A General Quadratic Performance Approach to Binary Distillation Control

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Abstract: High purity distillation control of a binary mixture in a tray column is considered in the paper at hand. The approach is based on an inferential control idea: dynamics within the column may be described as movements of concentration waves; the position of the wave front on the one hand side can be inferred from few temperature measurements, on the other hand the position implies the product concentrations. Dynamics of wave propagation is derived by simplification of a first principles model of the column. The resulting descriptor model is the basis for a recent LMI based controller design scheme that provides general quadratic performance for descriptor systems.

Keywords: Quadratic performance; descriptor system; binary distillation; inferential Control.

1. INTRODUCTION

Distillation is one of the most common separation processes in the chemical industries and it is also one of the most energy consuming ones. Therefore the control of this kind of processes has been a focus of process control for many years. Most approaches toward control of distillation columns are based on linear models which are based on identification techniques (e.g. Skogestad et al. [1988], Allgöwer and Raisch [1992]). The disadvantage of identified models is the missing physical interpretation. First principle models on the other hand are rather complex and typically not suitable for a direct model based controller computation.

In the paper at hand a reduced model for a distillation column is derived in descriptor form. The control problem is captured as a generalized quadratic performance problem. A solution to this problem is briefly reviewed (see Rehm and Allgöwer [2002] for details) and applied to the problem at hand.

The resulting controller is tested by means of a high order nonlinear model of the distillation process.

2. DESCRIPTOR MODEL

Separation of a binary mixture in a 40 tray distillation column with one feed stream is considered. A schematic representation of the process is given in the left part of Fig. 1. Exemplary the separation of two alcohols (Methanol,n-Propanol) is taken into account. The mixture is fed in the column with the feed flow rate F. Feed flow rate F and feed composition x_F (molar fraction) are determined by upstream processes.

The stationary feed flow rate and feed composition are corrupted by disturbances. The feed stream separates the column into rectifying- (upper part of the column) and stripping section (lower part of the column). Separation is achieved due to intensive heat and mass transfer between liquid flow and countercurrently rising vapor flow.

At the bottom of the column the liquid flow splits up into a liquid product stream which is removed with flow rate B from the column and a stream which is, after being heated in the reboiler, recirculated back to the column as vapor flow with flow rate V.

At the top of the column the vapor flow with the accumulated more volatile product is completely condensed in the condenser. The condensate is partly pumped back in the column with a flow rate L (reflux stream) and is partly removed as the distillate product with a flow rate D (Deshpande [1985]).

We consider the distillation column in "LV" configuration, that is: liquid flow rate L and vapor flow rate V are considered to be control inputs. Measured variables are the concentrations on trays 14 and 28.

2.1 Control Objectives

The main control objective is to stabilize the product concentrations at the top and bottom of the column at their stationary values. Additionally the deviations from the stationary values due to disturbances in the feed flow should be small.

Table 1. Notation for model variables

x_i	... liquid concentration of the more volatile component on the i^{th} tray
y_i	... vapor concentration of the more volatile component on the i^{th} tray
n_i	... liquid holdup of the i^{th} tray
$(\cdot)_{B,M,D}$	... corresponding quantities of reboiler feed tray, and condensor

