Boundary geometric control of co-current heat exchanger

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Abstract: A control strategy is proposed to control the internal fluid temperature at the outlet of a co-current heat exchanger by manipulating the inlet external fluid temperature. The dynamic model of the heat exchanger is given by two partial differential equations. Based on nonlinear geometric control, a state-feedback law that ensures a desired performance of a measured output defined as spatial average temperature of the internal fluid is derived. Then, in order to control the outlet internal fluid temperature, a control strategy is proposed where an external controller is introduced to provide the set point of the considered measured output by taking as input the error between the outlet internal fluid temperature and its desired set point. The validity of the proposed control design and strategy is examined in simulation by considering the tracking and perturbation rejection problems.

Keywords: distributed parameter system, partial differential equation, geometric control, characteristic index,PI controller, co-current heat exchanger.

1. INTRODUCTION

As a thermal device, heat exchangers are widely used in process industries both for cooling and heating operations. The dynamic behavior of the heat exchanger is modeled by a set of partial differential equations (PDE) that describe the spatio-temporal variation of the temperatures. Thus, the need to find the best operating conditions for the heat exchangers and to improve their effectiveness lead to take into account their distributed nature. In this context, good performances can be attained using more efficient control strategy based on the direct use of the distributed parameter model rather than a reduced or a lumped model (Ray, 1989; Christofides, 2001).

Heat exchangers can be classified into two major types according to their flow arrangement: co-current and counter-current heat exchangers. For the first one, the two fluids travel in the same direction. By contrast, for the second one, the fluids move in opposite directions.

In the control problem of tube heat exchangers, the variable which is manipulated, theoretically, is the thermal power at the inlet of the outer tube, i.e. grossly the product of a flow rate and a difference of temperature. In practice, to control the outlet temperature of a heat exchanger, two possible strategies which are not equivalent exist. The first one is to use the inlet temperature of the external fluid, while the second is to manipulate its flow rate.

When the flow rate is considered as a manipulated variable, if it becomes too low, the flow regime in the outer tube can be laminar instead of turbulent, which affects the parameters of the models, in particular the heat transfer coefficient (Xuan and Roetzel, 1993). So the tuning of the controller should vary with the flow rate, which is a difficult task (Abdelghani-Idrissi et al., 2001; Arbaoui et al., 2007). In addition, by manipulating the flow rate, a minimum bound is to set on this input. With the temperature as a manipulated input, it is possible to work at a constant large flow rate and the hydrodynamic regime is invariable. Physically, manipulating the temperature is almost possible if this latter is the outlet of a process with fast dynamics like plate heat exchangers. Potential flow rate variations will be assumed as a disturbance that affects the system and needs to be rejected by the designed controller.

Control of counter-current heat exchanger has attracted much attention, and several strategies are proposed based either on the PDE model or ODE model (see e.g. Maidi et al. (2008a) for more references) compared to the co-current heat exchanger for which few methods have been proposed in the literature. Derese and Noldus (1980) addressed the problem of controlling of the co-current heat exchanger using dynamical lumped parameter controllers designed based on technical frequency domain specifications. Based on the conjugate gradient method (CGM) of minimization, Huang and Yeh (2003) proposed an algorithm for determining an optimal external distributed heat-flux of a steady state co-current heat exchanger.

In this paper, a control strategy is proposed to control the outlet internal fluid temperature of a co-current heat exchanger by manipulating the inlet external fluid temperature. The designed approach is based on the use of the PDE model that describes the dynamic behavior. The idea is to design a state-feedback control that allows controlling the average temperature of the internal tube of the heat exchanger, assumed as the measured output. As it will be demonstrated, the direct design of a control law by considering the outlet temperature as the controlled variable is a difficult task due to the fact that the process is infinite-dimensional. Then, in order to control the outlet fluid temperature, a control strategy is proposed where a PI controller is introduced to provide the set point of the measured

