Cascade Hybrid Control for Anaerobic Digestion Systems ⋆

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Abstract: A cascade hybrid controller is proposed to guarantee the operational stability of anaerobic digestion systems and at the same time to assure a satisfactory remotion. The effluent chemical oxygen demand (COD) concentration and the dilution rate are taken respectively as the regulated and the manipulated variables. This controller is composed by a continuous inner loop which controls the concentration of volatile fatty acids and updates its reference at each sampling period according to the available discrete information of the COD, enhancing the robustness of the closed-loop with respect of influent disturbances. The performance of the proposed control scheme is illustrated via numerical simulations and compared with a discrete controller which only uses the COD information.

Keywords: Cascade control, anaerobic digestion, continuous and discrete measurements

1. INTRODUCTION

The stringent regulation policies imposed in chemical and biological processes has brought abut the need of more efficient monitoring, decision and control systems. Nowadays, it is not enough to regulate readily available variables such as pH or temperature, to guarantee both product quality and process safety (Alcaraz-González and González-Álvarez, 2007). In the particular case of anaerobic digestion, one must pay attention to certain operating conditions that may lead the system to the eventual crash even under tightly controlled pH and temperature conditions (Dochain, 2008). The last two decades have seen an increasing interest to improve the operation of bioprocesses by applying advanced control schemes. In particular, Anaerobic digestion (AD) has regained the interest of the wastewater treatment scientific and industrial community to reduce and transform the organic matter from industrial and municipal effluents into a high-energy gas (Henze et al., 1997). Nevertheless, its widespread application has been limited, because of the difficulties involved in achieving the stable operation of the AD process, which cannot be guaranteed by regulating temperature and pH, because the microbial community within the AD process is quite complex (Méndez-Acosta et al., 2008). In addition, the behavior of such a process may be affected by the substrate composition and inhibition by substrates or products, for example the accumulation of volatile fatty acids (VFA). Moreover, it is well-known that, to guarantee the so-called operational stability (Hill et al., 1987) and to avoid the eventual crash of the anaerobic digester, the organic matter in the liquid phase must be kept in a set of predetermined values.

Over the past decade, the regulation of the organic matter has been addressed by proposing many control techniques to keep certain operating variables which are readily available (such as the chemical oxygen demand (COD) and the biogas production) at a predetermined value (Schügerl, 2001; Alvarez-Ramírez et al., 2002; Puñal et al., 2002; Ahring and Angelidaki, 1997). For example, Alvarez-Ramírez et al. (2002) designed a continuous PI feedback controller to regulate COD concentration which uses the VFA as a secondary measurement that is incorporated into the feedback loop scheme to enhance the robustness of the control scheme with respect of influent disturbances. Nevertheless, the problem of the operational instability due to the accumulation of volatile fatty acids remains open (Méndez-Acosta et al., 2008). In this context, the regulation of the VFA concentration as a controlled variable seems to be very promising, because the operational stability of the AD process is largely dependent on the accumulation of VFA. For instance, some authors recommend a VFA concentration below 1.5 g/l (25 mmol/l) (Angelidaki et al., 2004). For this reason, it is necessary to design multiobjective controls which fulfill environmental regulations about COD effluents and at the same time guarantee the operational stability.

In this work we propose the regulation of COD concentration using a cascade scheme which assures the robustness and the operational stability via VFA continuous regulation, where the VFA reference is updated at each sampling period according to the available discrete information of the COD, in contrast with the continuous information used by Alvarez-Ramírez et al. (2002). The proposed scheme is a cascade hybrid control since is composed by a continuous and a discrete part. The paper is organized as follows. In section 2 the AD model is presented. Then, in section 3 the controller is designed and in section 4 it is implemented

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and tested via numerical simulations. Finally, the paper is closed with some concluding remarks.

