Comparison of discrete and continuous-discrete observers for composition estimation in distillation columns

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Abstract: In this paper we present a high-gain observer implemented in its discrete and continuous-discrete versions in order to compare the performance of both algorithms. The comparison is made considering the sampling time used to perform the observer's correction stage in order to establish that the continuous-discrete observer is the best option when a low sampling time is used. Under this condition the continuous discrete observer can process data performing a reliable on-line estimation of the system (a slow dynamics of the process is required). We apply both algorithms to a distillation column that uses the Ethanol-Water binary mixture.

Keywords: High-gain observer, discrete, continuous-discrete, distillation columns.

1. INTRODUCTION

The knowledge of state variables is often required in order to apply the advanced concepts of control and fault diagnosis to practical applications, specially in the chemical process industry. A method to obtain such variables, consists of combining a priori knowledge about physical systems with experimental data to provide an on-line estimator (observer).

The main control problems of distillation columns are caused due to tight interactions between the process variables, nonlinearities of the process, process and measurement delays and the large number of variables involved (see Murray-Gunther (2003)). For these reasons, a significant amount of effort has been devoted to develop algorithms that provide accurate parameter identification and state estimation (state observers) to reconstruct the product composition dynamics by secondary measurements (e.g., temperatures and flows) (see Quintero-Mármol et al. (1991), Deza et al. (1991)), and most recently in Bahar et al. (2006)Jana et al. (2006). The proposed observers are used to estimate unmeasured state variables from on-line and/or off-line measurements, see e.g. Bakir et al. (2005), Hammouri et al. (2006), Yildiz et al. (2005), Astorga et al. (2002) and Nadri et al. (2004).

Estimators are generally dynamic systems obtained from a nominal model by adding a correction term which is proportional to some output deviation. In other words, given a nominal model:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{y}(\mathbf{t}) = \mathbf{h}(x(t)) \end{cases} \quad (1)$$

The state $x(t)$ belongs to an open subset $\mathbf{V}$ of $\mathbb{R}^n$, the input u(t) belongs to a Borelian subset $\mathbf{U}$ of $\mathbb{R}^m$ and the output $y(t) \in \mathbb{R}^p$. An observer for the system represented by Eq. (1) is generally a dynamic system of the form:

$$\begin{cases} \dot{\hat{x}}(t) = \mathbf{f}(\hat{\mathbf{x}}(t), \mathbf{u}(t)) + \mathbf{k}(\mathbf{t})[h(\hat{\mathbf{x}}(\mathbf{t})) - \mathbf{y}(\mathbf{t})] \\ \dot{\mathbf{r}} = \mathbf{F}(\mathbf{r}(\mathbf{t}), \mathbf{u}(\mathbf{t}), \mathbf{y}(\mathbf{t}), \hat{\mathbf{x}}(\mathbf{t})) \\ \mathbf{k}(\mathbf{t}) = \varphi(\mathbf{r}(\mathbf{t})) \end{cases} \quad (2)$$

$r(t)$ and $k(t)$ are called indifferently the gain of the observer (see Hammouri et al. (2002)). An interesting class of nonlinear systems consists of those systems which are observable for every input, called *uniformly observable systems*. For this class of nonlinear systems, we can design an observer whose gain does not depend on the inputs (see Bornard and Hammouri (1991) and Gauthier et al. (1992)). For such systems a canonical (called triangular) form is designed in order to develop an observer. To ensure mathematical convergence, a particular high-gain is required (see Hammouri et al. (2002)).

However, using a high-gain observer may generate the so-called peak phenomena (overshoot problem); moreover, the estimator becomes noise sensitive. Due to nonlinearity of the system, the choice of the gain which gives the best compromise between fast convergence, the noise rejection and the attenuation of the peak phenomena becomes a difficult task, and only simulations allow to determine a possible gain. This paper aims to present a high-gain observer status in its discrete and continuous-discrete versions. We apply this algorithms to a binary distillation column that uses the binary mixture Ethanol-Water.

2. THE BINARY DISTILLATION COLUMN MODEL

The binary distillation column model is derived from the binary distillation column scheme shown in Fig. 1. There are three principal stages considered for a distillation column (condenser, tray and boiler); for every stage, the balances of energy and material should be formulated as well as the equilibrium conditions of the mixture.