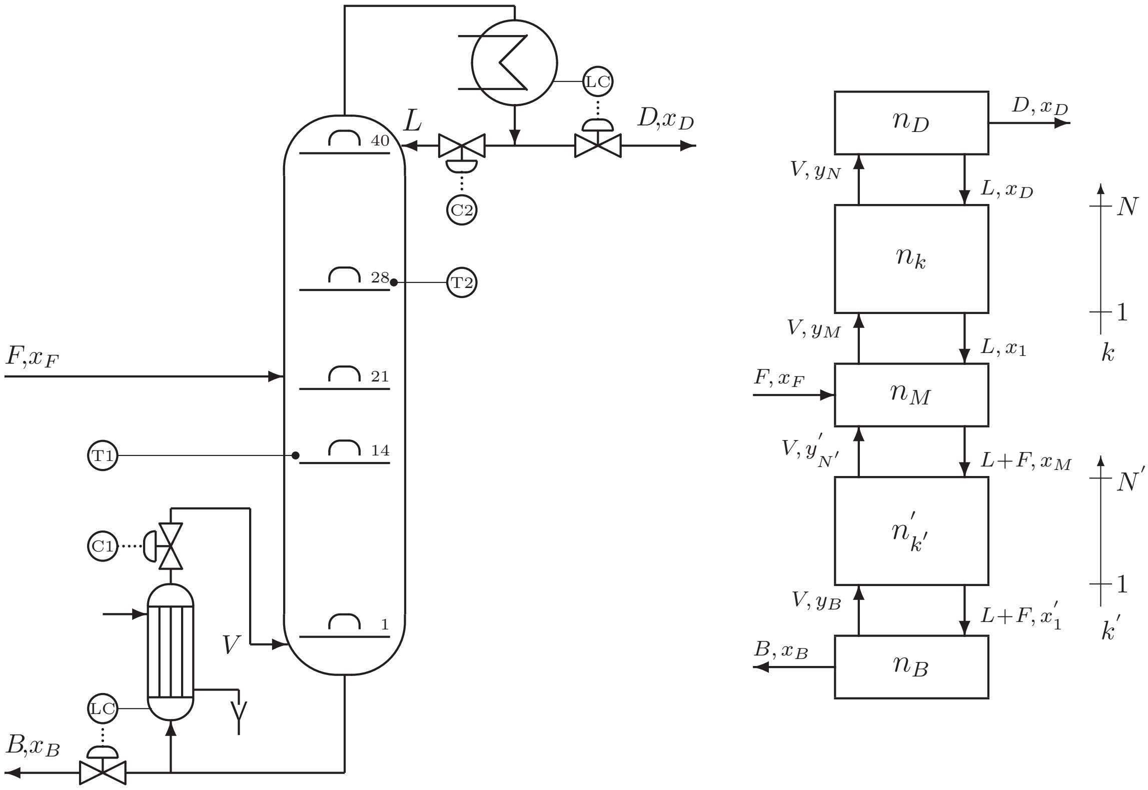

Fig. 1. Scheme of considered 40 tray distillation column (left) and subsystem structure for reduced modelling (right).

2.2 Reference Dynamics

A relatively detailed nonlinear model (CMO model without pressure losses, energy balances, and hydrodynamics (Deshpande [1985])) is used for simulation studies. The modelling equations describe the liquid concentrations of the more volatile component and are derived from the mass balance for every tray and for reboiler and condensor.

$$y_i = \frac{\alpha x_i}{1 + (\alpha - 1)x_i}\,, \quad \alpha = const. \tag{1}$$

The most important source of nonlinearity in the model are the equations (1) describing the vapor-liquid equilibrium (constant relative volatility α). The resulting model consists of 42 first order differential equations (40 equations from the intermediate trays plus two equations from reboiler and condensor).

2.3 Reduced Dynamics (Descriptor Model)

Starting point for the development of a reduced model in descriptor form of the distillation column is the fact (Retzbach [1986]) that qualitatively the behaviour of the column towards changes in the input values (V, L, F, x_F) can be regarded as motion and distortion of the stationary concentration profile (concentration versus tray number).

Instead of having detailed mass balances for rectifying and stripping section, the idea for a reduced model is thus to capture dynamics just by one position variable for a suitable concentration profile in every column section Due to (1) it is sufficient to consider a moving concentration profile only for the lighter component (measured in molar fractions, denoted by x in the following). Therefore the reduced model will contain two positions (s_r for the rectifying section and s_s in the stripping section) and three

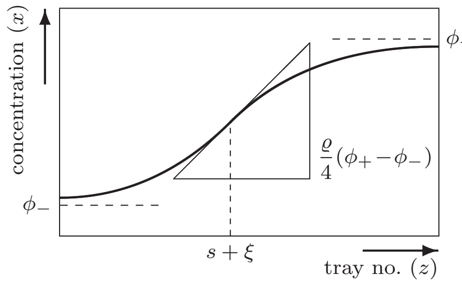

Fig. 2. Illustration of the shape parameters in function (2)

concentrations (concentration x_B in the reboiler, x_M for the feed tray, and x_D in the condenser) as state variables.

