An Internal Model Control Approach to Mid-Ranging Control

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Existing tuning rules for mid-ranging control can be improved. In this paper a novel strategy for midranging control based on Internal Model Control (IMC) principles is presented. The design reformulates mid-ranging control specifications in terms of classical bandwidth and sensitivity requirements. The performance of this design is demonstrated through simulation studies. The overall benefits of the IMC design are that it provides transparent and flexible tuning, and that it offers a natural framework for antiwindup. Both classical IMC and modified IMC structures are considered for anti-windup. Their performance during saturation is demonstrated through simulation studies, where minimal degradation is observed.

Keywords: control system analysis, control system design, controller, constraints, saturation

1. INTRODUCTION

The term mid-ranging control typically refers to the class of control problems where two control inputs i.e. actuators are manipulated to control one output. Furthermore there is the condition that one input should return to its midpoint or some setpoint. The inputs usually differ in their dynamic effect on the output and in the relative cost of manipulating each one with the faster input normally being more costly to use than the slower input (Henson et al., 1995). Therefore mid-ranging control schemes seek to manipulate both inputs upon an upset but then gradually reset or mid-range the fast input to its desired setpoint (Allison and Ogawa, 2003).

Mid-ranging control is commonly implemented using the architecture shown in Fig. 1 where u_1 is the fast input and u_2 is the slow input. This structure is referred to as Valve Position Control (VPC) and C_1 is usually chosen as a PI controller and C_2 as an I-only controller. The VPC method for mid-ranging has been found to be sub-optimal (Allison and Isaksson, 1998, Allison and Ogawa, 2003). As such, improvements to the approach of mid-ranging control problems are suggested by many authors. Model predictive control (MPC) has also been suggested by Allison and Isaksson (1998) as an advantageous approach to mid-ranging given that it is inherently a multi-variable control problem. Henson et al. (1995) also propose MPC as well as a Direct Synthesis approach for the design of habituating controllers. The habituating control described by Henson et al. (1995) is essentially a mid-ranging control problem. Allison and Ogawa (2003) put forward a Modified Valve Position Control (MVPC) scheme which combines the simplicity of conventional VPC with the systematic tuning of Direct Synthesis. Allison and Ogawa (2003) compare the performance of MVPC with that of both conventional VPC and Direct Synthesis.



Fig. 1 Block diagram of VPC strategy (Allison and Ogawa, 2003)

For many applications MVPC works fine and has the advantage that it can be implemented using the standard VPC structure in Fig. 1 with PID control blocks. However, MVPC is not optimal; Henson et al. (1995) show that better performance (and implicitly better robustness) can be obtained by using a more general structure which includes both feedforward and feedback elements. This is acknowledged by Allison and Ogawa (2003). The Direct Synthesis design, unlike MVPC, also allows enhanced performance such as decoupling between u_{1r} and y. MVPC does not achieve this decoupled response though u_1 tracks changes in u_{1r} correctly.

The mid-ranging design proposed by Henson et al. (1995) uses both feedback and pre-filters. The design criteria are focused on:

- obtaining a desired response from y_r to y
- obtaining a desired response from u_{1r} to u₁
- obtaining a decoupled response from u_{1_r} to y.

In this paper, a similar general structure is utilised where the decoupling can be achieved through the use of pre-filters.

Magnitude



Fig. 2 Desired frequency response of complementary sensitivities

The mid-ranging design proposed in this paper focuses on the respective disturbance responses of y, u_1 and u_2 and exploits both the structure and tuning methodology of internal model control (IMC). This makes the design trade-offs transparent.

Allison and Ogawa (2003) do not discuss anti-windup for MVPC. However Haugwitz et al. (2005) have shown that for some applications an additional feedback block can significantly improve the anti-windup performance of MVPC. The IMC structure provides a natural framework for both anti-windup (discussed in this paper) and robustness analysis (see Morari and Zafiriou, 1989, for the general case).

In Sections 2 to 5 the IMC tuning method for mid-ranging is presented followed by simulation studies in Section 6 that demonstrate the performance of IMC compared to Direct Synthesis. Anti-windup in IMC mid-ranging is discussed in Section 7 with an example that shows how the classical IMC structure presented in the previous sections and a modified IMC structure perform during saturation of the inputs.

2. MID-RANGING CONTROL OBJECTIVES

The plant model is (see Fig. 1),

$$y = G_1 u_1 + G_2 u_2 + d$$

where u_1 is the fast input and u_2 is the slow input. The objective is to use the both inputs to control y and mid-range u_1 i.e. to return u_1 to its setpoint, u_{1r} .