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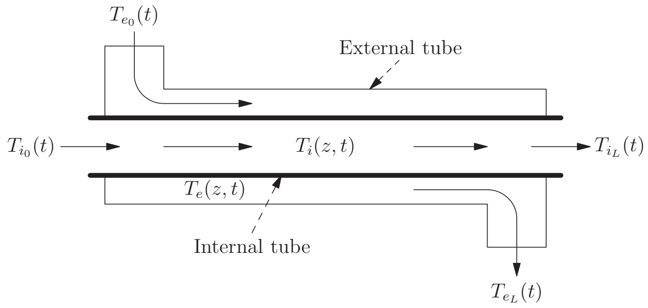

Fig. 1. co-current heat exchanger.

output (spatial average temperature). The design of the state-feedback control law makes use of the concept of characteristic index (Christofides and Daoutidis, 1996), which characterize the spatiotemporal interactions between the controlled and manipulated variables.

The paper is organized as follows. In section 2, the studied co-current heat exchanger is presented and its dynamic model given as a system of two PDEs. Section 3 concerns the formulation of the control problem and the design methodology. Section 4 is dedicated to simulation results concerning tracking and perturbation rejection problems. Finally, a conclusion ends the article.

2. CO-CURRENT HEAT EXCHANGER DYNAMIC MODEL

2.1 Description of the heat exchanger

The process studied in this work corresponds to a tubular co-current heat exchanger (Fig. 1). A fluid of constant density ρ_i and of heat capacity C_{p_i} flows through the internal tube of a heat exchanger, of length L, with a constant velocity v_i. This fluid enters at temperature T_{i_0} and exchanges heat with the an external fluid or non condensating vapor fluid, of constant density ρ_e and of heat capacity C_{p_e}, which flows in the same direction in the jacket with a velocity v_e. This fluid enters at temperature T_{e_0}. At the outlet of the exchanger, the internal fluid leaves at temperature T_{i_L}. In the present study, the internal and external cross sections S_i and S_e of the heat exchanger are supposed to be uniform and the surface area used for the heat transfer per unit length is $\mathcal{A}$. Both temperatures T_i of the internal fluid and T_e of the external fluid depend on time and spatial position along the tube.

The energy balance of the heat exchanger, after classical simplifying hypotheses (Ray and Ogunnaike, 1994), gives the following partial differential equation for the internal tube (internal fluid)

$$\frac{\partial T_i(z,t)}{\partial t} = -v_i \frac{\partial T_i(z,t)}{\partial z} + h_i \left[T_e(z,t) - T_i(z,t)\right] \qquad (1)$$

and the following partial differential equation for the jacket (external fluid)

$$\frac{\partial T_e(z,t)}{\partial t} = -v_e \frac{\partial T_e(z,t)}{\partial z} + h_e \left[T_i(z,t) - T_e(z,t)\right] \qquad (2)$$

where $h_i = \dfrac{U_i\,\mathcal{A}}{\rho_i\,S_i\,C_{p_i}}, \quad h_e = \dfrac{U_e\,\mathcal{A}}{\rho_e\,S_e\,C_{p_e}}.$

T_i and T_e are the temperatures of the internal and external fluids, respectively, h_i and h_e are the heat transfer coefficients, v_i and v_e are the velocities, U_i and U_e are the overall heat transfer coefficients, $\mathcal{A}$ is the surface area devoted to heat transfer.

Each PDE requires an initial condition and a boundary condition to be fully defined. The studied heat exchanger is of co-current type. For Eq. (1) describing the temperature of the internal fluid, the boundary condition is usually specified at $z = 0$ as the temperature of the fluid entering the tube is in general known and measurable. Thus, at $z = 0$, it gives

$$T_i(0,t) = T_{i_0}(t) \qquad (3)$$

and most often the initial condition is some given temperature profile at $t = 0$

$$T_i(z,0) = T_i^*(z) \qquad (4)$$

Similarly, for Eq. (2), describing the distribution of temperature of the external fluid in the jacket, the boundary condition is the temperature of the entering fluid T_{e_0}, specified at $z = 0$, consequently

$$T_e(0,t) = T_{e_0}(t) \qquad (5)$$

while the initial condition is some given temperature profile at $t = 0$

$$T_e(z,0) = T_e^*(z) \qquad (6)$$

Eqs. (1)-(6) constitute the dynamic model of the co-current heat exchanger.

