Multi-step Prediction Error Approach for MPC Performance Monitoring ⋆

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Abstract: Performance monitoring of model predictive control systems (MPC) has received a great interest from both academia and industry. In recent years some novel approaches for multivariate control performance monitoring have been developed without the requirement of process models or interactor matrices. Among them the prediction error approach has been shown to be a promising one, but it is k-step prediction based and may not be fully comparable with the MPC objective that is multi-step prediction based. This paper develops a multi-step prediction error approach for performance monitoring of model predictive control systems, and demonstrates its application in an industrial MPC performance monitoring and diagnosis problem.

Keywords: Multivariable control systems, Model predictive control, Performance evaluation, Performance monitoring, Prediction error methods.

1. INTRODUCTION

Since early work of Harris (1989), research on control performance assessment (CPA) has achieved a great progress and continues to be an active area. There is a great demand from industry for this research to produce practical solutions, particularly for MPC monitoring. Many algorithms in CPA including commercial software have been developed. There are several interesting reviews addressing related research achievements in different stages (Harris et al., 1999; Huang et al., 1999; Jelali , 2006; Qin , 2007).

Even with great achievements, multivariable CPA still has a number of stumbling blocks in practical applications. Recently some progress has been made towards this direction (Jelali , 2006; Huang et al., 2006). In particular, performance assessment of model predictive control (MPC) has been an interest since MPC is the most effective and widely used advanced multivariate control strategies in modern industries. With the existence of the constraints and economic optimization, the existing CPA is not directly applicable to its performance assessment (Xu et al., 2007).

For multivariable CPA to be practical, it must reduce *a priori* knowledge requirement. Traditional approaches for the multivariable CPA with minimum variance control as the benchmark need to estimate the interactor matrices, which is equivalent to knowing the process model (Huang et al., 1999) or at least the first few Markov parameter matrices. Recently, some new methods have been developed to address the multivariable CPA problems with only the input/output data (Jelali , 2006; Huang et al., 2006).

What simple index may be considered as a measure or one of the most important MPC performance measures? Consider that, if a closed-loop output is highly predictable, one should be able to do better, i.e. to compensate the predictable content by a well designed controller. This is the principle of predictive control. Should a better controller be implemented, the closed-loop output would have been less predictable. Therefore, high predictability of a closed-loop output implies high potential to improve its performance by controller re-tuning and/or re-design, or in other words, the existing controller may not have been satisfactory in terms of exploring its potential.

However, the CPA approach based on the prediction error has an equivalence to minimum variance based performance measure (Huang et al., 2008). Thus it may not be fully comparable with the MPC objective. Motivated by the prediction-error approach of (Huang et al., 2006; Zhao et al., 2008) and multi-step identification of Shook et al. (1992), this paper further develops closed-loop prediction-error measures based on multi-step prediction that is more relevant to model predictive control. Furthermore, applications of the proposed performance measures for an industrial model predictive control system are reported in this paper.

The remainder of this paper is organized as follows: Section 2 revisits the concept of prediction-error and closed-loop potentials for CPA. Section 3 introduces the

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multi-step prediction error. Based on it, new potential measures are defined for the MPC controller performance assessment in Section 4. This is followed by an industrial case study in Section 5 to illustrate the utility of the new performance measures. Finally the conclusion is drawn in Section 6.

2. REVISIT OF CLOSED-LOOP POTENTIAL FOR MULTIVARIATE CPA

In this section, we shall revisit the concepts of prediction error and closed-loop potentials as defined in Huang et al. (2006).

For a multivariable process, the closed-loop output driven by white noise can be described by a time series model:

$$Y_t = G_{cl} a_t \tag{1}$$

where G_{cl} is the time series model and a_t is white noise with mean zero and covariance Σ_a.

Transfer the above time series model to a moving average (MA) form:

$$Y_t = \sum_{k=0}^{\infty} F_k a(t-k) = F_0 a_t + F_1 a_{t-1} + \cdots + F_{i-1} a_{t-(i-1)} + F_i a_{(t-i)} + \cdots \tag{2}$$

Note that this time series model can be estimated without any a priori knowledge about the process.