The transfer function between y_r and y can be defined as the fast complementary sensitivity, T_f . The response from y_r to y when u_1 is set to zero can be defined as the slow complementary sensitivity, T_s (corresponding to the control action with the slow actuator alone). These are chosen to produce desired responses to setpoint changes such that the frequency response looks like Fig. 2. The proposed IMC midranging design is to specify not only T_f but also T_s . With two degrees of freedom, the rest follows as illustrated in Sections 3 and 4.

3. IMC STRUCTURE FOR MID-RANGING

Firstly the general IMC structure shown in Fig. 3 is considered. Assuming that the model is perfect $(G = \tilde{G})$, y and u can be derived as:

$$y = Gu + d \tag{1}$$



Fig. 3 IMC mid-ranging structure

$$u = Q(y_r - y + Gu) + Pu_{1_r}$$
(2)

Equations (1) and (2) can expressed as:

$$u = Qy_r - Qd + Pu_{1r} \tag{3}$$

$$y = GQy_r + (I - GQ)d + GPu_{1r}$$
⁽⁴⁾

For classical feedback control,

$$u = C(y_r - y) + \hat{P}u_{1_r}$$

where C is the equivalent feedback controller and \hat{P} is the equivalent pre-filter.

C can be found by expressing (2) as:

$$u = (I - QG)^{-1}Q(y_r - y) + (I - QG)^{-1}Pu_{1_r}$$
(5)

This gives:

$$C = (I - QG)^{-1}Q = Q(I - GQ)^{-1}$$
(6)

For mid-ranging the following are defined as:

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
, $G = \begin{bmatrix} G_1 & G_2 \end{bmatrix}$, $Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$ and $P = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$

4. IMC MID-RANGING DESIGN

From (4), T_f and T_s can be expressed in terms of Q_1 and Q_2 . This gives the conditions for the controller design:

Controller Design Conditions: $T_f = GQ = G_1Q_1 + G_2Q_2$ $T_s = G_2Q_2$

The sensitivity, S_f is defined as $S_f = 1 - T_f$ such that (4) can be expressed as,

$$y = T_f y_r + S_f d + GP u_{1_r}$$

To achieve the control objectives,

$$T_f|_{ss} = G_1 Q_1 + G_2 Q_2|_{ss} = 1$$

 $Q_1|_{ss} = 0$

This means that $T_s|_{ss} = G_2 Q_2|_{ss} = 1$.

The decoupling between u_{1r} and y is achieved through the use of pre-filters. From (4), to obtain a decoupled response between u_{1r} and y, the following condition must be satisfied:

Pre-filters Design Conditions: $GP = G_1P_1 + G_2P_2 = 0$

The design algorithm is then as follows:

- 1. Choose T_s
- 2. Q_2 is then designed such that $Q_2 = \frac{T_s}{G_2}$
- 3. Choose T_f
- 4. Q_1 is then designed such that $Q_1 = \frac{(T_f T_s)}{G_1}$
- 5. To design pre-filters, GP = 0 must be satisfied. There are different ways to achieve this:
 - a. Let $P_1 = 1$ and $P_2 = -G_1/G_2$
 - b. Let $P_1 = G_2/G_2|_{ss}$ and $P_2 = -G_1/G_2|_{ss}$
 - c. Let $G_2 = G_2^-G_2^+$ where G_2^+ is minimum phase and G_2^- is non-minimum phase, then let $P_1 = HG_2^-$ where $HG_2^-|_{ss} = 1$ and $P_2 = -HG_1/G_2^+$

 G_1 and G_2 can be either minimum phase or non-minimum phase. This design above can be extended for non-minimum phase systems as follows:

- 1. Choose T_s such that $T_s = T_s^{-}\tilde{T}_s$ where T_s^{-} includes the non-minimum phase components (including delays) of both G_1 and G_2 i.e. so that T_s/G_1 and T_s/G_2 are both casual and stable.
- 2. Q_2 is then designed such that $Q_2 = \frac{T_s}{G_2}$
- 3. Choose $T_f = T_f^{-}\tilde{T}_f$ where T_f^{-} includes the nonminimum phase component (including delay) of G_1 i.e. so that T_f/G_1 is casual and stable.
- 4. Q_1 is then chosen as $Q_1 = \frac{(T_f T_s)}{G_1}$.
- 5. P_1 and P_2 are chosen as before.