3. CONTROL OF THE CO-CURRENT HEAT EXCHANGER

3.1 Control problem formulation

As indicated above, to control the outlet internal temperature T_{i_L}, two manipulated variables are possible, either the inlet external fluid temperature T_{e_0} or the flow rate represented by the velocity v_e. In this work, the temperature T_{e_0}, corresponding to the boundary condition (5), is taken as a manipulated variable to easily control the outlet internal fluid temperature T_{i_L} since the hydrodynamic regime remains invariable. Now, due to Eq. (2), it is noticeable that by manipulating the boundary condition of the jacket, given by Eq. (5), a variation of the temperature of the external fluid T_e along the jacket results. Thus, by denoting as u the control variable and y the controlled variable, the model of the heat exchanger (1)-(6) takes the following form

$$\frac{\partial T_i(z,t)}{\partial t} = -v_i \frac{\partial T_i(z,t)}{\partial z} + h_i \left[T_e(z,t) - T_i(z,t)\right] \qquad (7)$$

$$\frac{\partial T_e(z,t)}{\partial t} = -v_e \frac{\partial T_e(z,t)}{\partial z} + h_e \left[T_i(z,t) - T_e(z,t)\right] \qquad (8)$$

$$T_i(0,t) = T_{i_0}(t) \qquad (9)$$

$$T_e(0,t) = T_{e_0}(t) = u(t) \qquad (10)$$

$$T_i(z,0) = T_i^*(z) \qquad (11)$$

$$T_e(z,0) = T_e^*(z) \qquad (12)$$

$$y(t) = \mathcal{C}\big(T_i(z,t)\big) = \int_0^L \delta(z-L)\,T_i(z,t)\,dz \qquad (13)$$

where $\mathcal{C}(\,.\,)$ is a bounded linear operator.

3.2 Design approach

Recently the nonlinear geometric control has proved to be very successful as a control approach of the linear and quasi-linear DSP (Christofides and Daoutidis, 1996; Gundepudi and Friedly, 1998; Christofides, 2001; Wu and Liou, 2001; Maidi et al., 2008a,b). The most important advantage of geometric control is that the control law can be designed using directly the PDE model, which leads to distributed control that increases the performances (Christofides, 2001). Thus, this theoretical approach will be used to derive a boundary control law for the co-current heat exchanger.

The manipulated variable $u(t)$ appears as an inhomogeneous part in the boundary condition (10), so in order to obtain the expression of the control law, we propose to insert the manipulated variable $u(t)$ through the use of Dirac delta function in the state equation (7) as follows

$$\frac{\partial T_e(z,t)}{\partial t} = -v_e \frac{\partial T_e(z,t)}{\partial z} + h_e\left[T_i(z,t) - T_e(z,t)\right] + v_e\,\delta(z)\,u(t) \tag{14}$$

so that the model will be affine with respect to the input $u(t)$.

Under these conditions, the boundary condition (10) becomes homogeneous,

$$T_e(0,t) = 0 \tag{15}$$

Now, as the open-loop system $u(t)$-$y(t)$ is infinite dimensional, the characteristic index σ does not exist. This can be easily verified by calculating the successive derivatives of the output (13) with respect to time. To overcome this problem, we propose to consider another measured output given as the average of the external fluid temperature, i.e.

$$y_m(t) = \mathcal{C}_m\big(T_i(z,t)\big) = \int_0^L c_m(z)\,T_i(z,t)\,dz \tag{16}$$

where $\mathcal{C}_m(\,.\,)$ is a bounded linear operator and $c_m(z)$ is a smooth positive function ($c_m(z) > 0$).