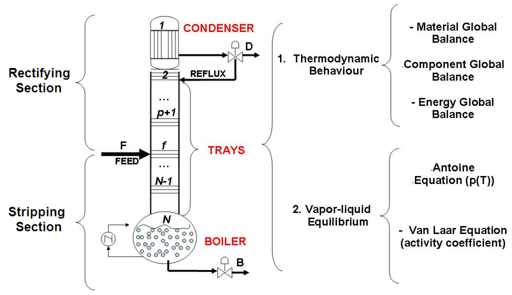

Fig. 1. Distillation column

2.1 General Aspects

The scheme of a typical distillation column is shown in Fig. 1. The binary feeding mixture (molar flow rate F) is introduced toward the middle of the column (the feeding tray). The distilled product (molar flow rate D), which mainly contains the light component, is removed from the top of the column. The bottom product (molar flow rate B), which contains the heavy component in greater concentration, is removed from the bottom of the column. Part of the overhead product is returned into the column to improve purity. Column stages are labeled with an ascendent numeration from the condenser to the boiler: $i = 1, ..., N$. The sections of the column are: the condenser, tray $i = 1$; the rectifying section, trays $i = 2, ..., f - 1$; the feeding tray, tray $i = f$; the stripping section, trays $i = f + 1, ..., N - 1$; the boiler, tray $i = N$.

2.2 Physic Behaviour

Due to its physical structure, a distillation column can be modeled as a set of interconnected stages using the mass balance and vapor-liquid equilibrium relation at every stage. The algebraic and differential equations of the model are formulated to calculate the light component composition of the mixture. The liquid phase and the vapor phase of the light component are designated by x and y, respectively.

Assumptions: The following assumptions, taken from Luyben (1992) and Halvorsen and Skogestad (2000) are considered in the model formulation:(**A1**) Constant pressure; (**A2**) Ideal Liquid-Vapor Equilibrium; (**A3**) Liquid-properties behave as a non-ideal mixture; (**A4**) Negligible molar vapor holdup compared to the molar liquid holdup; (**A5**) Boiler as a theoretical tray; (**A6**) Total condenser; (**A7**) Constant liquid volumetric hold up.

Vapor-liquid equilibrium: If a vapor and a liquid are in intimate contact for a long period of time, equilibrium is attained between the two phases. This concept of vapor-liquid equilibrium is fundamental to model distillation columns. If the vapor-liquid equilibrium exists, then the vapor composition y_i and the liquid composition x_i can be computed by means of correlating equations of the form $(y_i^{eq}, x_i^{eq}) = K_i(T_i, P_T)$, where T_i is the temperature, P_T is the total pressure, y_i^{eq} and x_i^{eq} are the vapor and liquid composition at the equilibrium phase respectively. The equilibrium constant K_i depends on the thermodynamical properties of the mixture.

The non-ideality of a binary mixture is due to different causes, the most frequent is the non-ideality of the liquid phase. In consequence, specially designed models are used to represent these non-idealities. For low pressure systems, the equation that represents the vapor composition is:

$$y_{i,j} P_T = P_{i,j}^{sat} x_{i,j} \gamma_{i,j} \tag{3}$$

where $j = 1$ if the component is ethanol and $j = 2$ if the component is water; $\gamma_{i,j}$ is the activity coefficient for every stage, it is a correction factor highly dependent on the concentration. In this work the vapor composition is calculated as a function of the light component. One method to determine this coefficient in every component of the mixture uses the Van Laar equation (see Perry (1999)).

Mass transfer effects: In order to deal with the mass transfer effects, the Murphree's efficiencies are introduced. The Murphree stage efficiency E_i is the ratio between the current change in vapor composition between two stages and the change that will occur if the vapor is in equilibrium with the liquid leaving the stage (see Murray-Gunther (2003)).

The molar flow rates: In the rectifying section the vapor molar flow V_R and the liquid molar flow L_R are :

$$\begin{cases} V_R = V_S + (1 - q_F)F, & i = 1, ..., f \\ L_R = (1 - R)V_R, & i = 1, ..., f - 1 \end{cases} \tag{4}$$

where

$$q_F = 1 + \frac{C_p(T_b - T_F)}{\lambda} \tag{5}$$

q_F describes the feeding condition. C_{p_j} is the specific heat, T_b is the boiling temperature, T_F is the feeding temperature and λ is the vaporization enthalpy for ethanol and water respectively.