The IMC approach concentrates on the disturbance responses i.e. the third column of the sensitivities matrices (17) and (28) in Henson et al. (1995) where nine sensitivities are discussed. Equation (7) shows the sensitivities for the IMC design.

$$\begin{bmatrix} y \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} GQ & GP & (I - GQ) \\ Q_1 & P_1 & -Q_1 \\ Q_2 & P_2 & -Q_2 \end{bmatrix} \begin{bmatrix} y_r \\ u_{1r} \\ d \end{bmatrix}$$
(7)



Fig. 4 Complementary sensitivities T_f (solid) and T_s (dashed) and sensitivity, S_f (dotted) for all examples.

From a classical point of view, the disturbance responses correspond to the sensitivity S_f and two control sensitivities, Q_1 and Q_2 . For the mid-ranging problem, it is required that the slow actuator should have a control sensitivity that is low bandwidth only; meanwhile the fast actuator should have a control sensitivity that is mid-frequency only and goes to zero at steady state, giving mid-ranging. IMC is advantageous because it gives the control sensitivities of the fast and slow actuators directly as Q_1 and Q_2 .

5. IMPLEMENTATION IN VPC STRUCTURE

The IMC mid-ranging design described in Section 4 can be extended for implementation using the VPC structure. This structure is similar to Fig. 1 but includes the pre-filters P_1 and P_2 . Without the pre-filters this design does not achieve the decoupled response between u_{1r} and y; it can only track u_1 to $u_1 = u_{1r}$.

For the VPC structure in Fig. 1,

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ -C_2 C_1 \end{bmatrix} (y_r - y)$$

From (6),

$$\begin{bmatrix} C_1 \\ -C_2 C_1 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \begin{bmatrix} I - \begin{bmatrix} G_1 & G_2 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}^{-1}$$

Therefore,

$$\begin{bmatrix} C_1 \\ -C_2 C_1 \end{bmatrix} = \begin{bmatrix} Q_1 / (I - G_1 Q_1 - G_2 Q_2) \\ Q_2 / (I - G_1 Q_1 - G_2 Q_2) \end{bmatrix}$$

This gives,

$$C_{1} = \frac{Q_{1}}{(I - G_{1}Q_{1} - G_{2}Q_{2})}$$
$$C_{2} = -\frac{Q_{2}}{Q_{1}}$$

6. SIMULATION STUDIES

The performance of the IMC design in the previous sections is demonstrated via simulation of four examples. The examples are all linear, stable systems. Examples 1, 2 and 3 are taken from Henson et al., (1995) to compare the performance of IMC controllers to the Direct Synthesis controllers. Therefore T_f and T_s for examples 1 to 3 are chosen to correspond to g_{y_d} and g_{u_d} given in Henson et al. (1995). In example 1, both G_1 and G_2 are minimum phase systems. For examples 2 and 3, G_1 is minimum phase whereas G_2 has a right half plane zero and a time delay respectively. Example 4 is an original example where both G_1 and G_2 are non-minimum phase. Additional first order setpoint filters are included in both control schemes according to Henson et al. (1995).

The process model transfer functions for the examples are given below.

Example 1:
$$y = \frac{1}{2s+1}u_1(s) + \frac{1}{5s+1}u_2(s) + \frac{1}{s+1}d(s)$$

Example 2: $y = \frac{1}{2s+1}u_1(s) + \frac{-2s+1}{(2s+1)^2}u_2(s) + \frac{1}{s+1}d(s)$
Example 3: $y = \frac{1}{2s+1}u_1(s) + \frac{e^{-2s}}{2s+1}u_2(s) + \frac{1}{s+1}d(s)$
Example 4: $y = \frac{-s+1}{(2s+1)^2}u_1(s) + \frac{-2s+1}{(2s+1)^2}u_2(s) + \frac{1}{s+1}d(s)$

 T_f and T_s are as follows:

Example 1:
$$T_f = \frac{1}{0.5s+1}$$
 and $T_s = \frac{2}{(2s+1)(s+2)}$
Example 2: $T_f = \frac{1}{s+1}$ and $T_s = \frac{-2s+1}{(2s+1)(s+1)^2}$
Example 3: $T_f = \frac{1}{s+1}$ and $T_s = \frac{e^{-2s}}{(s+1)^2}$
Example 4: $T_f = \frac{-s+1}{(s+1)^2}$ and $T_s = \frac{(-s+1)(-2s+1)}{(2s+1)(s+1)^3}$

The frequency response of sensitivity, S_f and the complementary sensitivities, T_f and T_s for the examples are shown in Fig. 4. T_f is chosen to determine the closed-loop bandwidth and T_s to determine the relative work done by u_1 and u_2 .