In this case, the derivative of the measured output (16) with respect to time yields

$$\begin{aligned}\frac{dy_m(t)}{dt} &= \int_0^L c_m(z)\,\frac{\partial T_i(z,t)}{\partial t}\,dz \\ &= \int_0^L c_m(z)\bigg(-v_i\frac{\partial T_i(z,t)}{\partial z} + h_i\left[T_e(z,t)\right.\\ &\qquad \left. - T_i(z,t)\right]\bigg)\,dz\end{aligned} \tag{17}$$

the characteristic index is greater than one. Performing one more differentiation, we obtain:

$$\begin{aligned}\frac{d^2y_m(t)}{dt^2} &= \int_0^L c_m(z)\bigg(-v_i\frac{\partial}{\partial t}\left(\frac{\partial T_i(z,t)}{\partial z}\right)\\ &\quad + h_i\left[\frac{\partial T_e(z,t)}{\partial t} - \frac{\partial T_i(z,t)}{\partial t}\right]\bigg)\,dz\end{aligned} \tag{18}$$

By substituting the term $\dfrac{\partial T_e(z,t)}{\partial t}$ by its expression given by (14) and after arrangement, equation (18) takes the form

$$\frac{d^2y_m(t)}{dt^2} = I_1 + h_i\,v_e\underbrace{\left[\int_0^L c_m(z)\delta(z)\,dz\right]}_{I_2}u(t) \tag{19}$$

where I_1 is the remaining term of the integral in equation (18). According to equation (19), it is clear that the input appears linearly.

Now, in order to have the control law $u(t)$ well-defined, the integral term I_2 must be different from zero. This condition ensures that the characteristic index of the measured output $y_m(t)$ with respect to the manipulated input $u(t)$ is equal to 2. The calculus of I_2 gives

$$I_2 = \int_0^L c_m(z)\,\delta(z)\,dz = \left. c_m(z)\right|_{z=0} \tag{20}$$

The condition on the characteristic index being equal to 2 is related to the choice of the function $c_m(z)$, i.e. the value of $c_m(z)$ should not be zero at $z=0$

$$I_2 = c_m(0) \neq 0 \tag{21}$$

Thus, by choosing a function $c_m(z) \geq 0$ that satisfies the condition (21), the characteristic index will be $\sigma = 2$. In summary, the modification of equation (14) by introduction of the manipulated input and the consideration of the new output (16) have ensured the existence of the characteristic index.

As $\sigma = 2$, this suggests requesting the following input-output response of the closed-loop system

$$\tau_2\frac{d^2y_m(t)}{dt^2} + \tau_1\frac{dy_m(t)}{dt} + y_m(t) = v(t) \tag{22}$$

Substituting (19) into equation (22), we obtain the following state-feedback control law

$$u(t) = \frac{1}{h_i\,v_e\,\tau_2\,I_2}\Big[v(t) - y_m(t) - \tau_1\,\dot{y}_m(t) - \tau_2\,I_1\Big] \tag{23}$$

where τ_1, τ_2 are adjustable controller parameters chosen to guarantee the input-output stability and to enforce the desired performance specifications for the output $y_m(t)$ (Christofides, 2001), and $v(t)$ is an external input.

The control robustness dealing with problems of model and parameter uncertainty and unmodeled dynamics, is provided in (23) through application of the linear control theory to the resulting linear [input $v(t)$-output $y_m(t)$] linear system to define the external input $v(t)$ by a robust controller. In this work, in order to ensure the robustness, i.e. to handle uncertainties and unmodeled dynamics, the external input $v(t)$ is defined by means of a PI controller (Kravaris and Kantor, 1990) as follows

$$v(t) = K_{c_m}\left[\big(y_m^d(t) - y_m(t)\big) + \frac{1}{\tau_{I_m}}\int_0^t \big(y_m^d(\xi) - y_m(\xi)\big)\,d\xi\right] \tag{24}$$

where K_{c_m}, τ_{I_m} are respectively the proportional gain, integral time constant of the PI controller, respectively. $y_m^d(t)$ is the set-point of the measured variable $y_m(t)$.