Here, only a sketch of the derivation of the reduced model in descriptor form is given, details can be found in Rehm [2004]. Furthermore we restrict ourselves to the presentation of the procedure for one column section, the deviation for the other section is completely analogous. The trays in this section are numbered by $z = 1, \ldots, N$ (see right side of Fig. 1). The concentration profile is modeled with the (continuous) function $x(z)$ (eq. (2), Fig. 2) which is well suited to describe the stationary profile in long packed columns (Kienle [1998]):

$$x(z) = \phi_- + \frac{\phi_+ - \phi_-}{1 + e^{-\varrho(z - s - \xi)}}. \tag{2}$$

With the least squares method the shape parameters ϕ_-, ϕ_+, ϱ, and ξ (see Fig. 2) are calculated such that $x(z)$ matches the stationary concentration profile ($s = 0$, i.e. s denotes the displacement relative to the stationary case) of the tray column for the discrete values $z = 1, \ldots, N$ in a least squares sense.

$$\begin{bmatrix} \begin{matrix}AY_1+Y_1^{\mathrm{T}}A^{\mathrm{T}}+\\ B_2\hat{C}_K+(B_2\hat{C}_K)^{\mathrm{T}}\end{matrix} & * & * & * \\ (A+B_2\hat{D}_KC_2)^{\mathrm{T}}+\hat{A}_K & \begin{matrix}X_1^{\mathrm{T}}A+\hat{B}_KC_2+\\ (X_1^{\mathrm{T}}A+\hat{B}_KC_2)^{\mathrm{T}}\end{matrix} & * & * \\ \begin{matrix}B_1^{\mathrm{T}}+D_{21}^{\mathrm{T}}\hat{D}_K^{\mathrm{T}}B_2^{\mathrm{T}}+\\ W_P^{\mathrm{T}}(C_1Y_1+D_{12}\hat{C}_K)\end{matrix} & \begin{matrix}B_1^{\mathrm{T}}X_1+D_{21}^{\mathrm{T}}\hat{B}_K^{\mathrm{T}}+\\ W_P^{\mathrm{T}}(C_1+D_{12}\hat{D}_KC_2)\end{matrix} & \begin{matrix}(D_{11}+D_{12}\hat{D}_KD_{21})^{\mathrm{T}}W_P+\\ W_P^{\mathrm{T}}(D_{11}+D_{12}\hat{D}_KD_{21})+V_P\end{matrix} & * \\ Q_P^{\mathrm{T}}(C_1Y_1+D_{12}\hat{C}_K) & Q_P^{\mathrm{T}}(C_1+D_{12}\hat{D}_KC_2) & Q_P^{\mathrm{T}}(D_{11}+D_{12}\hat{D}_KD_{21}) & -\Sigma_P \end{bmatrix} < 0. \tag{3}$$

$$\begin{aligned} Y_1 &:= RE^{\mathrm{T}}+E^{\mathrm{T}\perp}W_Y,\ R>0, \\ X_1 &:= SE+E^{\perp}W_X, \quad S>0, \end{aligned} \quad \begin{bmatrix} R & E^{+} \\ E^{\mathrm{T}+} & S \end{bmatrix} > 0 \tag{4}$$

$$\begin{pmatrix} * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & 0 & * & 0 & 0 \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * \end{pmatrix} \begin{pmatrix} \frac{d\Delta x_B}{dt} \\ \frac{d\Delta s_r}{dt} \\ \frac{d\Delta x_M}{dt} \\ \frac{d\Delta s_s}{dt} \\ \frac{d\Delta x_D}{dt} \end{pmatrix} = \begin{pmatrix} * & * & * & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & * & * & * \end{pmatrix} \begin{pmatrix} \Delta x_B \\ \Delta s_r \\ \Delta x_M \\ \Delta s_s \\ \Delta x_D \end{pmatrix} + \begin{pmatrix} 0 & * \\ 0 & * \\ * & * \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta x_F \\ \Delta F \end{pmatrix} + \begin{pmatrix} * & * \\ * & * \\ * & * \\ * & * \\ 0 & * \end{pmatrix} \begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix}$$