F is the molar flow of the feeding stream:

$$F = F_V [\rho_1 w_1 + \rho_2(1 - w_1)] \left(\frac{x_f}{M_{W_1}} + \frac{1 - x_f}{M_{W_2}} \right) \tag{6}$$

where F_V is the volumetric flow of the feeding stream, ρ_j is the density of the component j, M_{W_j} is the molecular weight, w_1 is the weight fraction of the light component given by:

$$w_1 = \frac{x_f \rho_1}{x_f \rho_1 + (1 - x_f)\rho_2} \tag{7}$$

and x_f is the molar composition of the feeding stream given by:

$$x_f = \left(\frac{V_1\rho_1/M_{W_1}}{V_1\rho_1/M_{W_1} + V_2\rho_2/M_{W_2}}\right) \quad (8)$$

where V_j is the initial volume of the component j on the feeding container.

The distilled product flow rate, $D = 0$ if the three-way ON-OFF reflux valve shown in 1) is totally closed ($r_v = 0$) and $D = V_R$ if this valve is totally open ($r_v = 1$)

In the stripping section (subindex $[\cdot]_S$ is used), the vapor molar flow V_S and the liquid molar flow L_S are, respectively

$$\begin{aligned} V_S &= \frac{Q_b}{\Delta H_1^{vap} x_{1,N} + \Delta H_2^{vap}(1 - x_{1,N}),} & i = f+1, ..., N \\ L_S &= L_R + q_F F, & i = f, ..., N \end{aligned} \quad (9)$$

where Q_b is the heating power on the boiler. Finally, the molar flow rate of the bottom product is:

$$B = (L_S - V_S)b_v$$

b_v is a binary variable representing the bottom-valve opening, *i.e.*: $b_v = 0$ if a batch distillation is performed and $b_v = 1$ if the bottom product is totally withdrawn from the boiler.

Molar hold-up: The molar hold-up for every stage must be determined from the distillation plant features and properties of the mixture. This quantity can be approximated as:

$$M_i = v_i \frac{1}{\dfrac{x_1 M_{W_1}}{\rho_1} + \dfrac{(1 - x_2)M_{W_2}}{\rho_2}} \quad (10)$$

2.3 The dynamic model

Tray and distilled product compositions are estimated by using the dynamic model based on material, component and energy balances. Taking into account the assumptions (**A1**) to (**A7**), a set of differential equations can be derived for the light component material balance as follows:

$$\begin{cases} \dfrac{dM_1}{dt} = V_2 - L_1 - D \\ \dfrac{dM_i}{dt} = V_{i+1} - L_i - V_i + L_{i-1} + \delta(i)F \\ \dfrac{dM_N}{dt} = L_{N-1} - V_N - B \end{cases} \quad (11)$$

for $i = 2, 3, ..., N-1$. M_i is the molar holdup of the boiler. The component balances for every stage are given by:

$$\begin{cases} \dfrac{d(M_1 x_1)}{dt} = V_2 y_2 - L_1 x_1 - D x_1 \\ \dfrac{d(M_i x_i)}{dt} = V_{i+1} y_{i+1} - L_i x_i - V_i y_i + L_{i-1} x_{i-1} + \delta(i) F x_F \\ \dfrac{d(M_N x_N)}{dt} = L_{N-1} x_{N-1} - V_N y_N - B x_N \end{cases} \quad (12)$$

where $\delta(i) = 1$ if $i = f$ and $\delta(i) = 0$ if $i \neq f$

The enthalpies of the process are considered constants, therefore the energy balance is not taken into account to develop this model. The state model presented in the following section is based on this dynamic model for the distillation column.

