the time constants are estimated as $A_d = 1$, $\mu = 1.0$, and $T_0 = 4.97$. Based on these values, the PID parameters are retuned, according to Table 1, as $K_p = 6.66$, $T_i = 1.24$, $T_d = 0.5$. The load disturbance step response provided by the new values of the PID controller parameters is shown as a solid line in Figure 6, where the load disturbance step response provided by the initial values is also plotted as a dashed line. The control signal is not shown for the sake of brevity, in any case there are no significant differences between the two cases. By retuning the controller, the performance index is improved from $J_{PID} = 0.32$ to $J_{PID} = 1.03$ while the integrated absolute error decreases from IAE $= 0.62$ to IAE $= 0.19$. It is worth noting that the same result is achieved if a set-point step response is employed for estimating the process parameters.

4.2 Example 2 - PI control

As a second example we consider the same process of Example 1, but the use of a PI controller is assumed. Initially, the controller parameters are selected as $K_p = 7$ and $T_i = 2$ (note that the controller is aggressive). Then, a unit step load disturbance is applied to the process and the amplitude of the disturbance, the gain of the process and the sum of the time constants are estimated as $A_d = 1$, $\mu = 0.99$, and $T_0 = 4.97$ (the same parameters are estimated by considering a set-point step response). Based on these values, the PI parameters are retuned, according to Table 1, as $K_p = 5.06$ and $T_i = 2.68$. The load disturbance step responses provided by the initial and new values of the PI controller parameters are shown in Figure 7 as a dashed and solid line respectively. As

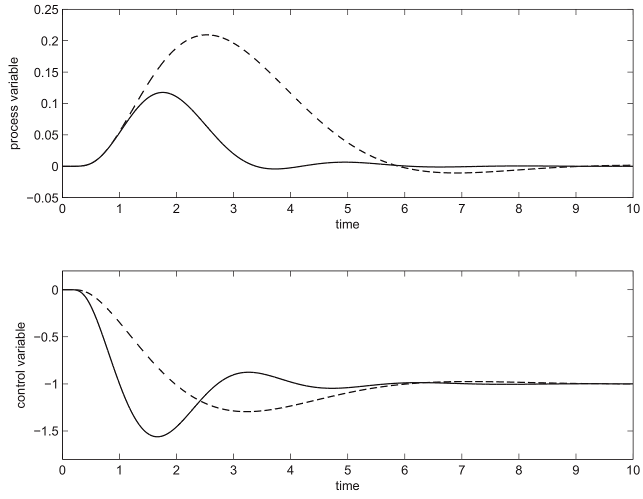

Fig. 6. Load disturbance step response for example 1. Dashed line: initial tuning. Solid line: automatic tuning.

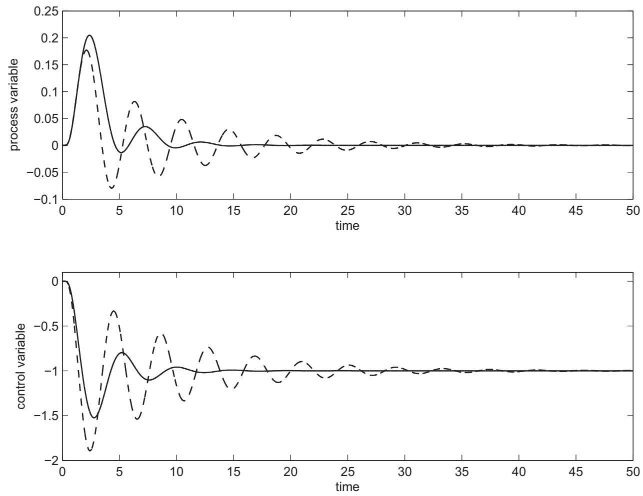

Fig. 7. Load disturbance step response for example 2. Dashed line: initial tuning. Solid line: automatic tuning.

in Example 1, retuning the controller allows to increase the performance. In particular, the performance index is improved from $J_{PI} = 0.64$ to $J_{PID} = 1.01$ while the integrated absolute error decreases from IAE $= 0.88$ to IAE $= 0.56$.

It turns out that the proposed autotuning method is effective and, by comparing these results with those of Example 1, it appears that the use of the derivative action allows to increase the controller performance significantly.

