Influence of Differences in System Dynamics in the context of Multi-unit Optimization

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Abstract: Extremum-seeking methods are unconstrained real-time optimization techniques that control the gradient to zero. The crucial difference between them lies in the gradient estimation method used. Multi-unit optimization technique proposes the use of a multiple units operated with an offset between them and the estimation of the gradient is by finite difference. Though this method gives fast convergence, the major bottleneck is that it assumes the units to be identical. This paper addresses the case where the static curves are indeed identical, while the dynamics are not so. It is shown that if all the units are stable, despite the difference in dynamics, the method would indeed converge to the true optimum. Also, it is shown that the difference in dynamics does not affect stability in the neighborhood of the optimum. In addition, this paper presents a possibility of replacing real units by static models in the calculation of the gradient. Experimental results are presented from a mixing system where an optimal temperature is sought.

Keywords: Real time optimization and control, multi-units optimization.

1. INTRODUCTION

Process optimization is a tool of choice to find the best operating point that balances conflicting objectives such as productivity, selectivity, and operating cost for continuous chemical process. To perform this optimization numerically, it is necessary to have a model of its operation. Though most processes are dynamic in nature, often a steady state model suffices since typically, for continuous process, one is interested in finding the best steady state operation point. However, due to process changes, the optimal operating point varies with time, and to reap the benefits, it is indeed crucial to track these changes.

Without being exhaustive, two main classes of techniques have been employed in the real-time optimization of continuous processes. The first class comprises of the repeated optimization techniques (Marlin & Hrymak, 2000) that alternate between the identification of a steady state model using measurements and numerical computation of the optimal input using the updated model. On the other hand, extremum-seeking methods (Ariyur & Krstic, 2003, Guay et al., 2004) treat the optimization problem as one of controlling the gradient to zero.

In the extremum-seeking framework, various methods have been used for the gradient determination. The perturbation method (Ariyur & Krstic, 2003) deduces the gradient by adding perturbation signal that is very slow compared to the process dynamics. The correlation between the input and output is used to estimate the gradient. In adaptive extremum seeking techniques (Guay et al., 2004), parameters of a dynamic model are adapted and the gradient is computed from the adapted model. All the above mentioned techniques have to respect time-scale separations between gradient estimation and the process dynamics, thereby leading to slow convergence.

The multi-unit optimization technique (Srinivasan, 2007) is an attempt to find another gradient method that would converge faster. The basic method uses two identical units operated with an offset between them and uses finite difference to estimate the gradient. However, the main drawback of this technique is that it needs multiple identical processes working in parallel, which is impossible to get in practice. It has been shown that, if the units are not identical, the stability of the scheme and the convergence toward the true optimum are not guaranteed (Woodward et al., 2009).

Further research (Woodward et al., 2009) has revealed a way to compensate the difference in the curves that represent the steady-state relationship between the input and the objective function. It uses translation in both the input direction and direction of the objective function so as to evaluate correctly the gradient and converge toward the true optimum.

The key advantage of multi-unit optimization technique is that a reliable gradient is available during the transient, and one need not have to wait for the steady state. This advantage arises from the fact that if the process dynamics are the same, the difference of the objective functions is rendered insensitive to the process dynamics. However, if the processes' dynamics are different, even if the static curves are identical, the gradient would be falsified. The stability and convergence to the true optimum are a priori not assured.

In this paper, the case of non-identical dynamics is considered. It is however assumed that the static curves are the same. Such a case occurs when the optimization objective is only a function of the system output whose dynamics is controlled by a controller with integrator. Results from stability analysis and the equilibrium point are presented. It is shown that the difference in dynamics does not change the equilibrium point but it indeed affects stability and the way it converges. The theoretical results are experimentally verified and the data are also presented in this paper.

This paper also analyses the possibility of replacing a physical unit by a mathematical model. This way, the multi-unit optimization runs with one physical system and one mathematical model. Here, experimental results with a physical dynamic system and a static mathematical model are presented to show that this option is indeed viable.