2.4 The state model

The distillation column is a process that belongs to a class of multi-variable nonlinear systems. The process inputs are the heating power applied on the boiler, and the opening period of the reflux valve, this is $\mathbf{u}(t) = [Q_b(t)\, r_v(t)]^T$. F y x_F are considered perturbations in the system, this is $\mathbf{d} = [F, x_F, b_v]$. A state representation can be obtained from Eqs. (11) and (12). Additionally, the nonlinear model has the following triangular form:

$$\begin{cases} \dot{\zeta}_1 = f_1(\zeta_1, \zeta_2, \mathbf{u}) \\ \dot{\zeta}_i = f_i(\zeta_1, ..., \zeta_i, \zeta_{i+1}, \mathbf{u}); \quad (i = 2, ..., f-2) \\ \dot{\zeta}_{f-1} = f_{f-1}(\zeta_1, ..., \zeta_{f-1}, \zeta_f, \mathbf{u}) \\ \dot{\zeta}_f = f_f(\zeta_{f-1}, ..., \zeta_N, \mathbf{u}, \mathbf{d}) \\ \dot{\zeta}_i = f_i(\zeta_{i-1}, ..., \zeta_N, \mathbf{u}, \mathbf{d}); \quad (i = f+1, ..., N-1) \\ \dot{\zeta}_N = f_N(\zeta_{N-1}, \zeta_N, \mathbf{u}, \mathbf{d}) \end{cases} \quad (13)$$

where, ζ represent the states of the process (the liquid compositions of the light component). Subindex f represents the feeding tray number in Eqs. (11) and (12). The model allows to calculate the flows B, D, V_i, L_i and T_i, x_i from inputs Q_b, r_v, b_v, F_V, x_F.

3. OBSERVER DESIGN FOR A CLASS OF NONLINEAR TRIANGULAR SYSTEMS

A special class of nonlinear systems consists of those which are observable for every input, called uniformly observable systems (see Hammouri et al. (2002)). For this class of nonlinear systems, an observer whose gain does not depend on the inputs can be designed. For such systems a canonical (triangular) form is used in order to design an observer. Due to the nonlinearity of the system, it is important to select the gain which gives the best compromise between fast convergence and accuracy. Consider the follow triangular system that can be rewritten in a compact form:

$$\begin{cases} \dot{\zeta}^1 = \mathbf{f^1}(\boldsymbol{\zeta}(t), \mathbf{u}(t)) \\ \dot{\zeta}^2 = \mathbf{f}^2(\boldsymbol{\zeta}(t), \mathbf{u}(t), \mathbf{d}(t), \varepsilon(t)) \\ \boldsymbol{\varrho}(\boldsymbol{t}) = (\varrho_1(t), \varrho_2(t))^T = (\mathbf{C}_{n_1}\boldsymbol{\zeta}^{\mathbf{1}}(t), \mathbf{C}_{n_2}\boldsymbol{\zeta}^{\mathbf{2}}(t))^T \end{cases} \quad (14)$$

where the states $\boldsymbol{\zeta}(t) = \left[\boldsymbol{\zeta}^1(t), \boldsymbol{\zeta}^2(t)\right]^T \in \mathbb{R}^n$ and $n = n_1 + n_2$; $\zeta^j = \left[\zeta_1^j, \zeta_2^j, \ldots, \zeta_{n_j}^j\right]^T \in \mathbb{R}^{n_j}$ for $j = 1, 2$; $y_j = \mathbf{C}_{n_j}\zeta^j = \zeta_1^j$ the first component of ζ^j; $\mathbf{C}_{n_j} = [1, 0, \ldots, 0]$; the input $\mathbf{u} \in \mathbb{R}^m$, and $\varepsilon(t)$ is an unknown and bounded function. The following assumptions are considered in order to design the observer:

- (**A8**) $\mathbf{f}^j$ is globally Lipschitz *w.r.t.* ζ.
- (**A9**) The state variables $\boldsymbol{\zeta}(t)$ are bounded

Considering the following notations:

i) $\mathbf{C}_{n_j} = [1, \ldots, 0] \in \mathbb{R}^{n_j}$ where n_j is the size for every state vector ζ^j.
ii)

$$\mathbf{A}_{n_j}(t) = \begin{bmatrix} 0 & a_1(t) & 0 & 0 \\ \vdots & & a_2(t) & 0 \\ 0 & & \ddots & a_{n_j-1}(t) \\ 0 & \ldots & 0 & 0 \end{bmatrix},$$

where $a_k(t)$, $k = 1, \ldots, n_{j-1}$ are bounded and unknown functions satisfying the following assumption:

- (**A10**) There are two finite real numbers α, β with $\alpha > 0$, $\beta > 0$ such that $\alpha \leq a_k(t) \leq \beta$.