$$\begin{pmatrix} \Delta x_{14} \\ \Delta x_{28} \end{pmatrix} = \begin{pmatrix} * & * & * & 0 & 0 \\ 0 & 0 & * & * & * \end{pmatrix} (\Delta x_B, \Delta s_r, \Delta x_M, \Delta s_s, \Delta x_D)^{\mathrm{T}} \tag{5}$$

However, while ϱ and ξ are kept constant, ϕ_- and ϕ_+ are used as adaptation parameters since concentration profiles not only move but also are distorted. Adaptation of these parameters is based on the requirement that (2) should also match the concentrations for the neighbouring systems when evaluated for $z = 0$ and $z = N+1$. This adaptation rule implies that the time derivatives of x_B, x_M, and x_D influence the dynamics of wave propagation. The linearisation of the overall reduced descriptor model of the distillation column is given in (5). Here "Δ" implies deviations from the stationary value while "$*$" denotes numerical entries. A detailed derivation of the model and numerical values are given in Rehm [2004].

3. CONTROLLER COMPUTATION

3.1 Synthesis for Generalized Quadratic Performance for Descriptor Systems

The idea of generalized quadratic performance (GQP) control is to impose a general quadratic constraint of the type

$$\int_0^T \begin{bmatrix} \boldsymbol{z}(t) \\ \boldsymbol{w}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} U_P & W_P \\ W_P^{\mathrm{T}} & V_P \end{bmatrix} \begin{bmatrix} \boldsymbol{z}(t) \\ \boldsymbol{w}(t) \end{bmatrix} dt \ll 0, \tag{6}$$

on the external input/output chanel $\boldsymbol{w} \to \boldsymbol{z}$ of a generalized plant description G_{cl}. Here the notation "$\ll 0$" means that $\int_0^T Q(\boldsymbol{w}(t), \boldsymbol{z}(t))\, dt \le -\epsilon \int_0^T \boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t))\, dt$ holds for all $\boldsymbol{w}(\cdot) \in L_2$ and some fixed $\epsilon > 0$.

The rather general GQP problem contains some important control problems as a special case if the objective parameters $U_P \ge 0$, $V_P = V_P^{\mathrm{T}}$, and W_P are chosen accordingly (Scherer et al. [1997]). For example

- the H_∞ constraint $\|G_{cl}\|_\infty < \gamma$, if U_P, V_P, and W_P are specified as $U_P = \frac{1}{\gamma}I$, $V_P = -\gamma I$, $W_P = 0$;
- the strict passivity constraint $G_{cl}(j\omega)+G_{cl}(j\omega)^* > 0$ for all $\omega \in \mathbb{R} \cup \{\infty\}$, when U_P, V_P, W_P are chosen as $U_P = 0$, $V_P = 0$, $W_P = -I$;
- sector constraints of the form
$$\int_0^T (\boldsymbol{z}(t)-\alpha\boldsymbol{w}(t))^{\mathrm{T}} (\boldsymbol{z}(t)-\beta\boldsymbol{w}(t))\, dt \ll 0 \tag{7}$$
for $U_P = I$, $V_P = -\alpha\beta I$, $W_P = -\frac{1}{2}(\alpha+\beta)I$.

