Identification of low-order unstable process model from closed-loop step test

Tao Liu, Furong Gao*

*Department of Chemical and Biomolecular Engineering, Hong Kong University of Science & Technology, Kowloon, Hong Kong (Tel: +852-2358-7139; e-mail address: kefgao@ust.hk)

Abstract: Based on a closed-loop step response test, control-oriented low-order model identification algorithms are proposed for unstable processes. By using a damping factor to the closed-loop step response for realization of the Laplace transform, an algorithm for estimating the process frequency response is developed in terms of the closed-loop control structure used for identification. Correspondingly, two model identification algorithms are derived analytically for obtaining the widely used low-order process models of first-order-plus-dead-time (FOPDT) and second-order-plus-dead-time (SOPDT), respectively. Illustrative examples from the recent literature are used to demonstrate the effectiveness and merits of the proposed identification algorithms.

1. INTRODUCTION

As model-based control strategies have demonstrated apparently improved set-point tracking and load disturbance rejection for open-loop unstable processes, control-oriented identification of low-order process model, e.g., first-order-plus-dead-time (FOPDT) or second-order-plus-dead-time (SOPDT), has been increasingly explored in the process control community (Seborg, Edgar, and Mellichamp, 2003; Liu and Gao, 2008a and 2008b). For safety and economic reasons, unstable processes are usually not allowed to be operated in an open-loop manner. Closed-loop identification methods have been therefore studied in the literature. One of the mostly used identification tests is the close-loop step response test, owing to its implemental simplicity. Based on closed-loop step test in terms of the internal model control (IMC) structure, Häggblom K. E. (1996) demonstrated that closed-loop identification facilitates better representation of the process dynamic response characteristics for closed-loop operation. Using a conventional proportional (P) or proportional-integral-derivative (PID) controller for closed-loop stabilization, recent closed-loop step identification methods can be seen in the references (Paraskevopoulos, Pasgianos and Arvanitis, 2004; Sree and Chidambaram, 2006; Cheres, 2006). Using relay feedback to yield sustained oscillation within an admissible fluctuation of the process output, identification methods based on the resulting limit cycle data have been developed in the papers (Shiu, Hwang and Li, 1998; Marchetti, Scali and Lewin, 2001; Vivek and Chidambaram, 2005; Liu and Gao, 2008a). It was, however, pointed out that the conventional relay feedback structure cannot guarantee periodic oscillation for unstable processes with large time delay (Tan, Wang and Lee, 1998; Thyagarajan and Yu; 2003). Some limiting conditions to form the limit cycle from relay feedback for an unstable process have been disclosed by Liu and Gao (2008b). Besides, using the pseudo-random binary sequence (PRBS) as excitation signal to the set-point, closed-loop identification methods for application of model predictive control (MPC) have been reported in the references (Sung et al, 2001; Saffer and Doyle, 2002; Bindlish, Rawlings and Young, 2003).

In this paper, identification algorithms based on a closed-loop step test are proposed for obtaining low-order models of FOPDT and SOPDT for tuning unstable processes. By using a damping factor to the closed-loop step response for realization of the Laplace transform, an algorithm is first given to estimate the closed-loop frequency response in terms of using a conventional P, PI or PID controller for closed-loop stabilization. Accordingly, the process frequency response can be analytically derived from the closed-loop frequency response with the knowledge of the controller. Then, two identification algorithms are analytically developed for obtaining FOPDT and SOPDT models, respectively. Both the algorithms can give good accuracy if the model structure matches the process. Measurement noise tests are also performed to demonstrate identification robustness of the proposed algorithms.

2. FREQUENCY RESPONSE ESTIMATION

It is commonly known that the Fourier transform of a step response does not exist due to $\Delta y(t) \neq 0$ for $t \to \infty$, where $\Delta y(t) = y(t) - y(t_0)$ and $y(t_0)$ denotes the initial steady output. However, by substituting $s = \alpha + j\omega$ into the Laplace transform to the step response,

$$\Delta Y(s) = \int_0^\infty \Delta y(t) e^{-st} dt \tag{1}$$

we can formulate

$$\Delta Y(\alpha + j\omega) = \int_0^\infty [\Delta y(t) e^{-\alpha t}] e^{-j\omega t} dt \tag{2}$$