2. MODEL PRESENTATION

There are hundreds of dynamical models available to describe anaerobic digestion, from the basic ones considering only one biomass (Andrews, 1968) to detailed models including several bacterial populations and several substrates. Among the complex models available in the literature, the IWA Anaerobic Digestion Model 1 (Batstone et al., 2002) has imposed itself as an useful tool to describe the behavior of a digestion plant with more insight into the process dynamics. However, its excessive complexity makes any advanced mathematical analysis of the model critical (Hess and Bernard, 2008). Thus it is considered a simplified macroscopic model of the anaerobic process based on 2 main reactions (Bernard et al., 2001), where the organic substrate (S_1) is degraded into volatile fatty acids (S_2) by acidogenic bacteria (X_1), and then the VFA are degraded into methane CH_4 and CO_2 by methanogenic bacteria (X_2). A mass balance model in a continuous stirred tank reactor (CSTR) can straightforwardly be derived (Bastin and Dochain, 1990; Bernard et al., 2001):

$$\dot{X}_1 = \mu_1(S_1)X_1 - \alpha D X_1 \tag{1a}$$
$$\dot{S}_1 = -k_1\mu_1(S_1)X_1 + D(S_{1in} - S_1) \tag{1b}$$
$$\dot{X}_2 = \mu_2(S_2)X_2 - \alpha D X_2 \tag{1c}$$
$$\dot{S}_2 = k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2 + D(S_{2in} - S_2) \tag{1d}$$

where D is the dilution rate, S_{1in} and S_{2in} are respectively, the concentrations of influent organic substrate and of influent VFA. The k_is are pseudo-stoichiometric coefficients associated to the bioreactions. Parameter $\alpha \in (0,1]$ represents the fraction of the biomass which is not retained in the digester. The specific bacterial growth rates, $\mu_1(S_1)$ and $\mu_2(S_2)$, are given by the nonlinear equations (Dochain and Vanrolleghem, 2001; Van-Impe et al., 1998):

$$\mu_1(S_1) = \mu_{\max 1}\frac{S_1}{S_1 + K_{S1}}$$
$$\mu_2(S_2) = \mu_{\max 2}\frac{S_2}{S_2 + K_{S2} + (S_2/K_{I2})^2}$$

where $\mu_{1\max}$, K_{S1}, $\mu_{2\max}$, K_{S2} and K_{I2} are the maximum bacterial growth rate and the half-saturation constant associated to the substrate S_1, the maximum bacterial growth rate in the absence of inhibition, and the saturation and inhibition constants associated to substrate S_2, respectively.

Hess and Bernard (2008) presented an extensive steady state analysis for system (1). They have found that, for a given dilution rate, the VFA concentration have three equilibrium points mathematically stables, but two of them are operationally unstable since one drives to substrate inhibition which can produce methanogenic bacterial extinction, which is the second operational unstable equilibrium. For this reason volatile fatty acids must be controlled, but at the same time, it should be guaranteed the COD remotion. In the next section we propose a control algorithm to achieve these goals.

3. CONTROL DESIGN

3.1 Problem formulation

Consider a nonlinear system describe by

$$\dot{x}(t) = f(x(t), u(t), \mu) \tag{2}$$
$$y_1(t) = C_1 x(t) \tag{3}$$
$$y_2(k\delta) = C_2 x(k\delta) \tag{4}$$

where $x \in \mathbb{R}^n$ represents the state vector, $u \in \mathbb{R}$ describes the input vector, $y_1 \in \mathbb{R}$ is a continuous measurement, while $y_2 \in \mathbb{R}$ is a discrete measurement obtained at each sampling period δ. Finally, $\mu \in \mathbb{R}^p$ is a parameter vector which may take values in a neighborhood $\mathcal{P} \in \mathbb{R}^p$ of the nominal ones, μ_0.

Given a constant value y_{2r}, it is desirable to regulate the output y_2 around y_{2r}, then, it can be defined the regulation error

$$e_2 = y_2 - y_{2r} \tag{5}$$

and the proposed *Regulation Problem* consists in finding, if possible, a controller which guarantees that $\lim_{t\to\infty} e_2(t) = 0$. However this controller must take into account both, continuous and discrete outputs (3)-(4) in order to guarantee the stability of the closed-loop system. The next assumption about the steady state around $e_2 = 0$ is instrumental in the controller design.