The rest of this paper is organized as follows. Section 2 of this paper presents briefly the standard multi-unit optimization technique. Section 3 presents the results of the analysis for stability and convergence when the units' dynamic are different. Section 4 presents the methodology and the results for the experimental trials.

2. MULTI-UNIT OPTIMIZATION

2.1 Problem formulation

Mathematically, a standard real time optimization problem is written as follows:

$$\begin{aligned} &\min_u \quad J = g(x,u) \\ &\dot{x} = f(x,u) = 0 \end{aligned} \tag{1}$$

J is a twice-differentiable function that is minimized and f represents the dynamics of a stable process. The states of the system are represented by the vector *x* and the inputs by the vector *u*. For the easing of presentation, inequality constraints are ignored.

In order to find the optimal input, it is easier to use the equality constraints to find an expression of x = h(u) and then substitute the same. This transforms the original problem into a unconstrained optimization problem, i.e. min J = p(u). Then, the necessary condition of optimality is then given by:

$$\partial p/\partial u\,|_{u*} = 0 \tag{2}$$

If it is assumed that the unconstrained optimization problem is convex, then the necessary condition indeed leads to the only minimum. Equation (2) is used in extremum-seeking methods to find the optimal point by gradient control.

2.2 Multi-unit optimization scheme

A schematic representation of a simplified version of the multi-unit optimization framework is shown in Fig. 1 (Srinivasan, 2007). The term "unit" is used to represent a real continuous chemical process, and here they are labeled "0" and "1". A difference of Δ between the inputs "u_0" and "u_1" is necessary to estimate the gradient by a first order finite difference equation $\partial J/\partial u = (J_1 - J_0)/\Delta$. Then, the method uses an integral controller with an appropriate value of gain to push the gradient towards 0. The gain K can be tuned to as a compromise between convergence speed and stability.

$$\dot{u}_0 = \dot{u}_1 = \frac{K}{\Delta}\left(J_1 - J_0\right) \tag{3a}$$

$$u_0 = u - \frac{\Delta}{2} \text{ and } u_1 = u + \frac{\Delta}{2} \tag{3b}$$

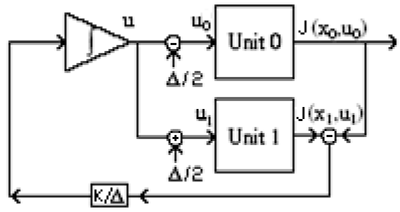

Fig. 1. Standard multi-unit optimization control loop.

In the case of multi-unit optimization, it is necessary to precise the dynamics of each of the units. The two units can be written mathematically as follows:

$$\begin{aligned} \dot{x}_0 &= f_0(x_0,u_0) \\ \dot{x}_1 &= f_1(x_1,u_1) \\ J_0 &= g_0(x_0,u_0) \\ J_1 &= g_1(x_1,u_1) \end{aligned} \tag{4}$$

The steady state for each unit can be described by:

$$\begin{aligned} x_0 &= h_0(u_0) \\ x_1 &= h_1(u_1) \end{aligned} \tag{5}$$

And the objective functions of each of the units at steady-state given by:

$$\begin{aligned} J_0 &= g_0(h_0(u_0),u_0) = p_0(u_0) \\ J_1 &= g_1(h_1(u_1),u_1) = p_1(u_1) \end{aligned} \tag{6}$$

3. MULTI-UNIT OPTIMIZATION WITH DIFFERENT DYNAMICS

In this section, the results of the analyses for stability and convergence are presented for the case when the units' dynamic are different but the static curve for each unit are identical. These analyses are made assuming that the technique is applied without any modification to compensate for the difference in dynamics.

3.1 Analysis for the equilibrium point

Here it is shown that if the scheme is stable, the system will converge toward the true optimum as long as the static curves between the units are identical. The difference between the dynamics does not bias the equilibrium point.