Note that if $\alpha > 0$, there exists $\Delta y(t) e^{-\alpha t} = 0$ for $t > t_N$, where t_N may be numerically determined using the condition of $\Delta y(t_N) e^{-\alpha t_N} \approx 0$, since $\Delta y(t)$ reaches a steady value after the closed-loop transient response to a step change of the set-point. Therefore, by regarding α as a damping factor to the closed-loop step response for Laplace transform, we may compute $\Delta Y(\alpha+j\omega)$ from the N points of step response data as

$$\Delta Y(\alpha+j\omega)=\int_0^{t_N}[y(t)e^{-\alpha t}]e^{-j\omega t}dt \tag{3}$$

For a closed-loop step test with initial steady state, i.e., $y(t)=r(t)=c$ for $t\le t_0$, where $r(t)$ denotes the set-point value, c is a constant and t_0 is the time for step test, we may formulate the step change of the set-point by using a time shift of t_0 (i.e., letting $t_0=0$) as

$$\Delta r(t)=\begin{cases}0, & t\le 0;\\ h, & t>0.\end{cases} \tag{4}$$

where h is the magnitude of the step change. Its Laplace transform for $s=\alpha+j\omega$ with $\alpha>0$ can be explicitly derived as

$$\Delta R(\alpha+j\omega)=\int_0^{\infty}he^{-(\alpha+j\omega)t}dt=\frac{h}{\alpha+j\omega} \tag{5}$$

Hence, the closed-loop frequency response can be derived using (3) and (5) as

$$T(\alpha+j\omega)=\frac{\alpha+j\omega}{h}\Delta Y(\alpha+j\omega),\ \alpha>0 \tag{6}$$

Note that $T(\alpha+j\omega)\to 0$ as $\alpha\to\infty$. On the contrary, $\alpha\to 0$ will cause t_N much larger for computation of (6). A proper choice of α is therefore required for implementation. Considering that all the closed-loop transient response data to a step change of the set-point should be used to procure good estimation of the closed-loop frequency response, the following constraint is suggested to choose α,

$$\Delta y(t_{\text{set}})e^{-\alpha t_{\text{set}}}>\delta \tag{7}$$

where $\Delta y(t_{\text{set}})$ denotes the steady-state output deviation to the step change in terms of the settling time (t_{set}), and δ is a threshold of the computational precision that may be practically taken smaller than 1×10^{-6}. It follows from (7) that

$$\alpha<\frac{1}{t_{\text{set}}}\ln\frac{\Delta y(t_{\text{set}})}{\delta} \tag{8}$$

To ensure computational efficiency with respect to the complex variable, $s=\alpha+j\omega$, for frequency response estimation, the lower bound of α may be simply taken as δ, if there exists no limit on the time length of the step test.

Once α is chosen in terms of the above guideline, the time length, t_N, may be determined from a numerical constraint for computation of (3), i.e.,

$$\Delta y(t_N)e^{-\alpha t_N}<\delta \tag{9}$$

which can be solved as

$$t_N>\frac{1}{\alpha}\ln\frac{\Delta y(t_N)}{\delta} \tag{10}$$

Note that there exists the following Laplace transform with initial steady closed-loop state of $y(0)=r(0)=c$, where c is a constant,

$$L[\int_0^t\Delta y(t)dt]=\frac{\Delta Y(s)}{s} \tag{11}$$

To guarantee identification robustness against measurement noise, we may compute the frequency response by

$$T(\alpha+j\omega)=\frac{\dfrac{\Delta Y(\alpha+j\omega)}{\alpha+j\omega}}{\dfrac{\Delta R(\alpha+j\omega)}{\alpha+j\omega}}=\frac{(\alpha+j\omega)^2}{h}\int_0^{t_N}[\int_0^t\Delta y(\tau)d\tau]e^{-\alpha t}e^{-j\omega t}dt \tag{12}$$

It can be seen from (12) that, rather than use individual output data measured from the step test, a time integral for each measurement point is used to compute the outer-layer integral for obtaining the frequency response estimation. This facilitates reducing measurement errors according to the statistic averaging principle.