Assumption 1. Given the constant reference y_{2r} there exists at least one linear vector $x_{ss}(\mu)$ and one scalar $u_{ss}(\mu)$ such that equations

$$0 = f(x_{ss}, u_{ss}, \mu), \tag{6}$$
$$0 = C_2 x_{ss} - y_{2r}, \tag{7}$$

hold. Additionally, the scalar y_{1ss} is defined such that

$$0 = C_1 x_{ss} - y_{1ss}. \tag{8}$$

Remark 2. x_{ss} represents the steady state vector, u_{ss} is the input necessary to achieve this steady state, while y_{1ss} denotes the constant value which reaches the output, y_1. Notice that, by using the central manifold theory (Isidori, 1995), it is evident that x_{ss}, u_{ss} and y_{1ss} will depend on both y_{2r} and the unknown parameter vector, μ. Additionally, assumption 1 can be reformulated in order to fix the value of y_{1ss} and find the solution of (6), (7) and (8) for x_{ss}, u_{ss} and y_{2r}. Finally, if for a given y_{2r}, the solution of (6) and (7) is not unique, neither is y_{1ss}.

In order to design a controller which solves the proposed regulation problem, some basic concepts will be presented in the next subsection.

3.2 Basic facts about jump observers

Consider the linear system

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad \forall t \in [0, \infty) \tag{9}$$
$$y(k\delta) = Cx(k\delta) \qquad k = 1, 2, 3, \ldots, \tag{10}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^q$ are the state, input and output vectors, respectively. In this case the outputs are obtained at each sampling time δ.

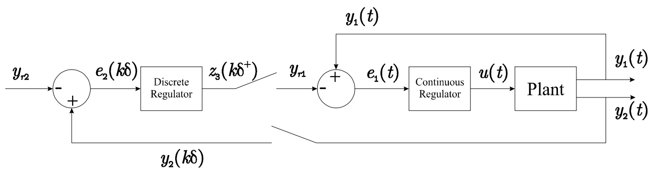

Fig. 1. Cascade proposed control scheme.

The usual way to estimate the unknown states of system (9) from output (10) consists in discretizing and designing a discrete observer. However, the observer thus obtained only provides information at each sampling period. Additionally, to obtain a discrete version of (9) it is necessary to have a well defined input in order to place the appropriate holder (for example a zero holder or a exponential holder), hence unexpected input variations during intersampling periods may produce a failure in the discrete observer (García-Sandoval, 2006). For this reason, an interesting problem would be to construct an observer of the form

$$\dot{z}(t) = Az(t) + Bu(t) \qquad \forall t \neq k\delta \tag{11}$$

$$z\left(k\delta^+\right) = z(k\delta) - G\left[y(k\delta) - Cz(k\delta)\right] \quad t = k\delta \tag{12}$$

where $z \in \mathbb{R}^n$ are the observer states and $z\left(k\delta^+\right)$ denotes the updated observer states at each sampling instant, i.e this is a continuous observer which updates its states at each sampling instant. The next lemma establishes conditions for the existence of such observer.

Lemma 3. Consider system (9)-(10) and suppose the pair $\left(e^{A\delta}, C\right)$ is observable, then an observer of the form (11)-(12), where the matrix gain G is such that matrix $(I + GC)\,e^{A\delta}$ is Schur, guarantees that $\lim_{t\to\infty}\left[x(t) - z(t)\right] = 0$.

Proof. See Appendix.

Remark 4. The main feature of observer (11)-(12) is that the intersampling state information is available at any time and it is not necessary to have a pre-established dynamic behavior for the input. Equation (11) can be seen as an continuous open loop observer in the intersampling period, whose states, according to (12), are updated at the sampling instant.

Observer (11)-(12) can be used to design a cascade controller which uses the continuous information in the inner loop and the discrete information in the external loop, as presented in the next subsection.

3.3 Proposed controller

In order to solve the regulation problem, it is proposed to design a cascade control scheme where the inner loop consists of a continuous controller which uses the input $u(t)$ to regulate the continuous output, y_1, around y_{1r} (notice that y_{1r} may not necessarily be equal to y_{1ss}, since y_{1ss} is initially unknown). Then, by using a jump observer, it is possible to devise an external loop controller which estimates the reference y_{1r} necessary to regulate the discrete output, y_2, around y_{2r}. The scheme is illustrated in Figure 1.