Lemma 1 *Under assumptions (**A8**) and (**A10**) exist a symmetric positive definite (S.P.D.) matrix* $\mathbf{S}_{n_j}$ *and a constant* $\mu > 0$ *s.t.:*

$$\forall t, \mathbf{S}_{n_j}\mathbf{A}_{n_j}(t) + \mathbf{A}_{n_j}^T(t)\mathbf{S}_{n_j} \leq -\mu\mathbf{I}_d \tag{15}$$

where $\mathbf{I}_d$ *is the identity matrix.*

Then,

$$\mathbf{S}_{n_j} = \begin{bmatrix} s_{11} & s_{12} & 0 & & 0 \\ s_{12} & s_{22} & \ddots & & \vdots \\ 0 & \ddots & & \ddots & 0 \\ \vdots & & & \ddots & s_{(n_j-1)n_j} \\ 0 & \ldots & 0 & s_{(n_j-1)n_j} & s_{n_j n_j} \end{bmatrix},$$

Assume that the system given in Eq. (14) satisfies hypothesis (**A8**) to (**A10**). Then the observer:

$$\begin{cases} \dot{\hat{\boldsymbol{\zeta}}}^1 = \mathbf{f}^1(\hat{\zeta}, \mathbf{u}) - r_1\boldsymbol{\Delta}_{\theta^{\delta_1}}\mathbf{S}_{n_1}^{-1}\mathbf{C}_{n_1}^T(\mathbf{C}_{n_1}\hat{\zeta}^1 - \varrho_1) \\ \dot{\hat{\boldsymbol{\zeta}}}^2 = \mathbf{f}^2(\hat{\zeta}, \mathbf{u}, \mathbf{d}) - r_2\boldsymbol{\Delta}_{\theta^{\delta_2}}\mathbf{S}_{n_2}^{-1}\mathbf{C}_{n_2}^T(\mathbf{C}_{n_2}\hat{\zeta}^2 - \varrho_2) \end{cases} \tag{16}$$

is an estimator for the system given in Eq. (14), where $r_1 > 0$, $r_2 > 0$; $\theta > 0$; $\boldsymbol{\Delta}_{\theta^{\delta_j}} = diag(\theta^{\delta_j}, \theta^{2\delta_j}, ..., \theta^{n_j\delta_j})$; $\delta_1 > 0$, $\delta_2 > 0$; $\mathbf{S}_{n_1}$ is given by **Lemma 1**.

The following theorem is given:

Theorem 1: *Denote by* ε *the upper bound of* $|\varepsilon(t)|$ *i.e.* $\varepsilon = \sup_{t\geq 0}|\varepsilon(t)|$, *then for* $r_1 > 0$, $r_2 > 0$, $\theta > 0$ *sufficiently large and* $\forall \delta_1 > 0$, $\delta_2 > 0$ *s.t.*

$$\frac{2n_1 - 1}{2n_2 + 1}\delta_1 < \delta_2 < \frac{2n_1 + 1}{2n_2 - 1}\delta_1; \tag{17}$$

$$|\hat{\zeta}(t) - \zeta(t)\| \leq \lambda e^{-\mu t} + \lambda'\varepsilon; \tag{18}$$

for some constants $\lambda > 0$, $\mu > 0$ *and* $\lambda' > 0$. *Moreover,* $\mu \to +\infty$ *as* $\theta \to 0$.

Remark 1. *If* $\varepsilon = 0$, *the system given in Eq. (16) becomes an exponential observer for the system given in Eq. (14). A proof of this result is given in Hammouri et al. (2002).*