We consider a generalized plant description Σ in descriptor form

$$\Sigma: \begin{aligned} E\dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + B_1\boldsymbol{w}(t) + B_2\boldsymbol{u}(t) \\ \boldsymbol{z}(t) &= C_1\boldsymbol{x}(t) + D_{11}\boldsymbol{w}(t) + D_{12}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) &= C_2\boldsymbol{x}(t) + D_{21}\boldsymbol{w}(t) \end{aligned} \tag{8}$$

with $\boldsymbol{x}(t) \in \mathbb{R}^{n_x}$, $\boldsymbol{w}(t) \in \mathbb{R}^{n_w}$, $\boldsymbol{u}(t) \in \mathbb{R}^{n_u}$, $\boldsymbol{z}(t) \in \mathbb{R}^{n_z}$, and $\boldsymbol{y}(t) \in \mathbb{R}^{n_y}$ denoting the generalized state variables, the external input variables, the control input variables, the external output variables, and the measurement variables, respectively. E and A are square constant matrices where, explicitly, E is allowed to be singular, i.e. $\mathrm{rank}(E) =: r \le n_x$. The remaining matrices are constant matrices of appropriate dimension.

The control problem is, for given matices $U_P \ge 0$, $U_P \in \mathbb{R}^{n_z \times n_z}$, $V_P = V_P^{\mathrm{T}} \in \mathbb{R}^{n_w \times n_w}$, and $W_P \in \mathbb{R}^{n_z \times n_w}$, to find a linear output feedback controller such that the undisturbed closed loop ($\boldsymbol{w} \equiv \boldsymbol{0}$) is an admissible system and such that the transfer matrix from the external input $\boldsymbol{w}$ to the external output $\boldsymbol{z}$ suffices a general quadratic performace bound (6).

The actual design problem therefore consists in the selection of matrices U_P, V_P, W_P such that the transfer matrix from $\boldsymbol{w}$ to $\boldsymbol{z}$ reflects the performance requirements (e.g. robustness, energy dissipation, ...). Since we aim at an admissible close loop, we assume in the following the corresponding necessary stabilizability/detectability properties for descriptor systems, namely stabilizability/detectability at infinity (see also Dai [1989]).

With a controller K_E,

$$K_E: \begin{array}{l} E\dot{\boldsymbol{\zeta}}(t) = A_K\boldsymbol{\zeta}(t) + B_K\boldsymbol{y}(t) \\ \boldsymbol{u}(t) = C_K\boldsymbol{\zeta}(t) + D_K\boldsymbol{y}(t),\ \boldsymbol{\zeta}(t) \in \mathbb{R}^{n_x} \end{array} \tag{9}$$

parametrized by A_K, B_K, C_K, D_K the closed loop system is given by

$$\begin{aligned} E_{cl}\dot{\boldsymbol{\xi}}(t) &= A_{cl}\boldsymbol{\xi}(t) + B_{cl}\boldsymbol{w}(t) \\ \boldsymbol{z}(t) &= C_{cl}\boldsymbol{\xi}(t) + D_{cl}\boldsymbol{w}(t), \qquad \boldsymbol{\xi}(t) \in \mathbb{R}^{2n_x}, \end{aligned} \tag{10}$$

$$E_{cl} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \qquad A_{cl} = \begin{bmatrix} A + B_2D_KC_2 & B_2C_K \\ B_KC_2 & A_K \end{bmatrix},$$

$$B_{cl} = \begin{bmatrix} B_1 + B_2D_KD_{21} \\ B_KD_{21} \end{bmatrix}, \quad C_{cl}^{\mathrm{T}} = \begin{bmatrix} C_1^{\mathrm{T}} + C_2^{\mathrm{T}}D_K^{\mathrm{T}}D_{12}^{\mathrm{T}} \\ C_K^{\mathrm{T}}D_{12}^{\mathrm{T}} \end{bmatrix},$$

$$D_{cl} = (D_{11} + D_{12}D_KD_{21}) \tag{11}$$

Then a sufficient condition for a controller K_E solving the GQP control problem for DAE systems is given by the following theorem:

Theorem 1. Consider a plant (8) and a controller (9). There exists a controller parameterization A_K, B_K, C_K, D_K such that the undisturbed (i.e. $\boldsymbol{w} \equiv 0$) closed loop system (10) is admissible with general quadratic performance if the LMIs (3), (4) [1] admit a solution $\{R, S, W_Y, W_X, \hat{A}_K, \hat{B}_K, \hat{C}_K, \hat{D}_K\}$.