Denote the n-th order derivative for a complex function of $F(s)$ with respect to s as

$$F^{(n)}(s)=\frac{d^{(n)}}{ds^n}F(s),\ n\ge 1. \tag{13}$$

It follows from (3) and (6) that

$$T^{(1)}(s)=\frac{1}{h}\int_0^{\infty}(1-st)\Delta y(t)e^{-st}dt \tag{14}$$

$$T^{(2)}(s)=\frac{1}{h}\int_0^{\infty}t(st-2)\Delta y(t)e^{-st}dt \tag{15}$$

Hence, by letting $s=\alpha$ and choosing α as well as that for computation of (3), the single integral in (14) and (15) can be computed numerically. The corresponding time lengths of t_N can be respectively determined using the numerical constraints,

$$(1-\alpha t_N)\Delta y(t_N)e^{-\alpha t_N}<\delta \tag{16}$$

$$t_N(\alpha t_N-2)\Delta y(t_N)e^{-\alpha t_N}<\delta \tag{17}$$

For a conventional PID controller used in the closed-loop structure to stabilize an unstable process for step test,

$$C(s)=k_{\text{C}}(1+\frac{1}{\tau_{\text{I}}s}+\frac{\tau_{\text{D}}s}{0.1\tau_{\text{D}}s+1}) \tag{18}$$

where k_{C} denotes the controller gain, τ_{I} the integral constant and τ_{D} the derivative constant, it can be derived that

$$C^{(1)}(s)=k_{\text{C}}[-\frac{1}{\tau_{\text{I}}s^2}+\frac{\tau_{\text{D}}}{(0.1\tau_{\text{D}}s+1)^2}] \tag{19}$$

$$C^{(2)}(s)=2k_{\text{C}}[\frac{1}{\tau_{\text{I}}s^3}-\frac{0.1\tau_{\text{D}}^2}{(0.1\tau_{\text{D}}s+1)^3}] \tag{20}$$

Note that the closed-loop transfer function can be derived as

$$T(s)=\frac{G(s)C(s)}{1+G(s)C(s)} \tag{21}$$

It follows from (21) that

$$G(s)=\frac{T(s)}{C(s)[1-T(s)]} \tag{22}$$

Its first and second derivatives can be derived accordingly as

$$G^{(1)}=\frac{T^{(1)}C+C^{(1)}T(T-1)}{C^2(1-T)^2} \tag{23}$$

$$G^{(2)}=\frac{CT^{(2)}+2C^{(1)}T^{(1)}T+C^{(2)}T(T-1)}{C^2(1-T)^2}$$

$$-\frac{2[CT^{(1)}+C^{(1)}T(T-1)][CC^{(1)}(1-T)-C^2T^{(1)}(1-T)]}{C^3(1-T)^3} \quad (24)$$

Therefore, by substituting $s=\alpha+j\omega_k$ ($k=1,2,\ldots,M.$), where M is the number of representative frequency response points in a user specified frequency range, the process frequency response can be numerically estimated for model fitting.

3. MODEL IDENTIFICATION ALGORITHMS

Low-order unstable process models of FOPDT and SOPDT are respectively in the form of

$$G_1(s)=\frac{k_p e^{-\theta s}}{\tau_p s-1} \quad (25)$$

$$G_2(s)=\frac{k_p e^{-\theta s}}{(\tau_1 s-1)(\tau_2 s+1)} \quad (26)$$

where k_p denotes the process gain, θ the process time delay and τ_p (or τ_1 and τ_2) the process time constant(s).

By regarding $s\in\mathbb{R}$ and taking the natural logarithm for both sides of (25) in terms of $0<s<\tau_p$, we obtain

$$\ln[-G_1(s)]=\ln(k_p)-\ln(1-\tau_p s)-\theta s \quad (27)$$

Subsequently, taking the first and second derivatives for both sides of (27) with respect to s yields

$$\frac{1}{G_1(s)}\frac{d}{ds}[G_1(s)]=\frac{\tau_p}{1-\tau_p s}-\theta \quad (28)$$

$$Q_2(s)=\frac{\tau_p^2}{(1-\tau_p s)^2} \quad (29)$$

where $Q_2(s)=d[Q_1(s)]/ds$ and $Q_1(s)$ is the left side of (28).

Substituting $s=\alpha$ into (29), it can be derived that

$$\tau_p=\begin{cases}\dfrac{\sqrt{Q_2}}{\alpha\sqrt{Q_2}-1} \text{ or } \dfrac{\sqrt{Q_2}}{\alpha\sqrt{Q_2}+1}, & \text{if } Q_2>\dfrac{1}{\alpha^2};\\[2ex] \dfrac{\sqrt{Q_2}}{\alpha\sqrt{Q_2}+1}, & \text{if } Q_2\le\dfrac{1}{\alpha^2}.\end{cases} \quad (30)$$

Note that for $Q_2\ge 1/\alpha^2$, we may determine a suitable solution based on model fitting accuracy for the closed-loop step response.