3.1 Application of the designed observer to a distillation column

In the previous sections Eqs. (14) to (16) describe the high-gain observer designed for a distillation column, as well as the appropriate model in which this observed is based. In this section, an observer synthesis to the following class of nonlinear systems is developed, which contains the model of binary distillation columns, considering the following notations:

$$\begin{cases} \zeta_i = \zeta_i^1; \;\; 1 \leq i \leq f - 1 \\ \zeta_{N-i+1} = \zeta_i^2; \;\; 1 \leq i \leq N - f + 1 \\ \zeta_F = \zeta_{N-f+2}^2 \end{cases} \tag{19}$$

then, the system given in Eq. (13) can be represented in the following compact form:

$$\begin{cases} \dot{\zeta}^1(t) = \mathbf{f}^1(\boldsymbol{\zeta}(t), \mathbf{u}(t), D(t)) \\ \dot{\zeta}^2(t) = \mathbf{f}^2(\boldsymbol{\zeta}(t), \mathbf{u}(t), \mathbf{d}(t), \varepsilon(t), B(t)) \\ \boldsymbol{\varrho}(t) = (\zeta_1^1, \zeta_1^2)^T = (\zeta_1, \zeta_N)^T \end{cases} \tag{20}$$

where $\varepsilon(t)$ is a bounded and unknown function *s.t.* $\dot{\zeta}_F = \varepsilon(t)$. The liquid flow rates in the stripping section and vapor flow rates in the rectifying section are not known variables. The holdup in the condenser, the boiler and trays (where it is assumed to be constant) is calculated . The weighted error between the estimated and the measured compositions is feed back to the model equation in order to correct the liquid flow rates in the stripping section and the vapor flow rates in the rectifying section (see Halvorsen and Skogestad (2000)).

The hypothesis (**A9**) and (**A10**) must be verified:

- (**A9**) is fulfilled since the flow rates are physically bounded.
- (**A10**) is satisfied because the liquid compositions $x_i \in [0, 1]$

Applying **Theorem 1**, the high-gain observer for the distillation column is:

$$\begin{cases} \dot{\hat{\boldsymbol{\zeta}}}^1 = \mathbf{f}^1(\hat{\zeta}, \mathbf{u}, D(t)) - \mathbf{Q}_{1\theta}(\mathbf{C}_{n_1}\hat{\boldsymbol{\zeta}}^1 - \varrho_1) \\ \dot{\hat{\boldsymbol{\zeta}}}^2 = \mathbf{f}^2(\hat{\zeta}, \mathbf{u}, B(t), \mathbf{d}, \varepsilon(t)) - \mathbf{Q}_{2\theta}(\mathbf{C}_{n_2}\hat{\boldsymbol{\zeta}}^2 - \varrho_2) \end{cases} \tag{21}$$

where $\mathbf{Q}_{j\theta} = r_j\boldsymbol{\Delta}_{\theta^{\delta_j}}\mathbf{S}_{n_j}^{-1}\mathbf{C}_{n_j}^T$, for $j = 1, 2$. The constant parameters are the same described in Eq. (16).

4. EXTENSION OF THE HIGH-GAIN OBSERVER TO THE CONTINUO-DISCRETE CASE

There are processes where the measurement of their variables are performed using long sampling times due to their slow dynamics. In Bahar et al. (2006) it is demonstrated that certain restrictions in the dimension of the sampling period used by a purely discrete observer exist. An alternative of solution for this problem is the use of continuous-discrete observers (see Hammouri et al. (2002)).

Consider a non-linear uniformly observable systems of the form:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{y}(\mathbf{t}) = \mathbf{h}(x(t)) \end{cases} \tag{22}$$

where $x(t) \in \mathbb{R}^n$, $u = (u_1, ..., u_n) \in \mathbb{R}^m$, are measurable inputs and $y \in \mathbb{R}$ is a measurable output. Using the model of the system with $u(t), y(t)$ as known measurements it is possible to estimate the state line in $x(t)$ of the system represented by Ec. 22, this task is performed by a recursive algorithm with the following structure:

i. A prediction period in the time interval $t \in [t_k, t_{k+1}]$*:*

$$\dot{\hat{x}}_k(t) = \mathbf{f}(\hat{\mathbf{x}}(t), \mathbf{u}(t)) \tag{23}$$

ii. A correction period in the time $t = t_{k+1}$*:*

$$\dot{\hat{x}}_{k+1}(t) = \hat{\mathbf{x}}_{\mathbf{k+1}}(-) - r\Delta_\theta S_\theta^{-1}C^T(C\hat{x}_{k+1}(-) - y_{k+1}) \tag{24}$$

To make the extension of the high-gain observer to continuous-discrete case, it is assumed that the observations are made at the time $k\triangle t$, where $\triangle t$ is the time between measurements and k is the instant in which the sample is taken. In this case, it is not considered a coordinates change because the triangular structure of the model studied in section 3 is used.