Remark. The preceding theorem constitutes also a necessary condition for the existence of a controller with GQP in the cases, where the corresponding analysis result is necessary for general quadratic performance, i.e. especially in the case of the H_∞ control problem. Therefore the results of Masubuchi et al. [1997] are included in Theorem 1 as a special case.

Theorem 1 is constructive: controller computation consists of three steps:

- Solution of the LMIs (3), (4). This is possible via effective numerical tools tailored for LMI problems arising from control theoretic problem setups (e.g. Gahinet et al. [1994], El Ghaoui et al. [1995]).
- Computation of non-singular matrices X_3, Y_3 such that

$$X_1Y_1 + X_2Y_3 = I \tag{12}$$
$$X_3Y_1 + X_4Y_3 = 0 \tag{13}$$

  hold together with the coupling condition $E^{\mathrm{T}}X_2 = X_3^{\mathrm{T}}E$. This is a essentially a factorization problem on the range of E which is always solvable provided (3), (4) have a solution.
- Solution of the linear equations

$$\begin{aligned} \hat{D}_K &:= D_K \\ \hat{C}_K &:= C_KY_3 + D_KC_2Y_1 \\ \hat{B}_K &:= X_3^{\mathrm{T}}B_K + X_1^{\mathrm{T}}B_2D_K \\ \hat{A}_K &:= X_1^{\mathrm{T}}(A + B_2D_KC_2)Y_1 + X_3^{\mathrm{T}}A_KY_3 + \\ &\quad + X_3^{\mathrm{T}}B_KC_2Y_1 + X_1^{\mathrm{T}}B_2C_KY_3 \end{aligned} \tag{14}$$

  for the controller matrices D_K, C_K B_K, A_K.

[1] Here E^+ denotes any generalized inverse with the property $EE^+E = E$ and "$*$" is used in order to indicate the symmetric expansion of a block matrix.

3.2 Distillation Control Problem as S/KS Mixed Sensitivity Problem

As a special case of generalized quadratic performance, the H_∞ control problem for the distillation problem is solved. The control objectives are translated into a mixed sensitivity set-up depicted in Fig. 4. with G representing the plant (reduced model in descriptor form), K the controller, and W_1, W_2, V frequency dependent weighting matrices. Controller design by "loop shaping" requires a selection of the weighting matrices such that the solution of the H_∞ control problem

$$\left\| \begin{matrix} W_1(I + GK)^{-1}V \\ -W_2K(I + GK)^{-1}V \end{matrix} \right\|_\infty \overset{!}{<} \gamma \tag{15}$$

results in a well behaved closed loop system. In this

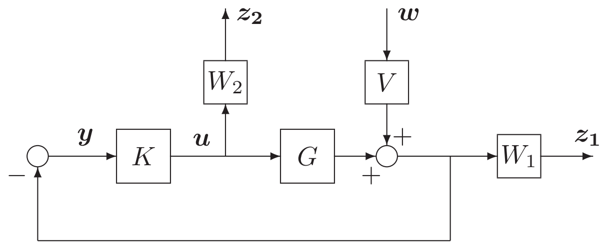

Fig. 4. Mixed sensitivity configuration

setup V can be interpreted as a filter which models the disturbance considered to be relevant for the problem at hand. With $S(s) := (I + GK)^{-1}$ being the sensitivity matrix of the closed loop the expression (15) with $\gamma = 1$ suggests to choose W_1 to be approximately the inverse of the wanted behavior for $S(s)$ and analogously W_2 to be the inverse of $K \cdot S$. General indications on selecting these weighting matrices can be found in Skogestad and Postlewaite [1996].