It should be noted that the above parameter estimation is without loss of generality since there exists $\alpha<\tau_p$ in general, which may be verified from the guideline for choosing α as given in the earlier section.

Consequently, the other two model parameters can be derived from (28) and (25) using $s=\alpha$ as

$$\theta=-Q_1(\alpha)+\frac{\tau_p}{1-\tau_p\alpha} \quad (31)$$

$$k_p=(\tau_p\alpha-1)G_1(\alpha)e^{\alpha\theta} \quad (32)$$

Hence, the above algorithm named **Algorithm-I** for obtaining a FOPDT model for an unstable process can be summarized as:

(i) Choose $s=\alpha$ and t_N to compute $T(\alpha)$, $T^{(1)}(\alpha)$ and $T^{(2)}(\alpha)$ in terms of (6) (or (12)), (14) and (15);
(ii) Compute $C(\alpha)$, $C^{(1)}(\alpha)$ and $C^{(2)}(\alpha)$ in terms of (18), (19) and (20);
(iii) Compute $G_1(\alpha)$, $G_1^{(1)}(\alpha)$ and $G_1^{(2)}(\alpha)$ in terms of (22), (23) and (24);
(iv) Compute $Q_1(\alpha)$ and $Q_2(\alpha)$ in terms of (28) and (29);
(v) Compute the process time constant, τ_p, from (30);
(vi) Compute the process time delay, θ, from (31);
(vii) Compute the process gain, k_p, from (32).

Following a similar procedure as above, taking the natural logarithm for both sides of (26) in terms of $0<s<\tau_1$, yields

$$\ln[-G_2(s)]=\ln(k_p)-\ln(1-\tau_1 s)-\ln(\tau_2 s+1)-\theta s \quad (33)$$

Accordingly, the first and second order derivatives for both sides of (33) with respect to s can be derived respectively as

$$\frac{1}{G_2(s)}\frac{d}{ds}[G_2(s)]=\frac{\tau_1}{1-\tau_1 s}-\frac{\tau_2}{\tau_2 s+1}-\theta \quad (34)$$

$$Q_2(s)=\frac{\tau_1^2}{(1-\tau_1 s)^2}+\frac{\tau_2^2}{(\tau_2 s+1)^2} \quad (35)$$

where $Q_2(s)=d[Q_1(s)]/ds$ and $Q_1(s)$ is the left side of (34).

Substituting $s=\alpha$ into (35) yields

$$Q_2(\alpha)=[2\alpha^2-\alpha^4Q_2(\alpha)]\tau_1^2\tau_2^2+[2\alpha-2\alpha^3Q_2(\alpha)](\tau_1^2\tau_2-\tau_1\tau_2^2)$$
$$+4\alpha^2Q_2(\alpha)\tau_1\tau_2+[1-\alpha^2Q_2(\alpha)](\tau_1^2+\tau_2^2)+2\alpha Q_2(\alpha)(\tau_1-\tau_2) \quad (36)$$

To solve τ_1 and τ_2 from (36), we reformulate (36) in the LS form of

$$\psi(\alpha)=\phi(\alpha)^T\gamma \quad (37)$$

where

$$\begin{cases}\psi(\alpha)=Q_2(\alpha),\\ \phi(\alpha)=[2\alpha^2-\alpha^4Q_2(\alpha),\ 2\alpha-2\alpha^3Q_2(\alpha),\ -\alpha^2Q_2(\alpha),\ 1,\ 2\alpha Q_2(\alpha)]^T,\\ \gamma=[\tau_1^2\tau_2^2,\ \tau_1^2\tau_2-\tau_1\tau_2^2,\ \tau_1^2+\tau_2^2-4\tau_1\tau_2,\ \tau_1^2+\tau_2^2,\ \tau_1-\tau_2]^T.\end{cases} \quad (38)$$

By choosing 5 different values of α in terms of the guideline given in (8) and denoting $\Psi=[\psi(\alpha_1),\ \psi(\alpha_2),\ldots,\ \psi(\alpha_5)]^T$ and $\Phi=[\phi(\alpha_1),\ \phi(\alpha_2),\ldots,\ \phi(\alpha_5)]^T$, an LS solution can be derived from the linear regression,

$$\gamma=(\Phi^T\Phi)^{-1}\Phi^T\Psi \quad (39)$$

It is obvious that all the columns of Φ are linearly independent with each other, such that Φ is guaranteed non-singular for computation of (39). Accordingly, there exists a unique solution of γ for parameter estimation.