Figs. 5 to 8 show setpoint and disturbance responses for the IMC controllers (solid line) and Direct Synthesis controllers, both parallel (dashed line) and series (dotted line).

It can be seen from Figs. 5 to 7 that identical output responses are obtained for Direct Synthesis and IMC schemes. This is expected since both pairs of controllers are tuned with the same time constants. The Direct Synthesis parallel architecture and IMC structure are identical when (6) is satisfied where C_1 and C_2 are the habituating controllers, $g_{c_{11}}$ and $g_{c_{21}}$ from Henson et al. (1995).



Fig. 5 Example 1: IMC (solid), Direct Synthesis parallel (dashed) and series (dotted) responses to unit changes in y_r at t=0, d at t=10 and u_{1_r} at t=20.



Fig. 6 Example 2: IMC (solid), Direct Synthesis parallel (dashed) and series (dotted) responses to unit changes in y_r at t=0, d at t=10 and u_{1_r} at t=20.



Fig. 7 Example 3: IMC (solid), Direct Synthesis parallel (dashed) and series (dotted) responses to unit changes in y_r at t=0, d at t=10 and u_{1_r} at t=20.



Fig. 8 Example 4: IMC (solid) response to unit changes in y_r at t=0, d at t=10 and u_{1_r} at t=20.

Direct Synthesis implemented in the series architecture responds differently from the IMC and the parallel architecture in example 1 because the relationship given by (11) in Henson et al. (1995) is not satisfied (because of the choice of $g_{c_{21}}$ in the design). For all examples, both methods produce completely decoupled response between u_{1r} and y. However the IMC design gives more flexibility since H in Section 4 can be adjusted to tune the response to allow faster setpoint tracking of u_{1r} .

From (7), Q_1 and Q_2 are directly related to the output setpoint and disturbance responses. Therefore it is simple to adjust these controllers because in the absence of model uncertainty, closed loop stability is automatically guaranteed as long as Q_1 and Q_2 are stable (Prett and Garcia, 1988). Henson et al. (1995) state that the controller $g_{c_{21}}$ can also be used to tune the responses of the two inputs to changes in y_r and d. The effect on closed loop performance of adjusting this tuning parameters is however not obvious. Neither is it obvious for what parameter values the closed loop system is stable.

7. ANTI-WINDUP IN MID-RANGING CONTROL

Haugwitz et al. (2005) propose anti-windup schemes for MVPC when u_1 saturates. Guidelines are given on how to tune mid-ranging controllers to maintain the same control action of u_2 in the saturated case as in the unsaturated case. Furthermore a modified anti-windup scheme is presented that achieves increased control action in u_2 to further reduce performance degradation. However, if the MVPC structure is to be modified, then significantly improved performance can be achieved for the saturated case by using the IMC approach in this paper.

In this section, the performance of the classical IMC structure used for mid-ranging (see Fig. 3) is considered when the inputs u_1 and u_2 saturate. This IMC structure works for most cases but as demonstrated by Zheng et al. (1994), sometimes IMC requires a modified structure. The modified IMC structure presented by Zheng et al. (1994) can be utilised for



Fig. 9 Modified IMC mid-ranging structure for anti-windup

the proposed IMC mid-ranging design as shown in Fig. 9. The performance during saturation of this modified IMC structure (Fig. 9) is also considered in this section.

7.1 Anti-windup in IMC

As with the controller design, firstly the general case is considered. For the unsaturated case in Fig. 9,

$$u = (1 + Q_b)^{-1}Q_f(y_r - y + Gu) + (1 + Q_b)^{-1} \tilde{P}u_{1_r}$$

From (2),

$$u = Q(y_r - y + Gu) + Pu_1,$$

Therefore it is desired that,

$$Q = (1 + Q_b)^{-1} Q_f$$
(7)
$$P = (1 + Q_b)^{-1} \tilde{P}$$

For mid-ranging, u, G, Q and P are defined as in Section 3. Additionally, \tilde{P} , Q_f and Q_b are as follows:

$$\tilde{P} = \begin{bmatrix} \tilde{P}_1 \\ \tilde{P}_2 \end{bmatrix}, \ Q_f = \begin{bmatrix} Q_{f_1} \\ Q_{f_2} \end{bmatrix} \text{ and } Q_b = \begin{bmatrix} Q_{b_1} & 0 \\ 0 & Q_{b_2} \end{bmatrix}.$$