Theorem 1: If (i) the scheme converges, (ii) the static curves of the two units are identical, then despite the difference in the dynamics, the steady state of the multi-units optimization control loop represents the real optimum as Δ tends to zero.

Proof: From (3a), it can be seen that at steady state:

$$\dot{u}_0 = \dot{u}_1 = 0 = \frac{K}{\Delta}\left(J_1 - J_0\right) = \frac{K}{\Delta}\left(p_1(u_1) - p_0(u_0)\right) \tag{7}$$

So, the equilibrium point is determined by:

$$p_0(u_0) = p_1(u_1) \tag{8}$$

As the static curves of two units are considered identical,

$$p_0(u) = p_1(u) = p(u) \tag{9}$$

Let $\bar{u}$ be the value of u at steady state. Then,

$$p\left(\bar{u} + \Delta/2\right) = p\left(\bar{u} - \Delta/2\right) \tag{11}$$

A second order Taylor expansion of the function p is considered:

$$p(\bar{u}) + p'(\bar{u})\frac{\Delta}{2} + p''(\bar{u})\frac{\Delta^2}{4} - p(\bar{u}) + p'(\bar{u})\frac{\Delta}{2} - p''(\bar{u})\frac{\Delta^2}{4} + O(\Delta^3) = 0 \tag{12}$$

$$p'(\bar{u}) = O(\Delta^2) \tag{13}$$

$$\lim_{\Delta \to 0} p'(\bar{u}) = \lim_{\Delta \to 0} O(\Delta^2) = 0 \tag{14}$$

Calculating the limit as Δ tends to zero, it is observed that the derivative becomes zero, indicating that the two units arrive at optimal point. ■

3.2 Analysis for the stability of the scheme.

To analyze the stability of the above scheme, the units are linearized around the current operating point. The transfer function representation is used and normalized transfer function that has a unit steady state gain is derived. Then, the characteristic equation of the loop is obtained and analyzed if the roots of this equation are in the left half of the complex plane.

Theorem 2: If an integral controller with gain K can stabilize the average of the two normalized dynamics, then for a small enough value of Δ, the scheme is locally asymptotically stable around the optimum.

Proof: In order to analyze locally the stability, it is necessary to linearize the dynamics of both units. With the linearization, it is possible to represent directly the relationship between u and J by a transfer function T(s). It is useful to rewrite T(s) as a combination of a dynamic term with a static gain of 1, labeled as N(s), and a static term which represents the steady state gain:

$$T_0(s) = N_0(s)p'(u_0) \text{ and } T_1(s) = N_1(s)p'(u_1) \tag{15}$$

Note that the static gain of the different units is given by the gradient (linearization) of the static curve p(u) at their respective operating points. The above decomposition separates the static behavior of the units from its dynamics. The condition on the characteristic equation for this loop to be stable is given by:

$$Z = 1 + \frac{K}{s\Delta}\left(T_1(s) - T_0(s)\right) = 0 \tag{16}$$

Considering a Taylor expansion approximation of p gives:

$$p'(u_0) = b - \frac{\Delta}{2}M \text{ and } p'(u_1) = b + \frac{\Delta}{2}M \tag{17}$$

where b = p'(u) and M = p"(u). Distributing and rearranging the terms in (16), one gets,

$$s + \frac{KM}{2}\left(N_1 + N_0\right) + \frac{Kb}{\Delta}\left(N_1 - N_0\right) = 0 \tag{18}$$

At the equilibrium point (which has been shown to be the optimum in Theorem 1), as Δ tends to zero, (b/Δ) goes to zero. Then, the third term of the characteristic equation disappears. So, around the optimum, the stability is no more influenced by the value of Δ, nor by the error in the dynamics. The stability of the system around the optimum is then determined by whether or not the integral controller with the given gain stabilizes the average dynamics. ■