Thus, the transfer function of the closed loop system is the following

$$\frac{Y_m(s)}{Y_m^d(s)} = \frac{K_{c_m}\left(\tau_{I_m}+1\right)}{\tau_{I_m}\,\tau_2\,s^3+\tau_{I_m}\,\tau_1\,s^2+\left(\tau_{I_m}+K_{c_m}\,\tau_1\right)+K_{c_m}} \quad (25)$$

The scalar parameters K_{c_m}, τ_{I_m} and τ are tuned in order for the denominator to approach a polynomial minimizing an ITAE criterion (Corriou, 2004) and it can be verified that the following polynomial is Hurwitz (the poles have a negative real part) to ensure the closed loop stability related to the roots of the characteristic equation

$$\tau_2\tau_{I_m}\,s^3+\tau_1\tau_{I_m}\,s^2+\left(\tau_{I_m}+\tau_1K_{c_m}\right)s+K_{c_m}=0 \quad (26)$$

At this point, it is clear that the control law (23) ensures the desired performances of the introduced measured output $y_m(t)$ rather the controlled output $y(t)$. Actually, the output $y_m(t)$ is introduced only in order to avoid the problem of non-existence of the characteristic index. In order to solve the formulated boundary control problem, i.e. controlling the output $y(t)$, we propose to keep the control law (23) derived for the measured output (16) with $c(z)$ satisfying the condition (21). Then, define the set point of the measured output $y_m(t)$, denoted by $y_m^d(t)$, by means of a PI controller taking as input the error $e(t) = y^d(t) - y(t)$, where $y^d(t)$ is the corresponding set point of the controlled variable $y(t)$. Note that another control technique can be adopted to provide the set point $y_m^d(t)$. The proposed global control strategy is summarized in Fig. 2.

The control law (23) requires that the complete state $T_i(z,t)$ must be available especially to evaluate the integral term I_1 and the measured output $y_m(t)$. From a practical point of view, this is impossible since the state $T_i(z,t)$ is infinite. Ray (1989) discusses some way that can provide the complete state of a distributed parameter system. The design of Kalman filter that estimates the whole state variables vector in the case of a counter-current heat exchanger has been studied by Maidi et al. (2008a). In this work, it is considered that the vector of state variables is fully available to clearly show the effectiveness and the contribution of the proposed control strategy.

Note that the choice of the function $c_m(z)$ is not unique. Nevertheless, the relation (21) shows that the function $c_m(z)$ is involved in the evaluation of the integral term I_1 and in calculating the measured output $y_m(t)$ and its derivative $\dot{y}_m(t)$, so it is suggested to choose a simple function for example $c_m(z) = L - z$. From a practical point of view, these calculations can be provided simply by a computer by processing the data measurements $T_i(z,t)$ and $T_e(z,t)$.

4. SIMULATION RESULTS

In this section, the performance of the proposed control strategy will be illustrated through application examples. For simulation purpose of the closed-loop system, the method of lines (Wouwer et al., 2004) is used by considering a number of discretization points $N = 100$. The control is held constant over the sampling period equal to 0.02 s in all simulation runs. The integral term I_1, the measured output $y_m(t)$ and its derivative $\dot{y}_m(t)$ involved in the control law (23) are evaluated numerically using the trapezoidal method. The terms involving differentiation according to the space variable z are evaluated by means of finite differences.

The heat exchanger parameters (Friedly, 1972) are $v_e = 2\,\mathrm{m\,.\,s^{-1}}$, $v_i = 1\,\mathrm{m\,.\,s^{-1}}$, $h_e = 1\,\mathrm{s^{-1}}$, $h_i = 1\,\mathrm{s^{-1}}$ and $L = 1\,\mathrm{m}$. For the internal PI controller, the tuning parameters obtained following the tuning procedure described at the end of the section 3.2 are $K_{c_m} = 0.0240$, $\tau_{I_m} = 0.0469$ s. The tuning parameters K_c and τ_I of the external PI controller that provide the set point $y_m^d(t)$ have been achieved by trial and error and observation of the obtained performance, so that the retained parameters are $K_c = 0.02$ and $T_i = 0.3$ s.