With the MA model, one can obtain the optimal ith step prediction:

$$Y_{t|t-i} = F_i a_{(t-i)} + F_{i+1} a_{(t-i-1)} + \cdots \tag{3}$$

and the prediction error:

$$e_{t|t-i} = Y_t - Y_{t|t-i} = F_0 a_t + F_1 a_{t-1} + \cdots + F_{i-1} a_{t-(i-1)} \tag{4}$$

where $F_0 = I$. The covariance of the prediction error can be calculated as

$$cov(e_{t|t-i}) = F_0 \Sigma_a F_0^T + F_1 \Sigma_a F_1^T + \cdots + F_{i-1} \Sigma_a F_{i-1}^T \tag{5}$$

Define its scalar measure:

$$s_i = tr(cov(e_{t|t-i})) = tr(F_0 \Sigma_a F_0^T + \cdots + F_{i-1} \Sigma_a F_{i-1}^T) \tag{6}$$

s_i is monotonically increasing with i, as $i \to \infty$, $e_{t|t-i} \to Y_t$, and $s_\infty = tr(cov(Y_t))$. If we plot s_i versus i, the plot reflects how the prediction error increases with the prediction horizon.

A closed-loop potential is defined in Huang et al. (2006) as:

$$p_i = \frac{s_\infty - s_i}{s_\infty} \tag{7}$$

The closed-loop potential can be interpreted as following (Huang et al., 2006): If a deadbeat control action can be applied from time i, then the sum of squared error (SSE) can be reduced by $100 \times p_i$ percent. From stochastic view point, if i is greater than the interactor order d, it is possible that the variance of the multivariate output can be reduced by $100 \times p_i$ percent of the current variance. Since the order of the actual interactor matrix may not be known, one can check the trajectory of the closed-loop potential versus a range of possible time lag d. As s_i is monotonically increasing with i, p_i is monotonically decreasing. When $i \to 0$, $s_0 = tr(cov(Y_t - Y_{t|t})) = 0$, $p_0 = 1$. Therefore, the index p_i starts from 1 at $i = 0$ and monotonically decreases to 0 at $i \to \infty$. Larger the closed-loop potential is, more potential the control performance can be improved.

From the potential plot we can draw the conclusion whether or how much the present closed-loop has potential to improve. Furthermore, with the plot, we can compare performance of a controller between different tuning parameters.

3. CLOSED-LOOP POTENTIAL MEASURES BASED ON MULTI-STEP PREDICTION

3.1 Multi-step optimal prediction and its scalar measure

It is well-known that minimum variance control is an aggressive control and not all controllers are designed towards minimum variance performance. Therefore, in addition to the measure of the optimal i-step prediction error s_i, which is associated with minimum variance performance, we consider a control that achieves optimal prediction performance over multi-steps, i.e. over a window from N_1 to N_2, where N_1 typically equals time delay d. In this way, we consider an optimum that is not based on a single prediction point but based on multiple prediction points.

For the multi-step optimal prediction problem, the minimization of the following multi-step prediction error is of interest (Shook et al., 1992; Huang et al., 2003):

$$s_{N_1,N_2} = \frac{1}{N_p} \sum_{j=N_1}^{N_2} E[Y_{t+j} - Y_{t+j|t}]^T [Y_{t+j} - Y_{t+j|t}] \tag{8}$$

where $Y_{t+j|t}$ is an optimal j-step ahead prediction, N_1 and N_2 are the minimum and maximum prediction step, $N_p = N_2 - N_1 + 1$, and s_{N_1,N_2} is defined as the scalar measure of the optimal multi-step prediction error (from N_1 to N_2). MPC attempts to minimize the error of multi-step predictions, i.e. from the first N_1 step to the N_2 step prediction. Thus the objective function (8) is MPC relevant.

It has been shown in Huang et al. (2003) that the objective function of multi-step prediction error is equivalent to the variance of filtered one-step prediction error:

$$\begin{aligned} s_{N_1,N_2} &= \frac{1}{N_2 - N_1 + 1} \sum_{n=N_1}^{N_2} E||[Y_{t+n} - Y_{t+n|t}]||^2 \\ &= E||[F_{N_1,N_2}(z^{-1})(Y_t - Y_{t|t-1})]||^2 \end{aligned} \tag{9}$$

where the filter $F_{N_1,N_2}(z^{-1})$ is the spectral factor of the following spectrum (Huang et al., 2003):

$$L_{N_1,N_2} = \frac{1}{N_2 - N_1 + 1} \sum_{n=N_1}^{N_2} ||F_n(e^{-nj\omega})||^2 \tag{10}$$

where

$$F_n(z^{-1}) = \sum_{i=0}^{n-1} F_n z^{-i}$$

If $N_1 = 1$ and $N_2 = k$, it is easy to show that $F_{1,k}(z^{-1})$ has the following form:

$$F_{1,k}(z^{-1}) = \tilde{F}_0 + \tilde{F}_1 z^{-1} + \cdots + \tilde{F}_{k-1} z^{-k+1} \tag{11}$$

where $\tilde{F}_i$ is to be determined next.