As the observer gain is constant, it is true that:

$$\forall t > 0, \mathbf{A}_k^T(t)\mathbf{S}_k + \mathbf{S}_k\mathbf{A}_k(t) - \rho\mathbf{C}_k^T\mathbf{C}_k \leq\leq -\mu\mathbf{I}_k \tag{25}$$

where S_k is a symmetric positive definite matrix with the following structure:

$$\mathbf{S}_k = \begin{bmatrix} s_{11} & s_{12} & 0 & & 0 \\ s_{12} & s_{22} & \ddots & & \vdots \\ 0 & \ddots & & \ddots & 0 \\ \vdots & & & \ddots & s_{(k-1)k} \\ 0 & \dots & 0 & s_{(k-1)k} & s_{kk} \end{bmatrix},$$

C_k is denoted as a vector of k elements:

$$\mathbf{C}_{n_j} = [1, \dots, 0] \tag{26}$$

and A_k is given by:

$$\mathbf{A}_k(t) = \begin{bmatrix} 0 & a_1(t) & 0 & 0 \\ \vdots & & a_2(t) & 0 \\ 0 & & \ddots & a_{k-1}(t) \\ 0 & \dots & 0 & 0 \end{bmatrix},$$

where the terms a_k may be unknown and satisfy the hypothesis **A9**.

5. EXPERIMENTAL VALIDATION OF THE OBSERVER

The distillation pilot plant, located at the Process Control Laboratory of the National Center of Technological Research and Development (CENIDET) in Cuernavaca, Morelos, México, was used to carry out the required experiments. It has twelve trays, where temperature measurements are available through 8 RTD's Pt-100 located at trays 1, 2, 4, 6, 7, 9, 11 and 12. Using these measurements and considering the equilibrium relation (see Section 2.2.2), the respective liquid compositions can be obtained.

The mixture used in these experiments was Ethanol(EtOH)-Water(H_2O) which is considered as a non-ideal mixture. The experimental validation of the observers is done considering: EtOH volume of 2000 ml, H_2O volume of 2000 ml and process total pressure of 105.86 kPa. The specifications of every component of the mixture can be found in Perry (1999). The experiment lasts 82 minutes, once it has reached the stable state. In minute 27 the system goes from total reflux to partial reflux. In minute 54, a change in the input Q_b is applied.

In the discrete observer the sampling time is used to estimate and correct. In the continuous-discrete observer the prediction and correction times can be different in order to use less data, therefore less processing time, performing a reliable online estimation. The observer estimates the liquid composition of the light component (EtOH) for every stage by having the temperature measurements on stage 1 (condenser) and stage 12 (boiler) only.

The high-gain observer is obtained by fixing $r_1 = r_2 = 25$; $\theta = 0.09$ and satisfying (16) with $\delta_1 = 1.2$, $\delta_2 = \frac{1}{2}\left[1 + \left(\frac{2n_1+1}{2n_2-1}\right)^2\right]\delta_1 = 0.0983$(where $n_1 = f - 1 = 6, n_2 = n - f + 2 = 6$). Finally Lemma 1 gives:

$$\mathbf{S}_{n_1} = \mathbf{S}_{n_2} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1.5 & 0 & 0 & 0 \\ 0 & -1.5 & 4 & -2 & 0 & 0 \\ 0 & 0 & -2 & 8 & -3 & 0 \\ 0 & 0 & 0 & -3 & 10.5 & -4 \\ 0 & 0 & 0 & 0 & -4 & 15.5 \end{bmatrix}$$

Fig. 2 shows a comparison between the experimental dat and the estimation performed by the discrete observer in plate 12 (boiler) using a sampling time of $3s$. If the sampling time is slightly increased to $5.4s$ the discrete observer can not perform an adequate estimation, as can be seen in Fig. 3.