Extending the general case before for mid-ranging gives:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \left(I + \begin{bmatrix} Q_{b_1} & 0 \\ 0 & Q_{b_2} \end{bmatrix} \right)^{-1} \begin{bmatrix} \overline{u}_1 \\ \overline{u}_2 \end{bmatrix}$$

$$\text{where } \begin{bmatrix} \overline{u}_1 \\ \overline{u}_2 \end{bmatrix} = \begin{bmatrix} \widetilde{P}_1 \\ \widetilde{P}_2 \end{bmatrix} u_{1r} + \begin{bmatrix} Q_{f_1} \\ Q_{f_2} \end{bmatrix} \left(y_r - y + \begin{bmatrix} G_1 & G_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \right)$$

Therefore,

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{Q_{f_1}}{1 + Q_{b_1}} \\ \frac{Q_{f_2}}{1 + Q_{b_2}} \end{bmatrix} (y_r - y + G_1 u_1 + G_2 u_2) + \begin{bmatrix} \frac{\tilde{P}_1}{1 + Q_{b_1}} \\ \frac{\tilde{P}_2}{1 + Q_{b_2}} \end{bmatrix} u_{1r}$$

This means that

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} \frac{Q_{f_1}}{1+Q_{b_1}} \\ \frac{Q_{f_2}}{1+Q_{b_2}} \end{bmatrix} \text{ and } \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} \frac{\tilde{P}_1}{1+Q_{b_1}} \\ \frac{\tilde{P}_2}{1+Q_{b_2}} \end{bmatrix}$$



Fig. 10 Example 1: Response of IMC scheme with no saturation (solid), $\lambda = 0$ (dashed) and $\lambda = 1$ (dotted) for a unit changes in y_r at t=10, d at t=30, u_{1_r} at t=50 and t= 70.

From Zheng et al. (1994), Q_f can usually be chosen by the following relationship:

 $Q_f = \lambda Q(\infty) + (1 - \lambda)Q$

- Condition 1: when $\lambda = 0$, then $Q_f = Q$ and from (7) $Q_b = 0$
- Condition 2: when $\lambda = 1$, then $Q_f = Q(\infty)$ and from (7) $Q_b = (Q(\infty) - Q)/Q$

The classical IMC structure as shown in Fig. 3 corresponds to the condition where $\lambda = 0$ and the modified IMC structure given by the block diagram in Fig. 9 corresponds to $\lambda = 1$. For condition 2 above, Q_{f_1} cannot be chosen to be $Q(\infty)$ because for the mid-ranging control problem it is required that $Q_{f_1}(0) = 0$. To ensure that $Q_{f_1}(0) = 0$, Q_f can be chosen such that,

$$Q_{f_1} = Q_1(\infty) \frac{s}{s + p_1}$$

where p_1 is a pole of Q_1 .

 Q_{f_2} can simply be chosen as $Q_{f_2} = Q_2(\infty)$. However, if Q_2 is strictly proper then Q_2 can be defined as:

$$Q_2 = Q_{2_b} \times Q_{2_s}$$

where Q_{2_b} is bi-proper and Q_{2_s} is strictly proper so that $Q_{f_2} = Q_{2_b}(\infty) \times Q_{2_s}$.

Fig. 10 shows the response of the system in example 1 defined in Section 6 when u_1 and u_2 saturate. Responses for both the classical IMC structure (Fig. 3) and the modified IMC structure (Fig. 9) are considered. For this example the response when $\lambda = 0$ and $\lambda = 1$ is similar because $Q_{f_1} = Q_1$.

When u_1 saturates, output tracking is still achieved by the slow actuator. The control action of u_2 is the same when u_1

saturates as in the unsaturated case. When u_2 saturates however, the fast actuator compromises between input and output tracking. Therefore the classical IMC structure offers acceptable performance during saturation. In some cases, the modified IMC structure offers even better performance with saturation of u_1 and u_2 .

8. CONCLUSION

Many mid-ranging designs have been proposed and of them, MVPC and Direct Synthesis are most appropriate to address the ad hoc nature of conventional mid-ranging tuning procedures. MVPC works sufficiently especially when restricted to conventional mid-ranging structure and PID control. However, when not restricted, the Direct Synthesis (Henson et al., 1995) and IMC approaches give better performance. The IMC mid-ranging design presented in this paper gives the same improved performance over MVPC as Direct Synthesis. Moreover, this approach gives insight to the design trade-offs of MVPC and Direct Synthesis by emphasis on bandwidth considerations. An added benefit of the IMC approach is the ability to integrate anti-windup to midranging control. Other mid-ranging approaches require additional control blocks and further modifications to achieve acceptable performance under saturation of the inputs. IMC mid-ranging is already a natural structure for anti-windup.

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