According to Eqn. 4, the optimal one step prediction error $Y_t - Y_{t|t-1} = a_t$, i.e. white noise. Thus

$$\begin{aligned} s_{1,k} = & \quad E[F_{1,k}(z^{-1})a_t]^T[F_{1,k}(z^{-1})a_t] \\ &= tr\{[(\tilde{F}_0 + \tilde{F}_1 z^{-1} + \cdots + \tilde{F}_{k-1}z^{-k+1})a_t]^T \\ &\quad [(\tilde{F}_0 + \tilde{F}_1 z^{-1} + \cdots + \tilde{F}_{k-1}z^{-k+1})a_t]\} \end{aligned}$$

which can be further written as

$$s_{1,k} = tr(\tilde{F}_0\Sigma_a\tilde{F}_0^T + \tilde{F}_1\Sigma_a\tilde{F}_1^T + \cdots + \tilde{F}_{k-1}\Sigma_a\tilde{F}_{k-1}^T) \quad (12)$$

In the next two sections we will derive univariate and multivariate expressions of the optimal multi-step prediction error, respectively.

3.2 The Univariate Process

For the univariate process, the terms F_i and $\tilde{F}_i$ are both scalars (hence we use f_i and $\tilde{f}_i$ to stand for the scalar values), so the scalar prediction error measures can be simplified to the following forms:

$$s_k = (f_0^2 + f_1^2 + \cdots + f_{k-1}^2)\sigma_a^2 \quad (13)$$

$$s_{1,k} = (\tilde{f}_0^2 + \tilde{f}_1^2 + \cdots + \tilde{f}_{m-1}^2)\sigma_a^2 \quad (14)$$

When $k = 1$, by definition, $s_{1,1}$ is the variance of one step prediction error; thus

$$s_1 = s_{1,1}$$

When $k = 2$, the following result could be obtained:

$$s_{1,2} = (\tilde{f}_0^2 + \tilde{f}_1^2)\sigma_a^2 = \frac{1}{2}(2f_0^2 + f_1^2)\sigma_a^2 = \frac{1}{2}(s_1 + s_2)$$

Similarly, when $N_1 = 1, N_2 = k$, we have

$$\begin{aligned} s_{1,k} &= \frac{\sigma_a^2}{k}\{kf_0^2 + (k-1)f_1^2 + \cdots + f_{k-1}^2\} \\ &= \frac{1}{k}\sum_{i=1}^{k} s_i \end{aligned} \quad (15)$$

Thus

$$\begin{aligned} s_{k,m} = & \frac{1}{m-k+1}[(m-k+1)(f_0^2 + f_1^2 + \cdots + f_{k-1}^2) \\ & + (m-k)f_k^2 + \cdots + f_{m-1}^2]\sigma_a^2 \end{aligned}$$

Proposition 1. For a univariate control loop, the measure of optimal multi-step prediction error from k to m ($s_{k,m}$) is no smaller than that of the optimal k-step prediction error (s_k), and the two measures are asymptotically equal, namely

$$s_{k,m} - s_k \geq 0 \quad (16)$$

$$\lim_{k\to\infty}\{s_{k,m} - s_k\} = 0 \quad (17)$$

Proof.