4.3 Example 3 - Measurement noise

As a third example we consider again the same process of Example 1, but the process output is corrupted with zero-mean white noise with a variance of $0.1 \cdot 10^{-3}$. The load disturbance step response obtained by selecting the controller parameters as $K_p = 3$, $T_i = 2$, and $T_d = 0.5$ is shown in Figure 8. In order to determine the performance index J_{PID} correctly, it is necessary to discard from the computation of the integrated absolute error those areas

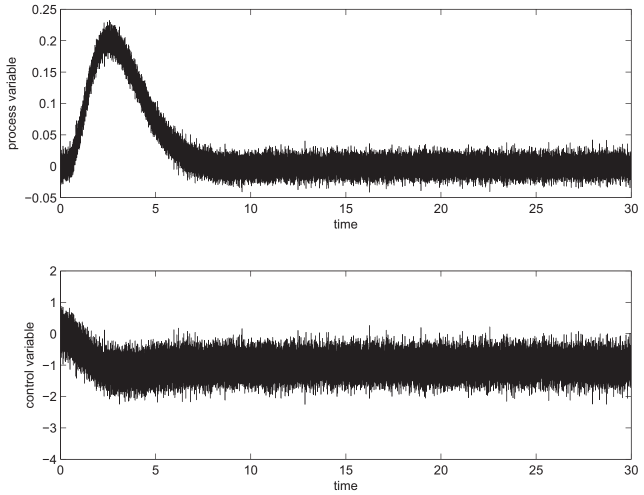

Fig. 8. Load disturbance step response for example 3 with $K_p = 3$, $T_i = 2$, and $T_d = 0.5$.

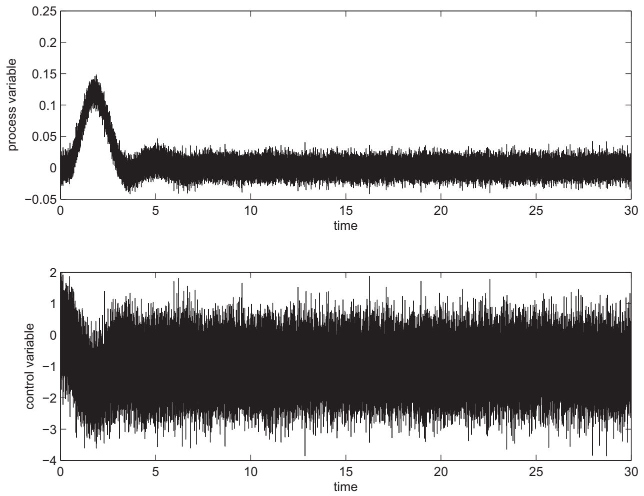

Fig. 9. Load disturbance step response for example 3 with $K_p = 6.87$, $T_i = 1.26$, and $T_d = 0.51$.

whose value is less than a predefined threshold (because they are actually due to the noise) (Visioli, 2006b). It results $J_{PID} = 0.29$, which suggests that the controller needs to be retuned. The gain of the process and the sum of the time constants are then estimated as $A_d = 1$, $\mu = 0.97$, and $T_0 = 5.06$ (once again, note that virtually the same values are obtained by considering a set-point step response). Based on these values, the PID parameters are retuned, according to Table 1, as $K_p = 6.87$, $T_i = 1.26$, and $T_d = 0.51$. The load disturbance step response obtained with the new PID controller is shown in Figure 9. In this case the performance index is $J_{PID} = 1.06$. By retuning the controller the integrated absolute error is decreased from IAE = 0.68 to IAE = 0.19. It turns out that the presence of noise does not impair the effectivess of the method, as expected because the considered variables are integrated.

5. CONCLUSIONS

In this paper we have proposed an automatic tuning methodology for distributed-lag processes based on a closed-loop experiment. Being based on the evaluation of a set-point or load disturbance step response, the technique can employ process routine operating data and can therefore be extended straightforwardly as a self-tuning method. Indeed, a performance index has been devised in order to assess the performance of the controller based on the achieved integrated absolute error. Illustrative examples have shown the effectiveness of the method and that it is robust to the measurement noise. Thus, the methodology appears to be suitable to implement in an industrial setting.

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