The next Theorem presents the proposed solution to the regulation problem.

Theorem 5. Consider assumption 1 holds. Assume the pairs

$$[A_0, B_0] \quad \text{and} \quad [A, C_{T1}]$$

with

$$A = \begin{pmatrix} A_0 & -B_0 \\ 0 & 0 \end{pmatrix}, \quad C_{T1} = (C_1 \ 0), \tag{13}$$

$$A_0 = \left[\frac{\partial f}{\partial x}\right]_{(x,u,\mu)=(0,0,0)} \quad \text{and} \quad B_0 = \left[\frac{\partial f}{\partial u}\right]_{(x,u,\mu)=(0,0,0)}$$

are stabilizable and detectable, respectively. Then a controller which solves the regulation problem for system (2) using outputs (3) and (4) is

$$\begin{aligned} \dot{z}_1(t) &= (A_0 - G_1C_{01})\,z_1(t) - B_0z_2(t) \\ &\quad + B_0u(t) + G_1e_1(t), \qquad \forall t \neq k\delta, \quad (14a) \\ \dot{z}_2(t) &= -G_2C_{01}z_1(t) + G_2e_1(t), \quad \forall t \neq k\delta, \quad (14b) \\ z_3\left(k\delta^+\right) &= G_dC_{02}z_1(k\delta) + z_3(k\delta) \qquad \forall t = k\delta, \\ &\quad - G_de_2(k\delta), \qquad k = 1,2,3,\ldots (14c) \\ u(t) &= Kz_1(t) + z_2(t) \qquad (14d) \end{aligned}$$

with

$$e_1(t) = y_1(t) - z_3\left(k\delta^+\right), \tag{15}$$

$$e_2(k\delta) = y_2(k\delta) - y_{2r}, \tag{16}$$

where K and $G = (G_1, G_2)^T$ are such that matrices $(A_0 + B_0K)$ and $(A - GC_{T1})$ are Hurwitz, while given matrices

$$\overline{A}_d = \begin{pmatrix} A_d & -M_d \\ 0 & 1 \end{pmatrix} \tag{17}$$

with

$$A_d = e^{(A - GC_{1T})\delta} \quad \text{and} \quad M_d = \int_0^\delta e^{(A - GC_{1T})\lambda}Gd\lambda, \tag{18}$$

the scalar G_d is such that $\left(I + \overline{G}_d\overline{C}_2\right)\overline{A}_d$ is Schur, where

$$\overline{G}_d = \begin{pmatrix} 0 \\ G_d \end{pmatrix}, \quad \overline{C}_2 = (C_{T2} \ 0) \quad \text{and} \quad C_{T2} = (C_2 \ 0), \tag{19}$$

obviously the pair $\left(\overline{A}_d, \overline{C}_2\right)$ must be detectable.

Proof. Given a reference y_{2r}, if the solution of (6), (7) and (8) is x_{ss}, u_{ss} and y_{1ss}, the linear version of system (2) around the pair $(x, u) = (x_{ss}, u_{ss})$ and the nominal values μ_0 is

$$\begin{aligned} \dot{\underline{x}}(t) &= A_0\underline{x}(t) + B_0u(t) - B_0u_{ss} \qquad (20a) \\ &\quad + \widehat{f}\left(\underline{x}(t), u(t), \mu\right) \\ y_1(t) &= C_1\underline{x}(t) + y_{1ss} \qquad (20b) \end{aligned}$$

where $\underline{x} = x - x_{ss}$, while

$$A_0 = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=x_{ss},\\ u=u_{ss}}}, \quad B_0 = \left.\frac{\partial f}{\partial u}\right|_{\substack{x=x_{ss},\\ u=u_{ss}}} \quad \text{and} \quad C_{01} = \left.\frac{\partial h_1}{\partial x}\right|_{x=x_{ss}},$$

are the linear approximations and $\widehat{f}(\underline{x}, u, \mu)$ contains the second or higher order terms. Since z_3 remains constant along each sampling instant, it can be considered that $\dot{z}_3 = 0 \ \forall t \neq k\delta$. Defining