Recall that the measures of the optimal k-step prediction error and optimal multi-step prediction error are respectively:

$$s_k = (f_0^2 + f_1^2 + \cdots + f_{k-1}^2)\sigma_a^2 \quad (18)$$

and

$$\begin{aligned} s_{k,m} = & \frac{1}{m-k+1}[(m-k+1)(f_0^2 + f_1^2 + \cdots + f_{k-1}^2) \\ & + (m-k)f_k^2 + \cdots + f_{m-1}^2]\sigma_a^2 \end{aligned} \quad (19)$$

Thus

$$\begin{aligned} s_{k,m} - s_k = & \frac{1}{m-k+1}[(m-k+1)(f_0^2 + f_1^2 + \cdots + f_{k-1}^2) \\ & + (m-k)f_k^2 + \cdots + f_{m-1}^2]\sigma_a^2 - (f_0^2 + f_1^2 + \cdots + f_{k-1}^2)\sigma_a^2 \\ \triangleq & \frac{1}{N_p}[(N_p-1)f_k^2 + (N_p-2)f_{k+1}^2 + \cdots + f_{m-1}^2]\sigma_a^2 \quad (20) \\ \geq & 0 \end{aligned}$$

where $N_p = m - k + 1$.

Consider a stable closed-loop response:

$$\begin{aligned} y_t = G_{cl}(z^{-1};\theta)e_t &= \frac{B(z^{-1})}{A(z^{-1})}e_t \\ &= \frac{b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}}{a_0 + a_1 z^{-1} + \cdots + a_n z^{-n}}e_t \end{aligned} \quad (21)$$

Write the above transfer function in the zero-pole form

$$y_t = \frac{b_0(1-\beta_1 z^{-1})(1-\beta_2 z^{-1})\cdots(1-\beta_m z^{-1})}{(1-\alpha_1 z^{-1})(1-\alpha_2 z^{-1})\cdots(1-\alpha_n z^{-1})}e_t \quad (22)$$

where $|\alpha_i| < 1$.

Partial fraction expansion of Eqn. (22) yields

$$\begin{aligned} y_t &= (\frac{c_1}{1-\alpha_1 z^{-1}} + \frac{c_2}{1-\alpha_2 z^{-1}} + \cdots + \frac{c_n}{1-\alpha_n z^{-1}})e_t \\ &= \sum_{p=1}^{n} c_p(1 + \alpha_p z^{-1} + \alpha_p^2 z^{-1} + \cdots)e_t \\ &\triangleq (f_0 + f_1 z^{-1} + \cdots + f_i z^{-i}\cdots)e_t \end{aligned} \quad (23)$$

where

$$f_i = \sum_{p=1}^{n} c_p \alpha_p^i \quad (24)$$

So, the *ith* term of Eqn. (18) can be calculated as

$$\begin{aligned} f_i^2 &= (\sum_{p=1}^{n} c_p\alpha_p^i)^2 \\ &= \sum_{p=1}^{n} c_p^2\alpha_p^{2i} + 2\sum_{p=1}^{n-1}\sum_{q=p+1}^{n} c_p c_q(\alpha_p\alpha_q)^i \end{aligned} \quad (25)$$

According to Eqn. (20)

$$s_{k,m} - s_k = \frac{1}{N_p}\sum_{i=k}^{m-1}(m-i)f_i^2\sigma_a^2 \quad (26)$$

Substituting Eqn. (25) in Eqn. (26), we obtain that

$$\begin{aligned} & s_{k,m} - s_k \\ =& \frac{\sigma_a^2}{N_p}\sum_{i=k}^{m-1}(m-i)(\sum_{p=1}^{n} c_p^2\alpha_p^{2i} + 2\sum_{p=1}^{n-1}\sum_{q=p+1}^{n} c_p c_q(\alpha_p\alpha_q)^i) \\ =& \frac{\sigma_a^2}{N_p}(m\sum_{p=1}^{n} c_p^2\sum_{i=k}^{m-1}\alpha_p^{2i} - \sum_{p=1}^{n} c_p^2\sum_{i=k}^{m-1} i\alpha_p^{2i} \\ & + \sum_{p=1}^{n-1}\sum_{q=p+1}^{n} 2mc_p c_q\sum_{i=k}^{m-1}(\alpha_p\alpha_q)^i \\ & - \sum_{p=1}^{n-1}\sum_{q=p+1}^{n} 2c_p c_q\sum_{i=k}^{m-1} i(\alpha_p\alpha_q)^i) \end{aligned} \quad (27)$$

where the terms $\sum_{i=k}^{m-1}\alpha_p^{2i}$, $\sum_{i=k}^{m-1} i\alpha_p^{2i}$, $\sum_{i=k}^{m-1}(\alpha_p\alpha_q)^i$ and $\sum_{i=k}^{m-1} i(\alpha_p\alpha_q)^i$ can be determined respectively as