The initial conditions $T_i(z,0)$ and $T_e(z,0)$ are the steady state profiles (Fig. 3) defined by $T_{i_0}(t) = 25$°C and $T_{e_0}(t) = 50$°C.

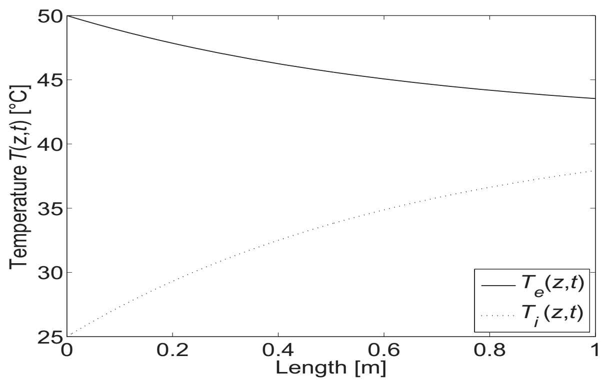

Fig. 3. Profiles of the temperatures of the internal and external fluids at steady state.

4.1 Tracking problem

In the first simulation run, the reference input tracking capabilities of the controller are studied. Thus, two step set points have been specified at times $t = 1$ s and $t = 30$ s corresponding respectively to $y^d(t) = 60$ °C and $y^d(t) = 30$ °C. On Fig. 4, it is clear that the output $y(t)$ (Fig. 4b) follows perfectly the imposed set point whereas the control moves of $u(t)$ are physically acceptable (Fig. 4c). In addition, the spatial profiles of temperature obtained at time $t = 60$ s is also realistic (Fig. 4d).

4.2 Disturbance rejection

The second performed test concerns the problem of disturbance rejection. The performances of the control strategy are thus evaluated with respect to changes of the internal fluid temperature at the inlet of the heat exchanger which is a disturbance for the process. For that reason, a step of -10% of the temperature of the entering internal fluid (at $z = 0$) is imposed as a disturbance at time $t = 30$ s, after having imposed a step set point at time $t = 1$ s corresponding to $y^d(t) = 60$ °C. From Fig. 5, it is clear that the controller behaves adequately to reject the disturbance effect and achieve perfectly the set point tracking (Fig. 5b). The dynamic behavior of the manipulated variable $u(t)$ (Fig. 5c) remains also physically admissible. Again, the profiles of temperatures at $t = 60$ s, after successively the step set point and the step disturbance, are typical of the behavior of a co-current heat exchanger (Fig. 5d).

5. CONCLUSION

In this paper, the geometric control of a co-current heat exchanger is investigated, and a control strategy is proposed to

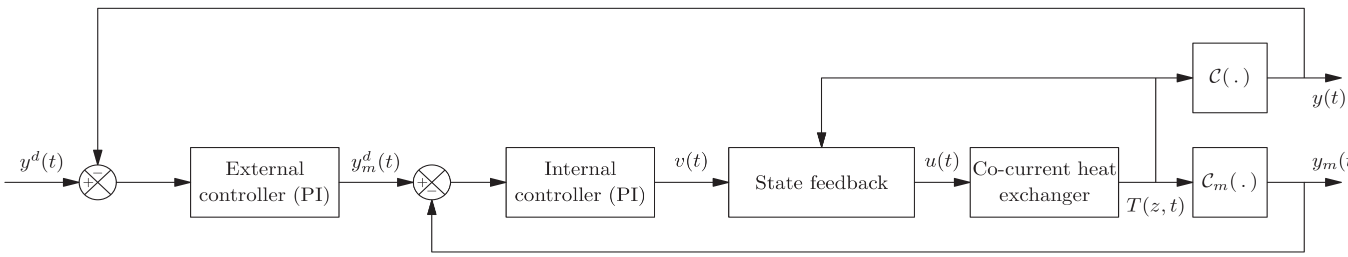

Fig. 2. Global control strategy of a co-current heat exchanger ($T(z,t) = [T_e(z,t)\, T_i(z,t)]^T$).