$$\begin{aligned} \xi_1 &= \underline{x} - z_1 \\ \xi_2 &= u_{ss} - z_2 \\ \xi_3 &= y_{1ss} - z_3 \end{aligned}$$

the linear approximation of the closed-loop system can be written as

$$\dot{\xi}(t) = \overline{A}\xi(t), \qquad \forall t \neq k\delta \tag{21}$$

$$\xi\left(k\delta^{+}\right) = \left(I + \overline{G}_d\overline{C}_2\right)\xi(k\delta), \quad \forall t = k\delta,\ k = 1,2,.. \tag{22}$$

where

$$\overline{A} = \begin{pmatrix} A - GC_{T1} & -G \\ 0 & 0 \end{pmatrix}$$

while A and C_{T1} are given in (13). Notice that $\underline{x}(k\delta) = \underline{x}\left(k\delta^{+}\right)$. Integrating (21) for $t \in \left(k\delta^{+}, (k+1)\delta\right)$,

$$\xi\left((k+1)\delta\right) = \overline{A}_d\xi\left(k\delta^{+}\right), \tag{23}$$

where $\overline{A}_d$ is described in (17), while A_d and M_d are defined in (18). Taking an increment in (22) and replacing (23),

$$\xi\left((k+1)\delta^{+}\right) = \left(I + \overline{G}_d\overline{C}_2\right)\overline{A}_d\xi\left(k\delta^{+}\right).$$

Since by hypothesis $(A - GC_{T1})$ is Hurwitz, then A_d is Schur and $\overline{A}_d$ have only one eigenvalue in the unitary circle. However, if G_d is such that $\left(I + \overline{G}_d\overline{C}_2\right)\overline{A}_d$ is Schur, it is guaranteed that $\xi\left(k\delta^{+}\right)$ is asymptotically stable. Therefore $\lim_{k\to\infty} z_3(k\delta) = y_{1ss}$, $\lim_{k\to\infty} z_2(k\delta) = u_{ss}$ and $\lim_{k\to\infty} z_1(k\delta) = \underline{x}(k\delta)$. Then by continuity $\lim_{t\to\infty} z_2(t) = u_{ss}$ and $\lim_{t\to\infty} z_1(t) = \underline{x}(t)$ and the closed-loop dynamics of (20) approach to

$$\underline{\dot{x}}(t) = (A_0 + B_0K)\underline{x}(t) + \widehat{f}\left(\underline{x}(t), K\underline{x}(t) + u_{ss}, \mu\right)$$

which is asymptotically stable in a neighborhood of the origin since $(A_0 + B_0K)$ is Hurwitz. Then $\lim_{t\to\infty}[x(t) - x_{ss}] = 0$ and this guarantees that $\lim_{t\to\infty}[y(t) - y_{2r}] = 0$, concluding the proof.

Remark 6. z_1 and z_2 represent the dynamic of the continuous regulator, while z_3 is the state of the discrete regulator. Notice that z_2 which is present in the input (14d) can be considered as an integral action since integrating (14b), which is precisely the dynamic of z_2, appears an integral of $e_1(t)$. Analogously, z_3 is also an discrete integral action which consider $e_2(k\delta)$. These variables give the robustness to the cascade controller.

4. CONTROL IMPLEMENTATION

Considering that the VFA concentration is measured continuously, while the COD concentration is measured at each sampling period δ, the outputs of system (1) are

$$y_1(t) = C_1x(t) \qquad \text{and} \qquad y_2(k\delta) = C_2x(k\delta),$$

where $x = \mathrm{col}(X_1, S_1, X_2, S_2)$ and

$$C_1 = (0\ 0\ 0\ 1) \quad , \quad C_2 = (0\ 1\ 0\ 0).$$

System (1) is similar to (2), hence controller (14) presented in Theorem 5 can be designed. The linear matrix approximations of system (1) are

$$A_0 = \mu_1(S_{1ss})\begin{pmatrix} 0 & \Psi_1 & 0 & 0 \\ -k_1 & -k_1\Psi_1 - \dfrac{1}{\alpha} & 0 & 0 \\ 0 & 0 & 0 & \Psi_2 \\ 0 & k_2\Psi_1 & -k_3\Psi_2 & -k_3\Psi_2 - \dfrac{1}{\alpha} \end{pmatrix},$$