$$\sum_{i=k}^{m-1} \alpha_p^{2i} = \frac{\alpha_p^{2k}(1-\alpha_p^{2(m-k)})}{1-\alpha_p^2} \tag{28}$$

$$\sum_{i=k}^{m-1} i\alpha_p^{2i} = \frac{\alpha_p^{2k}(1-\alpha_p^{2(m-k)})}{(1-\alpha_p^2)^2} + \frac{(k-1)\alpha_p^{2k}-(m-1)\alpha_p^{2m}}{(1-\alpha_p^2)} \tag{29}$$

$$\sum_{i=k}^{m-1} (\alpha_p\alpha_q)^i = \frac{(\alpha_p\alpha_q)^k(1-(\alpha_p\alpha_q)^{m-k})}{1-\alpha_p\alpha_q} \tag{30}$$

$$\sum_{i=k}^{m-1} i(\alpha_p\alpha_q)^i = \frac{(\alpha_p\alpha_q)^k(1-(\alpha_p\alpha_q)^{m-k})}{(1-\alpha_p\alpha_q)^2} + \frac{(k-1)(\alpha_p\alpha_q)^k-(m-1)(\alpha_p\alpha_q)^m}{(1-\alpha_p\alpha_q)} \tag{31}$$

Substituting the above four equations in Eqn. (27) yields

$$\begin{aligned} &s_{k,m} - s_k \\ =&\frac{\sigma_a^2}{N_p}(\sum_{p=1}^{n} c_p^2(-\frac{\alpha_p^{2k}(1-\alpha_p^{2(m-k)})}{(1-\alpha_p^2)^2} + \frac{N_k\alpha_p^{2k}-\alpha_m^2}{1-\alpha_p^2}) \\ &+\sum_{p=1}^{n-1}\sum_{q=p+1}^{n} 2c_pc_q(-\frac{(\alpha_p\alpha_q)^k(1-(\alpha_p\alpha_q)^{m-k})}{(1-\alpha_p\alpha_q)^2} \\ &+\frac{N_k(\alpha_p\alpha_q)^k-(\alpha_p\alpha_q)^m}{1-\alpha_p\alpha_q})) \\ \triangleq&\sum_{p=1}^{n} c_p^2 \times Sum1 + \sum_{p=1}^{n-1}\sum_{q=p+1}^{n}(2c_pc_q)\times Sum2 \end{aligned} \tag{32}$$

where

$$Sum1 = \frac{\sigma_a^2}{N_p}[-\frac{\alpha_p^{2k}(1-\alpha_p^{2(m-k)})}{(1-\alpha_p^2)^2} + \frac{N_p\alpha_p^{2k}-\alpha_m^2}{1-\alpha_p^2}]$$

$$Sum2 = \frac{\sigma_a^2}{N_p}[-\frac{(\alpha_p\alpha_q)^k(1-(\alpha_p\alpha_q)^{m-k})}{(1-\alpha_p\alpha_q)^2} + \frac{N_p(\alpha_p\alpha_q)^k-(\alpha_p\alpha_q)^m}{1-\alpha_p\alpha_q}]$$

When $k \to \infty$, $m \to \infty$ since $m \geq k$. Let $m-k = P \geq 0$. Consequently, $N_p = m-k+1 = P+1$. The limits of $Sum1$ and $Sum2$ can be obtained:

$$\begin{aligned}\lim_{k\to\infty} Sum1 =& \lim_{k\to\infty}[-\frac{\alpha_p^{2k}(1-\alpha_p^{2(m-k)})}{N_p(1-\alpha_p^2)^2} + \frac{N_p\alpha_p^{2k}-\alpha_p^{2m}}{N_p(1-\alpha_p^2)}]\sigma_a^2 \\ =&0\end{aligned} \tag{33}$$

As $|\alpha_i| < 1$, obviously $|\alpha_p\alpha_q| < 1$. Similarly,

$$\begin{aligned}&\lim_{k\to\infty} Sum2 \\ =&\lim_{k\to\infty}(-\frac{(\alpha_p\alpha_q)^k(1-(\alpha_p\alpha_q)^{m-k})}{N_p(1-\alpha_p\alpha_q)^2} + \frac{N_p(\alpha_p\alpha_q)^k-(\alpha_p\alpha_q)^m}{N_p(1-\alpha_p\alpha_q)})\sigma_a^2 \\ =&0\end{aligned} \tag{34}$$