In case of the distillation control problem at hand an indirect approach is taken: with stabilizing the measured concentrations x_{14}, x_{28} also the stationary profiles are fixed and thus approximately also the product concentrations. In order to realize this idea the descriptor S/KS H_∞ control problem depicted in Figure 4 (with G being the descriptor model (5)) is solved by the outlined descriptor GQP synthesis procedure with specification of W, Q, and Σ as for the H_∞ set-up. The synthesis LMIs are jointly optimized with respect to γ. A final value of $\gamma = 1.01$ shows that the control objectives are approximately met.

The resulting controller has a dynamical order of 9, i.e. equal to the order of the generalized plant description. After removing the fastest two eigen-modes of the controller in order to avoid numerical problems due to stiffness, the controller is tested in simulation studies with the nonlinear CMO model of the distillation column.

4. RESULTS

In Figure 5 the stationary concentration profiles for various severe persistent disturbances for the closed loop are shown. It can be seen that the controller is able to stabilize the profile position although the disturbances result in a distortion of the stationary profile in the vicinity of the feed tray.

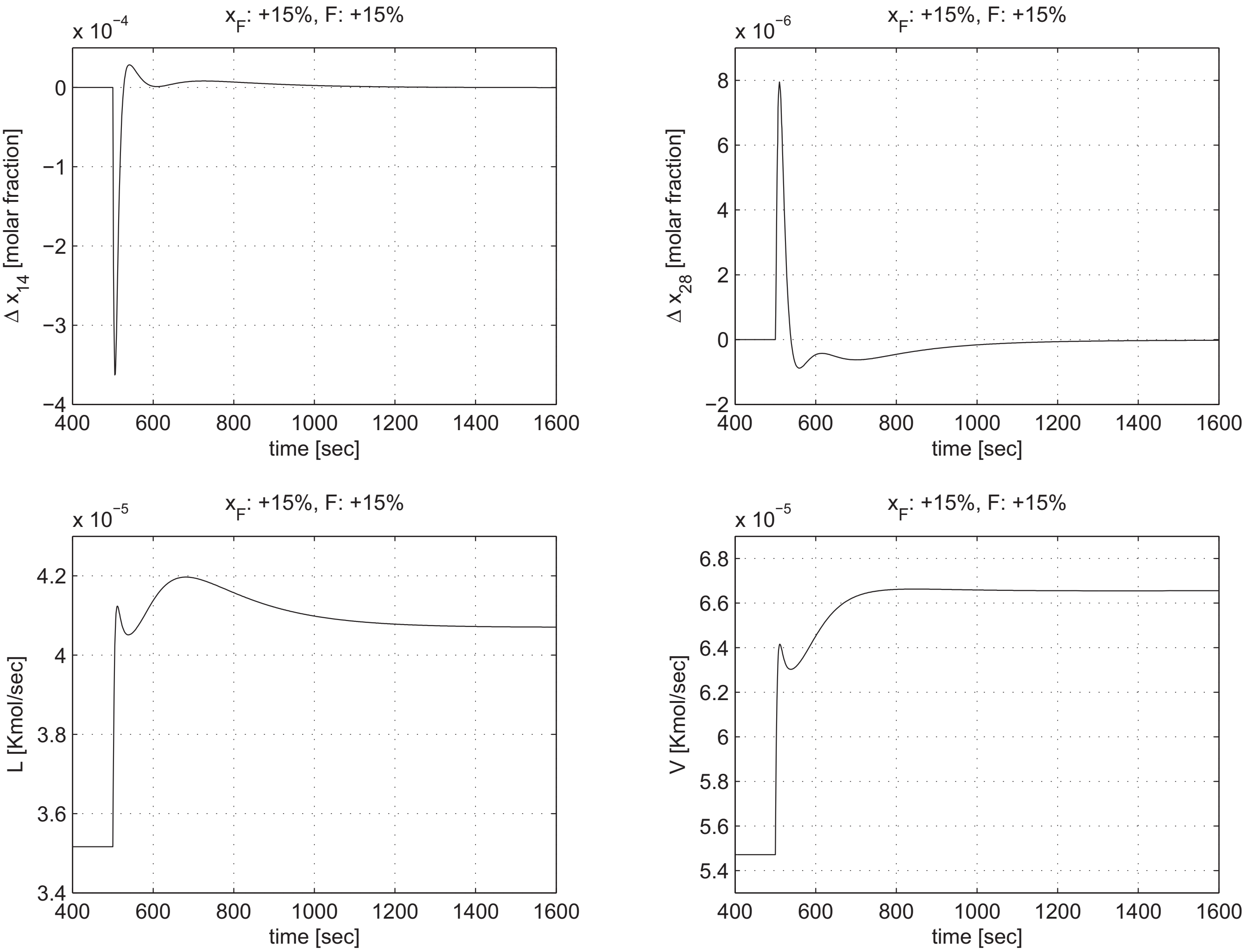