Then, the model parameters can be retrieved from γ as

$$\begin{cases}\tau_1=\dfrac{\gamma(5)}{2}+\dfrac{1}{2}\sqrt{\gamma^2(5)+4\dfrac{\gamma(2)}{\gamma(5)}}\\ \tau_2=\tau_1-\gamma(5)\end{cases} \quad (40)$$

Note that there exist three redundant fitting conditions in the parameter estimation of γ, which can be surely satisfied if the model structure matches the process to be identified. To procure fitting accuracy for a high-order process, we may use $\gamma(1)$, $\gamma(3)$ and $\gamma(4)$ together with $\gamma(2)$ and $\gamma(5)$ to derive an LS fitting solution for parameter estimation in terms of using the natural logarithm, i.e.,

$$\begin{bmatrix} 2 & 2 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ln \tau_1 \\ \ln \tau_2 \end{bmatrix} = \begin{bmatrix} \ln \gamma(1) \\ \ln[\frac{\gamma(4)-\gamma(3)}{4}] \\ \ln[\frac{\gamma(5)}{2}+\frac{1}{2}\sqrt{\gamma^2(5)+4\frac{\gamma(2)}{\gamma(5)}}] \\ \ln[\frac{\gamma(5)}{2}+\frac{1}{2}\sqrt{\gamma^2(5)+4\frac{\gamma(2)}{\gamma(5)}}-\gamma(5)] \end{bmatrix} \quad (41)$$

Consequently, the other two model parameters can be derived from (34) and (26) as

$$\theta = -Q_1(\alpha) + \frac{\tau_1}{1-\tau_1\alpha} - \frac{\tau_2}{\tau_2\alpha+1} \quad (42)$$

$$k_p = (\tau_1\alpha - 1)(\tau_2\alpha+1)G_2(\alpha)e^{\alpha\theta} \quad (43)$$

Hence, the above algorithm named **Algorithm-II** for obtaining an SOPDT model for an unstable process can be summarized as:

(i) Choose $s=\alpha$ and t_N to compute $T(\alpha)$, $T^{(1)}(\alpha)$ and $T^{(2)}(\alpha)$ in terms of (6) (or (12)), (14) and (15);
(ii) Compute $C(\alpha)$, $C^{(1)}(\alpha)$ and $C^{(2)}(\alpha)$ in terms of (18), (19) and (20);
(iii) Compute $G_2(\alpha)$, $G_2^{(1)}(\alpha)$ and $G_2^{(2)}(\alpha)$ in terms of (22), (23) and (24);
(iv) Compute $Q_1(\alpha)$ and $Q_2(\alpha)$ in terms of (34) and (35);
(v) Compute the time constants, τ_1 and τ_2, from (40) (or (41));
(vi) Compute the process time delay, θ, from (42);
(vii) Compute the process gain, k_p, from (43).

4. ILLUSTRATION

Example 1. Consider the FOPDT unstable process studied in the recent literature (Padhy and Majhi, 2006),

$$G(s) = \frac{1}{s-1}e^{-0.8s}$$

Based on relay feedback test with two P controllers, Padhy and Majhi (2006) derived a FOPDT model, $G_m = 1.0e^{-0.8033s}/(1.0007s-1)$. For illustration, the unity feedback control structure with a proportional controller of $k_c = 1.2$, which is equivalent to that of Padhy and Majhi (2006), is used for closed-loop step test. By adding a step change with a magnitude of $h = 0.05$ to the set-point, the closed-loop step response is shown in Fig.1. According to the guidelines given in (8) and (10), $\alpha = 0.1$ and $t_N = 150\,(\mathrm{s})$ are chosen to use the proposed Algorithm-I, resulting in a FOPDT model listed in Table 1, which indicates high accuracy. The fitting error is given in terms of the closed-loop transient response in the time interval [0, 50]s.