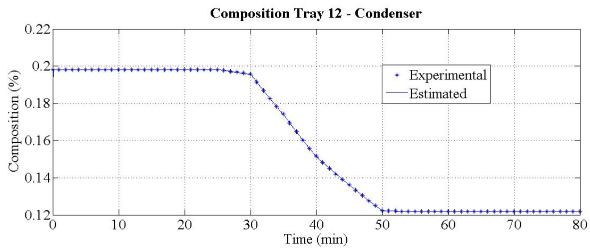

Fig. 2. Composition estimation of tray 12 by the discrete observer using a sampling time of 3 sec

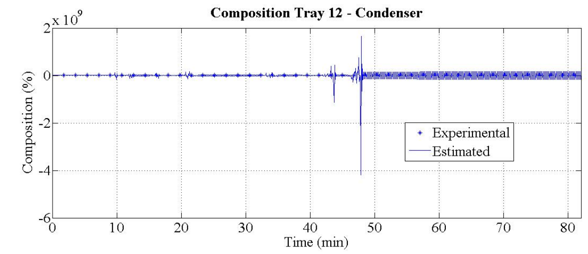

Fig. 3. Composition estimation of tray 12 by the discrete observer using a sampling time of 5.4 sec

In the continuous-discrete observer, a fixed prediction time of $3s$ is used, but different correction times are used in order to validate its performance. Figs. 4 to 6 show the experimental and estimated data when the correction time is 15 seconds, 30 seconds and 1 minute, respectively. In these figures it can be seen the good tracking and quickly convergence of the observer to the experimental data. The observer estimates the compositions of the plant adequately, under different conditions of correction time, having a maximum error of 0.03 and a minimum error 0.0001 between the estimated and the experimental data (the Euclidiean norm was used to estimate the error).

6. CONCLUSIONS

In order to validate the performance of the high-gain observer versions: discrete and continuous-discrete, some experiments were conducted under similar conditions. First,

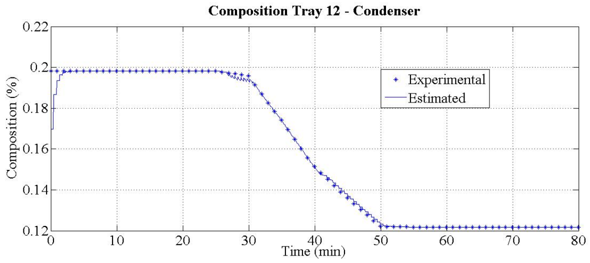

Fig. 4. Composition estimation of tray 12 by the continuous-discrete observer using a correction time of 15 sec

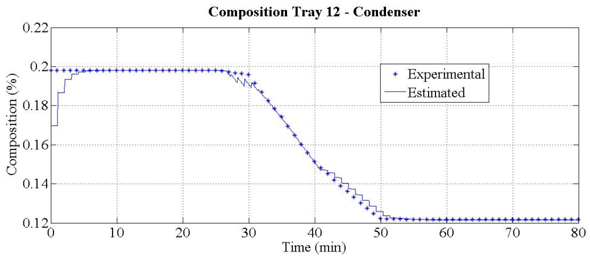

Fig. 5. Composition estimation of tray 12 by the continuous-discrete observer using a correction time of 30 sec

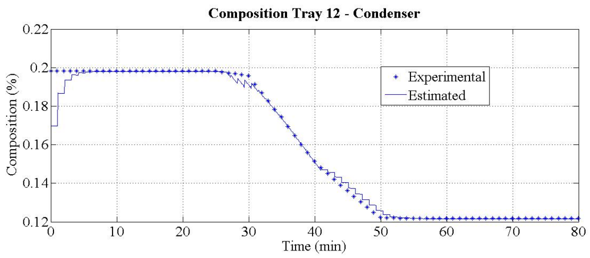

Fig. 6. Composition estimation of tray 12 by the continuous-discrete observer using a correction time of 60 sec

the purely discrete observer was validated, in order to perform, later, an adequate comparison with the continuous-discrete observer and analyze their response. Both validation use the same component specifications and same experimental inputs.

As can be seen in the presented figures the continuous-discrete observer presents a good tracking and quickly convergence to the experimental data, in spite of the sampling time used in the correction stage unlike the purely discrete observer, where the sampling time affects considerably its performance. Therefore, it can be assumed that the continuous-discrete observer is a suitable option to estimate the desired variables when the measurements of the system are performed using a long sampling time due to the slow dynamics of the process, which is the case of a distillation column where the compositions of the light component in a binary mixture of Ethanol-Water are estimated accurately.

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