$$B_0 = \left(-\alpha X_{1ss}\ (S_{1in} - S_{1ss})\ -\alpha X_{2ss}\ (S_{2in} - S_{2ss})\right)^T,$$

where $\Psi_1 = \frac{K_{S1}}{\mu_{\max 1}}\frac{\mu_1(S_{1ss})}{S_{1ss}^2}X_{1ss}$, $\Psi_2 = \frac{\mu_2(S_{2ss})}{S_{2ss}^2}X_{2ss}\cdot\left[\frac{K_{S2} - (S_{2ss}/K_{I2})^2}{\mu_{\max 2}}\right]$, while the steady state is described by

$$\alpha D_{ss} = \mu_1(S_{1ss}) = \mu_2(S_{2ss}),$$

$$X_{1ss} = \alpha^{-1}k_1^{-1}(S_{1in} - S_{1ss}),$$

$$X_{2ss} = \alpha^{-1}k_3^{-1}\left[k_1^{-1}k_2(S_{1in} - S_{1ss}) + (S_{2in} - S_{2ss})\right],$$

where it must be fixed one variable (which might be any of this: D_{ss}, S_{1ss} or S_{2ss}) in order to estimate the others.

Table 1. Nominal parameter values and variations.

Parameter	Nominal value	% of variation from nominal value $t < 10$	$10 \le t < 30$	$t \ge 30$
$\mu_{\max 1}$	$1.2\,\mathrm{d}^{-1}$	−10	−20	−20
$\mu_{\max 2}$	$0.744\,\mathrm{d}^{-1}$	−7	−10	−10
K_{S1}	7.1 g/l	−5	10	10
K_{S2}	9.28 mmol/l	10	10	10
K_{I2}	16 mmol/l	−15	−15	−15
k_1	42.14 g/g	−12	−12	−12
k_2	116.5 mmol/g	15	15	15
k_3	268 mmol/g	13	13	13
α	0.5	0	15	15
S_{1i}	30 g/l	25	10	10
S_{2i}	750 mmol/l	−5	−5	50

In order to analyze the closed-loop dynamic behavior, the parameters reported in Table 1 where taken from (Bernard et al., 2001). Following Theorem 5 and considering a sample period of 1day, the next matrices were obtained

$$A_0 = \begin{pmatrix} 0 & 0.1207 & 0 & 0 \\ -13.1687 & -5.7096 & 0 & 0 \\ 0 & 0 & 0 & 0.1554 \\ 36.4063 & 14.0570 & -83.7500 & -42.2704 \end{pmatrix},$$

$$B_0 = (-0.6526\ 27.5000\ -3.0566\ 743.1464)^T,$$

while using LQR techniques the next feedback parameters were obtained

$$K = (-0.0007\ -0.0044\ -0.0162\ -0.0083),$$

$$G = (-0.1132\ 5.3922\ -0.8990\ 88.2809\ -10.0000)^T,$$

$$G_d = 2.5.$$

Since it is desirable to guarantee the operational stability, it was set that VFA reference must be bounded by the next domain: $y_{1r} \in (0, 12)$ mmol/l, while $D \in (0.05, 1)\ \mathrm{d}^{-1}$.

4.1 Simulation results

To verify if the proposed controller enhances the robustness of the closed-loop with respect of influent disturbances, a discrete controller was designed with the same sampling period to regulate the COD concentration. Figure 2 presents the dynamic response for both controllers. The simulation was carried out using steady state initial conditions and nominal parameter values (see Table 1). At time $t = 5$ d a step in the reference from 2.5 to 2 COD g/l was induced. As can be seen in Figure 2a both controllers can handle this step approximately with the same dynamic response, however, at time $t = 15$ d a drastic overload in the VFA influent concentration was induced. Since this overload does not affect the acidogenesis phase, the discrete controller remains unaltered as well as its COD concentration, while the cascade controller detects a change in the VFA concentration and modify the dilution