control the outlet internal fluid temperature. The main idea consists in inserting the manipulated variable, i.e. the inlet external fluid temperature, in the state equations of the heat exchanger by means of a Dirac function. Furthermore, the spatial average temperature of the internal fluid has been introduced, as measured output, in order to ensure the existence of the characteristic index. Then, to achieve a desired performance of the outlet internal fluid temperature, a control strategy is proposed where a PI external controller is introduced to provide the set point of the introduced measured output by taking as input the error between the outlet internal fluid temperature and its desired set point. The effectiveness of the proposed design and control strategy is demonstrated through numerical experiments. The simulation results show that the control strategy behaves correctly and ensures a satisfactory tracking and disturbance rejection.

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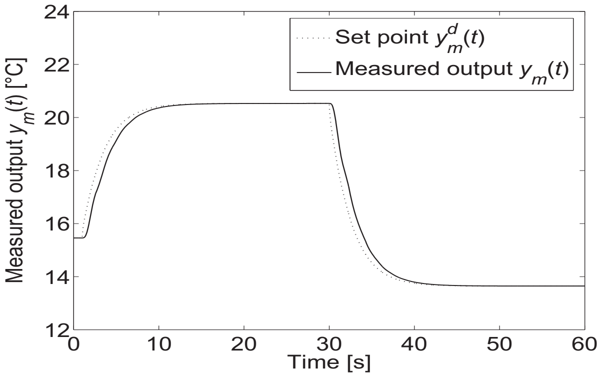
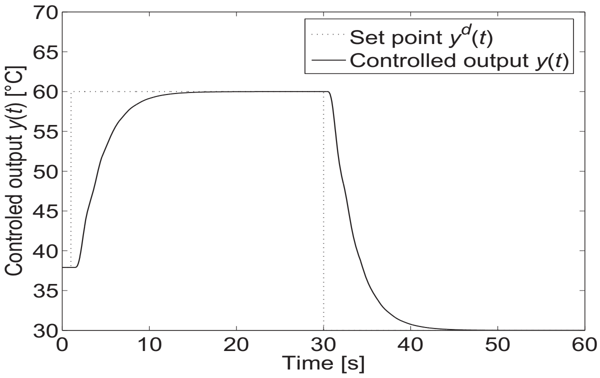
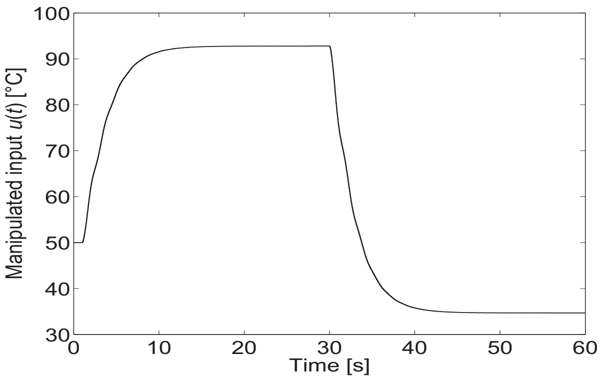


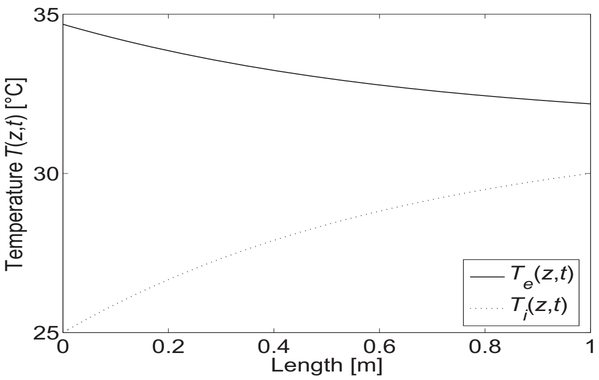
(a) Measured output $y_m(t)$.