To demonstrate identification robustness against measurement noise, assume that a random noise of $N(0, \sigma_N^2 = 0.035\%)$, causing the noise-to-signal ratio (NSR) to 5%, is added to the output measurement which is then used for feedback control. By performing 100 Monte-Carlo tests in terms of varying the ‘seed’ of the noise generator from 1 to 100, the identified results are listed in Table 1, where the model parameters are respectively the mean of 100 Monte-Carlo tests, and the values in the adjacent parentheses are the sample standard deviation. The results for the noise levels of NSR=10% and 20% are also listed in Table 1 to show the achievable identification accuracy and robustness.

Example 2. Consider the SOPDT unstable process studied by Cheres (2006) and Sree and Chidambaram (2006),

$$G(s) = \frac{1}{(2s-1)(0.5s+1)}e^{-0.5s}$$

Based on a closed-loop step test in terms of a PID controller ($k_c = 2.71$, $\tau_I = 4.43$ and $\tau_D = 0.319$) and a unity step change to the set-point, Cheres (2006) derived only a referential FOPDT model for controller tuning, and so was done in Sree and Chidambaram (2006). By performing the same closed-loop step test, the proposed Algorithm-II based on the choice of $\alpha = 0.1, 0.15, 0.2, 0.25, 0.3$ and $t_N = 300$ (s) gives a SOPDT model listed in Table 1, again demonstrating good accuracy. The fitting error is given in terms of the closed-loop transient response in the time interval [0, 30]s.

To demonstrate identification robustness against measurement noise, 100 Monte Carlo tests are performed in terms of NSR=5%, 10% and 20%, respectively. The identified results are listed in Table 1 for comparison, which indicate again that good identification accuracy and robustness is therefore obtained.

5. CONCLUSIONS

Low-order model identification methods have been increasingly appealed for improving control system design to operate unstable processes. By applying a damping factor to the closed-loop step response for realization of the Laplace transform, a frequency response estimation algorithm has been proposed for model fitting. Based on the process frequency response estimated from the closed-loop step response with the knowledge of the controller, two model identification algorithms have been analytically developed for obtaining the widely used low-order process models of FOPDT and SOPDT for practical applications. Two illustrative examples from the recent literature have been performed to demonstrate the achievable accuracy of the proposed algorithms. The results under Monte Carlo noise tests have also demonstrated good identification robustness of the proposed algorithms.

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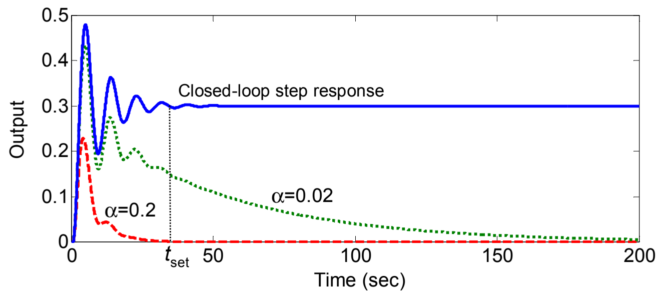

Fig. 1. Illustration of choosing α for example 1

Table 1. Closed-loop step response identification under different measurement noise levels

NSR	Example 1	*err*	Example 2	*err*
0	$\frac{1.0000e^{-0.8008s}}{1.0006s-1}$	8.46×10^{-8}	$\frac{0.9999e^{-0.4996s}}{(2.0000s-1)(0.5000s+1)}$	1.47×10^{-8}
5%	$\frac{1.0001(\pm0.0011)e^{-0.8019(\pm0.045)s}}{1.0015(\pm0.033)s-1}$	4.15×10^{-7}	$\frac{0.9999(\pm0.0025)e^{-0.4998(\pm0.0082)s}}{[2.0002(\pm0.0037)s-1][0.5004(\pm0.0098)s-1]}$	1.72×10^{-7}
10%	$\frac{1.0003(\pm0.0021)e^{-0.8018(\pm0.089)s}}{1.0013(\pm0.066)s-1}$	5.08×10^{-7}	$\frac{0.9999(\pm0.0054)e^{-0.4992(\pm0.023)s}}{[2.0005(\pm0.0087)s-1][0.5009(\pm0.034)s-1]}$	7.61×10^{-7}
20%	$\frac{1.0006(\pm0.0041)e^{-0.7971(\pm0.183)s}}{0.9973(\pm0.136)s-1}$	2.86×10^{-6}	$\frac{0.9995(\pm0.0097)e^{-0.4985(\pm0.026)s}}{[2.0008(\pm0.017)s-1][0.5011(\pm0.026)s-1]}$	2.79×10^{-6}