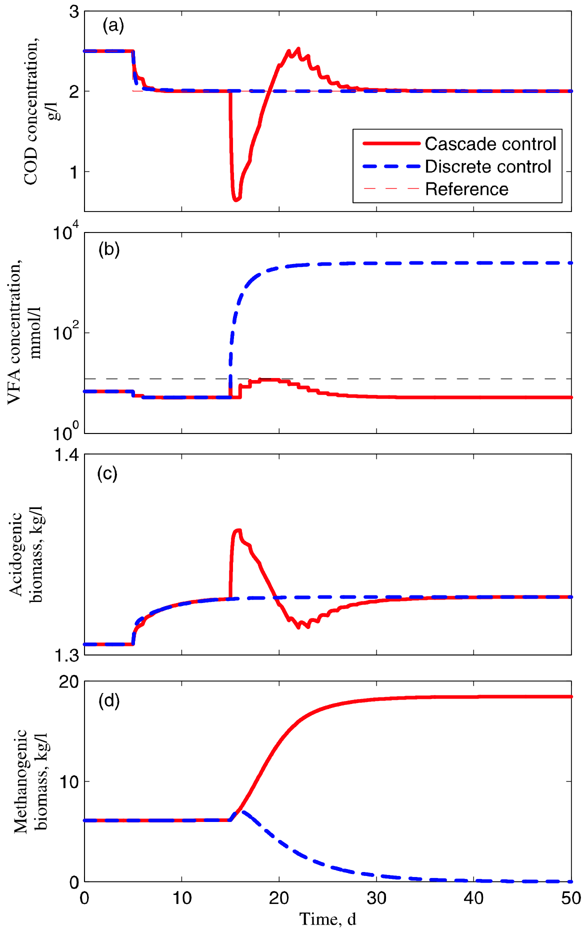

Fig. 2. Comparison of the cascade control with a traditional COD regulator. (a) COD concentration. (b) VFA concentration. (c) Acidogenic biomass. (d) Methanogenic biomass.

rate (see Figure 3). This induced a variation on the COD concentration which is corrected by the cascade controller in approximately 10 days. As can be seen in Figure 2b the VFA concentration increases in both cases, however, since for the cascade controller y_{1r} is bonded, at approximately $t = 25\,\text{d}$, this reference reached the upper bound and the inner loop did not allow it to keep increasing, while for the discrete controller it keeps increasing, producing an acidification and consequently, a substrate inhibition which finally causes the methanogenic bacterial death, as shown in Figure 2d. This simulation remarks the capabilty of the cascade controller to guarantee the operational stability at the expense of a small temporary variation on the COD concentration.

Another simulation for the cascade controller was carried out, in order to verify the robustness to parametric and load variations as reported in Table 1, notice that variations up to 50% were induced. Figures 4 and 5 present the dynamic response. It is evident the robustness of the closed-loop with respect to load disturbances and several parametric variations.

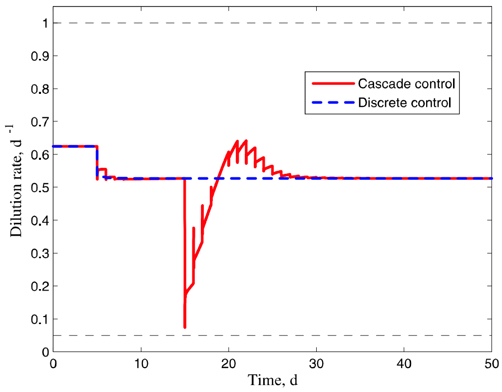

Fig. 3. Dilution rate for the cascade control and a traditional COD regulation.

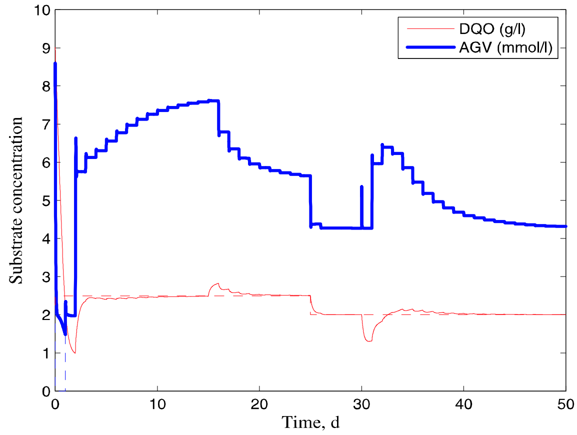

Fig. 4. Substrate concentrations.