Consequently,

$$\begin{aligned}&\lim_{k\to\infty}\{s_{k,m} - s_k\} \\ =&\sum_{p=1}^{n} c_p^2 \times Sum1 + \sum_{p=1}^{n-1}\sum_{q=p+1}^{n}(2c_pc_q)\times Sum2 \\ =&0\end{aligned} \tag{35}$$

3.3 The multivariate process

Following a similar procedure as that for the univariate process, the following measure of optimal multi-step prediction error can be derived:

$$\begin{aligned} s_{k,\,m} &= tr(\tilde{F}_0\Sigma_a\tilde{F}_0^T + \tilde{F}_1\Sigma_a\tilde{F}_1^T + \cdots + \tilde{F}_{m-1}\Sigma_a\tilde{F}_{m-1}^T) \\ &= \sum_{p=0}^{k-1}\sum_{j=1}^{N}\sum_{i=1}^{N} f_{ijp}^2\sigma_j^2 + \sum_{p=k}^{m-1}\frac{m-p}{m-k+1}\sum_{j=1}^{N}\sum_{i=1}^{N} f_{ijp}^2\sigma_j^2 \end{aligned} \tag{36}$$

A same proposition as in the univariate case can be proved, but is omitted in this shorter version due to space limit.

4. CLOSED-LOOP POTENTIALS BASED ON THE MULTI-STEP PREDICTION

Based on the multi-step prediction error derived in the above section, the following closed-loop potential measure is defined for performance assessment of MPC:

$$p_{k,\,m} = \frac{s_{\infty,\,\infty+N_p} - s_{k,\,m}}{s_{\infty,\,\infty+N_p}} \tag{37}$$

where $N_p = P+1$ and P represents the prediction horizon. It can be shown that $s_{\infty,\,\infty+N_p} = s_\infty = trace\{cov(Y_t)\}$.

Here we use the multi-step prediction error scalar measure $s_{k,\,m}$ instead of the k-step prediction error scalar measure s_k of Huang et al. (2006) to derive closed-loop potential. As has been proven in the last section, with a fixed prediction horizon P, i.e. $m-k = P = N_p - 1$, the scalar measure $s_{k,\,m}$ is monotonically increasing with k. So $p_{k,\,m}$ is monotonically decreasing. When $k = 0$, $s_{k,\,m} \geq= s_0 = 0$, so $p_{0,\,N_p} \leq 1$. Besides, as $s_{\infty,\,\infty+N_p} \geq s_{k,\,m}$, consequently, $0 \leq p_{k,\,m} \leq 1$. According to its definition we can see that the $p_{k,\,m}$ is dimensionless and in addition, the potential measure is more relevant to the MPC control strategy as it is multi-step prediction based. The actual process time delay (that corresponds to k) may not be known in practice. So the trajectory of the potential measure with a range of k will be more useful to assess the performance of the controller.

It is also desirable to know details about the performance of each output. So, the individual scalar potential measure is required. By replacing the operator $tr(\cdot)$ in Eqn. (36) with $diag(\cdot)$, the individual scalar measure is defined as:

$$s_{k,\,m}^{ind} = diag(\tilde{F}_0\Sigma_a\tilde{F}_0^T + \tilde{F}_1\Sigma_a\tilde{F}_1^T + \cdots + \tilde{F}_{m-1}\Sigma_a\tilde{F}_{m-1}^T)$$

where $s_{k,\,m}^{ind} \in \mathbb{R}^{N\times 1}$ and N represents the number of controlled variables.

The ith component of $s_{k,\,m}^{ind}$ can be obtained:

$$s_{k,\,m}^{ind}(i) = \sum_{p=0}^{k-1}\sum_{j=1}^{N} f_{jip}^2\sigma_i^2 + \sum_{p=k}^{m-1}\frac{m-p}{m-k+1}\sum_{j=1}^{N} f_{jip}^2\sigma_i^2 \tag{38}$$

As a result, the individual potential measure can be defined as:

$$p_{k,\,m}^{ind}(i) = \frac{s_{\infty,\,\infty+N_p}^{ind}(i) - s_{k,\,m}^{ind}(i)}{s_{\infty,\,\infty+N_p}^{ind}(i)} \tag{39}$$

where $s_{\infty,\,\infty+N_p}^{ind} = diag(s_{\infty,\,\infty+N_p}) = diag(cov(Y_t))$ and $1 \leq i \leq N$.