(b) Controlled output $y(t)$.

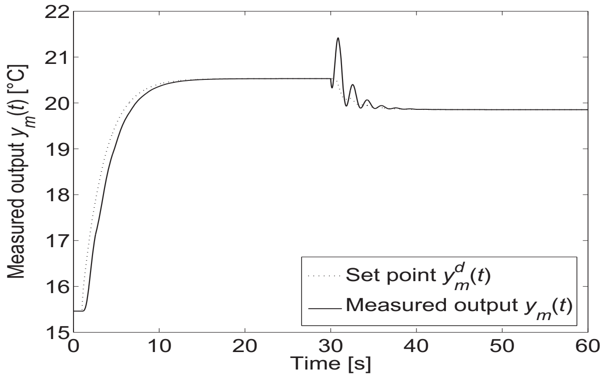


(c) Manipulated variable $u(t) = T_e(0, t)$.



(d) Spatial profiles $t = 60\,$s.

Fig. 4. Set point tracking.



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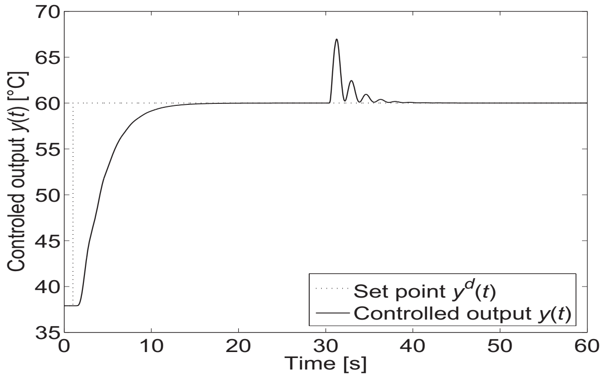
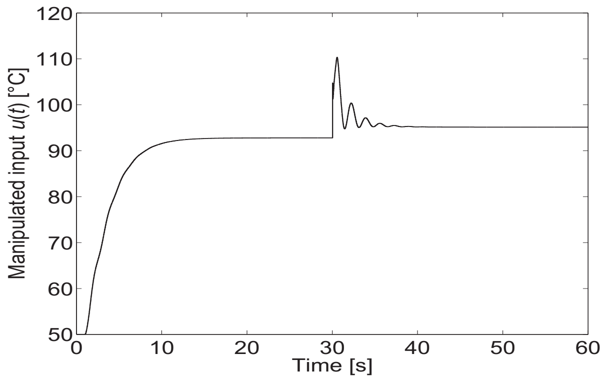
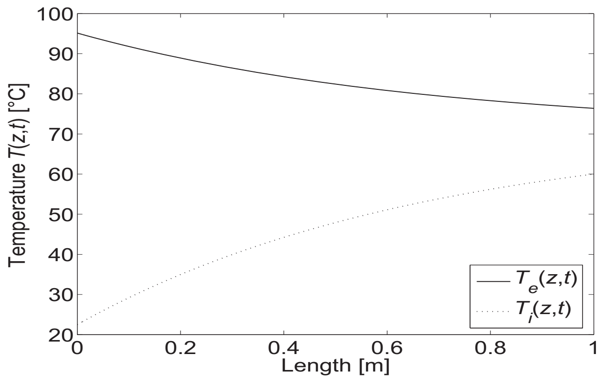


(b) Controlled output $y(t)$.



(c) Manipulated variable $u(t) = T_e(0, t)$



(d) Spatial profile at $t = 60\,$s.

Fig. 5. Disturbance rejection.