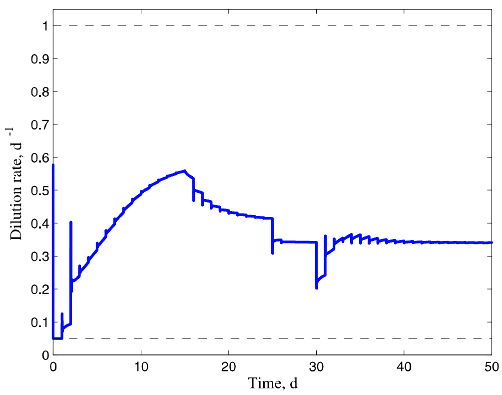

Fig. 5. Dilution rate.

5. CONCLUSION

A cascade scheme was proposed to regulate the COD concentration of AD processes. Although the control law is based on a linear approximation its robustness is conferred by integral actions of the controller. The cascade scheme also presents advantages since, additionally to the COD regulation, can be used to guarantee the operational stability, i.e. to avoid the AGV inhibition, at the expense of a small temporary variation on the COD concentration. Real time implementation are been carried out and results will appear soon. Although the design of the control law (14) was motivated by the control of AD systems, it can be applied to another systems. As future work this theory will be extended to the case of not constant references.

(Chapter head:)*

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Appendix A. OUTLINE OF PROOF

Proof. [Lemma 3] Let us define

$$\xi(t) = x(t) - z(t), \quad \text{and } \xi\left(k\delta^{+}\right) = x(k\delta) - z\left(k\delta^{+}\right),$$

where $\xi(t)$ represents the continuous error and $\xi\left(k\delta^{+}\right)$ is the updated error for each sampling period. Note that $x\left(k\delta^{+}\right) = x(k\delta)$ since system (9) is continuous. Now

$$\dot{\xi}(t) = A\xi(t) \qquad \forall t \neq k\delta \tag{A.1}$$

$$\xi\left(k\delta^{+}\right) = (I + GC)\,\xi(k\delta) \qquad t = k\delta. \tag{A.2}$$

Solving (A.1) for $t \in \left[k\delta^{+}, (k+1)\delta\right]$, it follows that

$$\xi(k+1) = A_d \xi\left(k\delta^{+}\right), \tag{A.3}$$

where $A_d = e^{A\delta}$. From (A.2) and (A.3) it is obtained

$$\xi\left((k+1)\delta^{+}\right) = (I + GC)\,\xi(k+1) = (I + GC)\,A_d \xi\left(k\delta^{+}\right),$$

and thus, if the pair (A_d, CA_d) is observable, then a matrix G can be calculated such that $A_d + GCA_d$ is Schur and the error $\xi\left(k\delta^{+}\right)$ will converge to zero, hence $\lim_{k\to\infty}\left[x(k\delta) - z\left(k\delta^{+}\right)\right] = 0$; then for $k\delta < t \leq (k+1)\delta$ the solution $z(t)$ converges to $x(t)$, that is $\lim\limits_{t\to\infty}[x(t) - z(t)] = 0$. On the other hand, to prove that the pair (A_d, CA_d) is observable if the pair (A_d, C) is observable, consider its observability matrix

$$\mathcal{O} = \begin{pmatrix} CA_d \\ CA_d^2 \\ \vdots \\ CA_d^n \end{pmatrix},$$

where $A_d \in \mathbb{R}^{n\times n}$, then using the Hamilton-Cailey theorem (Kailath, 1980)

$$A_d^n = a_0 I + a_1 A_d + \cdots + a_{n-1} A_d^{n-1},$$

the observability matrix becomes

$$\mathcal{O} = \begin{pmatrix} CA_d \\ CA_d^2 \\ \vdots \\ a_0 C + a_1 CA_d + \cdots + a_{n-1} CA_d^{n-1} \end{pmatrix}.$$

Since A_d is obtained through a discretization of matrix A then $a_0 \neq 0$ and $\mathcal{O}$ has full rank if the pair (A_d, C) is observable.