Fig. 3. Step responses for the controlled distillation column (+15% increase in feed flow rate F and +15% increase in feed concentration x_F with respect to stationary values at $t = 500sec$). Top: deviations from the steady state for the controlled variables x_{14} and x_{28}. Bottom: control variables, i.e. liquid flow rate L and vapor flow rate V.

In Figure 3 a detailed view on the control variables and the error in the controlled variables is given for a mutual step in the feed flow rate and feed composition. The plots show a fast transient behavior and small deviations. Furthermore no excessive action in the control variables is needed.

5. CONCLUSION

The generalized quadratic performance control problem for descriptor systems is solved for a reduced model of a distillation control problem. The resulting controller shows rather good results for a nonlinear reference model. The descriptor problem formulation is a direct result of reduced modeling. Furthermore, also standard approaches to build generalized plant descriptions easily fit into the descriptor system set-up. This was demonstrated by means of a S/KS - control problem formulation that directly leads to a descriptor model.

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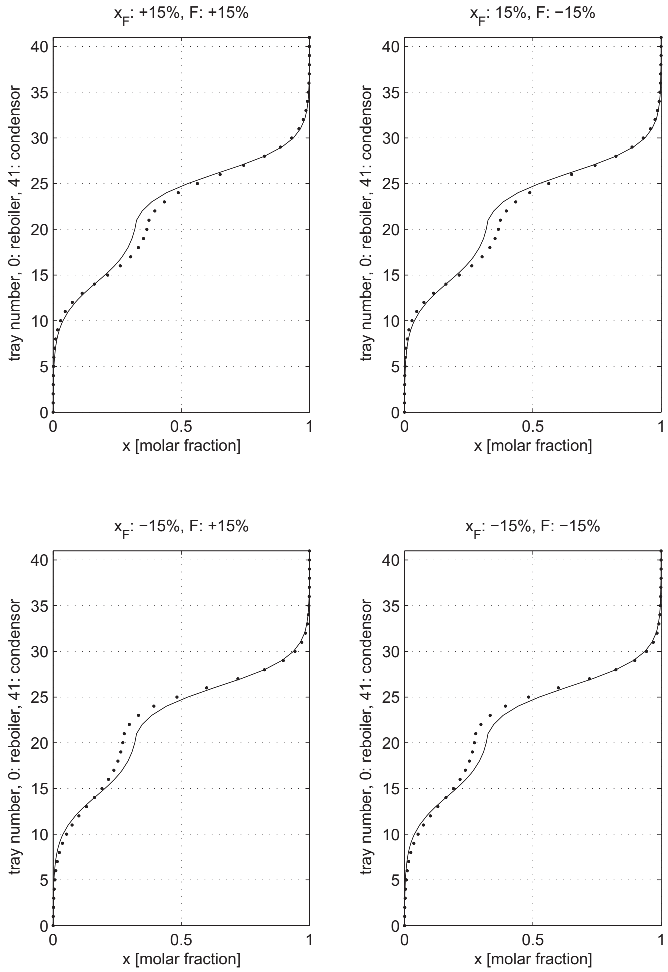

Fig. 5. Liquid concentration profiles in controlled distillation column. Solid lines: undisturbed stationary profile; dotted lines: new stationary profile for non-vanishing disturbances in feed flow rate F and feed concentration x_F (+/- 15% with respect to stationary values).

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