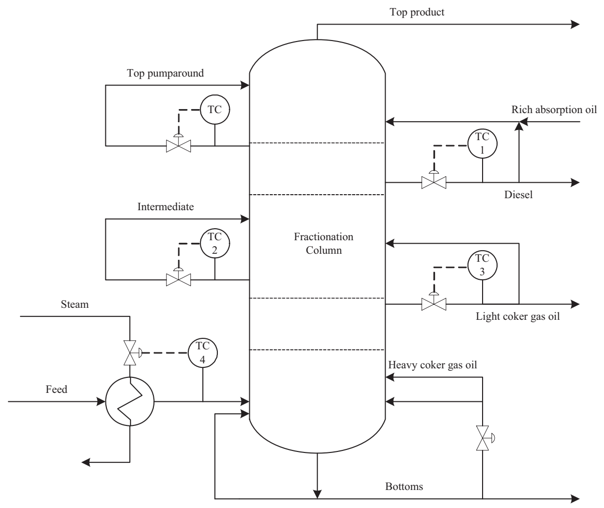

Fig. 1. Schematic diagram of the fractionation column in the delayed coking unit.

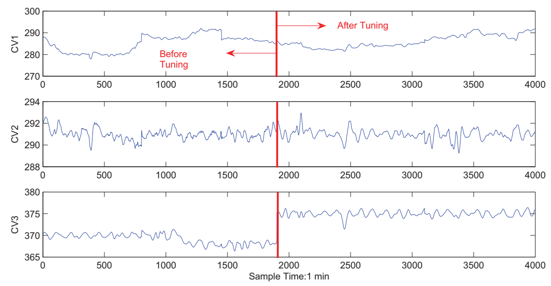

Fig. 2. Output data set under MPC controller.

5. INDUSTRIAL APPLICATION

In this section the proposed multi-step closed-loop potential measures will be applied to evaluate the performance of an industrial control system.

5.1 Process description

This is a control performance assessment and diagnosis problem for a MPC control system in a delayed coking refinery unit. Fig. 1 is a simplified process flow chart.

The control system consists of three manipulated variables (MVs), three controlled variables (CVs) and one disturbance variable (DV), the temperature of the feedstocks. A description of process variables and their corresponding tag names and the parameters for the MPC design is shown in Table 1.

Two different closed-loop operation data sets are collected with 1 min sampling interval under different MPC controller tunings as shown in Fig. 2. All the data is selected without the drum events to avoid unusual upset. The first part of the data from 1 to 1900 is selected before the controller tuning and the rest part is selected after the controller tuning.

5.2 Performance assessment

By using the proposed approach, the scalar potential measure trajectories for each data set are generated and

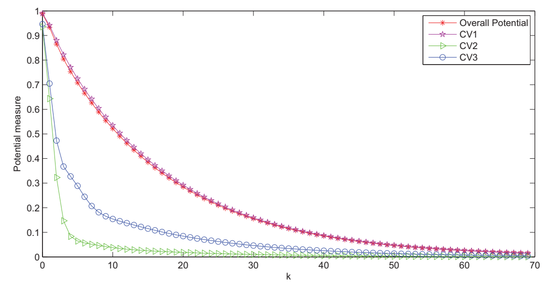

Fig. 3. Scalar potential measures of the system before controller tuning.

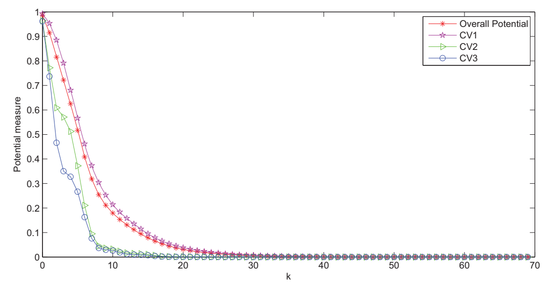

Fig. 4. Scalar potential measures of the system after controller tuning.

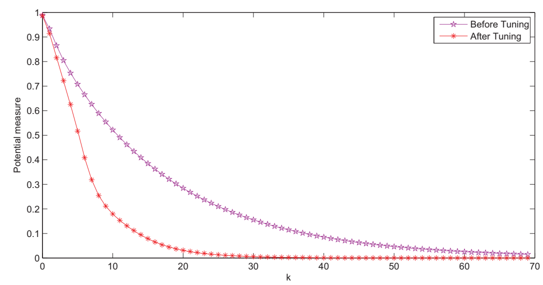

Fig. 5. Overall scalar potential measures of the system under different controller tunings.

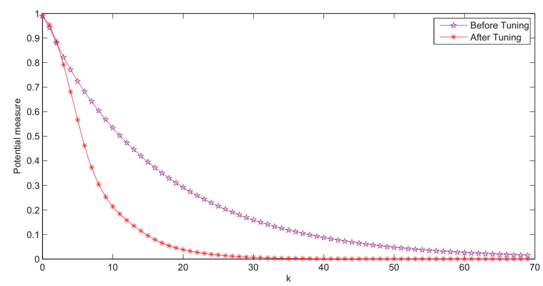

Fig. 6. Individual scalar potential measures of the system under different controller tunings of CV1.

shown in Fig. 3 and 4, respectively. The comparisons of the overall potential and individual potential for each CV are displayed from Fig. 5 to Fig. 8.

With these figures, the following performance analysis conclusions can be obtained:

Table 1. List of process variables and their corresponding tag names and parameters for MPC design.

No.	Tag	Weight	Horizon	Operation Range
CV1	Temperature of diesel	10	20	275-295
CV2	Temperature in the intermediate	1	20	285-295
CV3	Temperature of light coker gas oil	1	20	360-380
MV1	Valve opening of diesel	0.1	3	0-100
MV2	Valve opening of intermediate reflux	0.1	3	0-100
MV3	Valve opening of light coker gas oil reflux	0.1	3	0-100

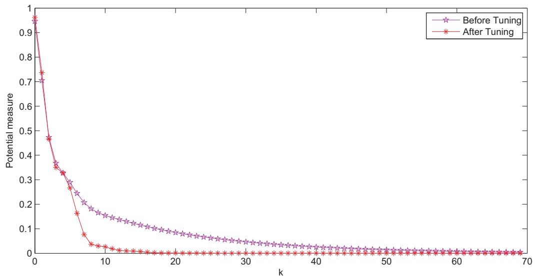

Fig. 7. Individual scalar potential measures of the system under different controller tunings of CV2.

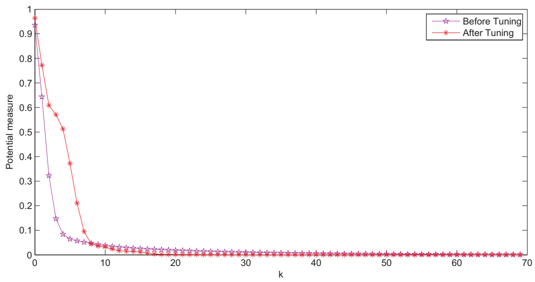

Fig. 8. Individual scalar potential measures of the system under different controller tunings of CV3.

- According to Fig 3, before the controller tuning, the overall scalar potential measure trajectory converges slowly and CV1 contributes the more potential to the overall potential than the other two CVs.
- According to Fig 4, after the controller tuning, the overall scalar potential measure trajectory converges fast and the three individual trajectories come close to each other, although the trajectory of CV1 still lies above the other two.
- According to Fig 5, there is significant improvement of the system's performance after the tuning as there is less potential after the controller tuning than that before tuning.
- According to Fig 6, 7 and 8, performance of both CV1 and CV2 is improved after the controller tuning; however, performance of CV3 is degraded after the tuning.
- The above results indicate that the improvement of the overall performance comes from the improvement of CV1 and CV2 but at some cost of CV3. As CV1 is the most important quality variable (the weight of CV1 is larger than those of the other two in Table 1.), it is worth to improve its performance by slightly deteriorating the performance of CV3.

In summary, after the controller tuning, there is significant improvement of the system's performance.

6. CONCLUSION

The closed-loop potentials are promising measures of model predictive control performance. However, they have certain limitations as they are originally defined. In this paper, new closed-loop potentials are proposed. The proposed performance potentials are multi-step prediction based and thus MPC relevant. Regardless of the dimension of the plant, the closed-loop potentials can be easily calculated, which facilitates the implementation, visualization, and interpretation. Industrial application demonstrates powerfulness of the the proposed performance measures.

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