Note that the difference in the dynamics would not affect the stability around the optimum. However, the difference in dynamics can affect the characteristic equation considerably when the system is far from the optimum. The characteristic equation (18) is a rich source of information from which two conclusions can be drawn:

Effect of gain K: The value of the gain is crucial to stability in all cases. If the normalized average dynamics is stable and minimum phase, then for small values of gain, the overall scheme would be stable. Moreover, if the value of K is small, it can be seen from (18) that the influence of the difference in dynamics is negligible. In other words, with a small gain, the inputs variations are slow compared to the dynamics, and the two units operate at their respective pseudo-steady states.

Evolution far from the optimum: When the starting point is far from optimum ("b" is not zero), it can be seen from (18) that the dynamic behavior is influenced by the sign of the ratio (b/Δ), rather than the sign of the individual components.

Without loss of generality, suppose unit 0 is faster than unit 1. A negative Δ and a negative b mean that the faster unit is closer to the optimum. The same situation occurs for a positive Δ and a positive b. These two cases would have similar adaptation characteristics, and are shown in Figure 3 with the red dot representing the faster unit 0 and the green representing the slower unit 1.

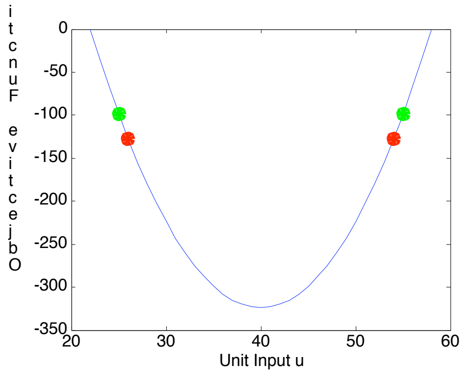

Fig. 3. Analysis of configurations with respect to dynamic behavior. Red faster than green leads to oscillations. Green faster than red leads to slow convergence.

In this configuration, since the system with a rapid dynamics is closer to the optimum, the difference between the outputs will be larger during the transients (for the same change in u). This will overestimate the gradient which, in the best case, will cause the system to converge with oscillations.

A negative Δ and positive b means that the slower unit is closer to the optimum. The same situation occurs for a positive Δ and a negative b. These two cases would have similar dynamics, and are shown in Figure 3 with the green dot representing the faster unit 0 and the red representing the slower unit 1.

In this configuration, since the system with a slower dynamics is closer to the optimum, the difference between the outputs will be smaller during the transients (for the same change in u). This will underestimate the gradient which will cause the system to converge slowly.

The interesting point is that even if there is an overestimation of the gradient (faster unit closer to the optimum), the system would not in general become unstable. This is due to the fact that once both the units overshoot the optimum, the situation is reversed, i.e., the slower unit is closed to the optimum. So, an under estimation of the gradient occurs, where the return back would be slow and sure.

4. EXPERIMENTAL RESULTS

4.1 Problem Formulation

In order to prove the theoretical results shown in the previous section, an experimental setup has been designed. The setup is composed of two tanks (units) whose temperature is controlled in order to minimize the criterion mentioned below.

$$\begin{aligned} &\min_{F_c} \quad J = (T_{out} - T_c)(T_{out} - T_h) \\ &\dot{T}_{out} = \frac{F_h}{V}(T_h - T_{out}) + \frac{F_c}{V}(T_c - T_{out}) = 0 \end{aligned} \tag{19}$$

Each unit is supplied with water by two pumps: a hot water pump and cold water pump. Hot water at temperature T_h and cold water at temperature T_c are added to these tanks with flow rates F_h and F_c respectively. The hot water pump is fixed with two heads in order to feed the same flow for both units. However, each unit has its own cold water pump (F_{c0} and F_{c1} being the decision variable). The temperatures in the units T_{out0} and T_{out1}, in the hot water tank and in the cold water tank are measured using thermistors. V is the volume of each of the units. The answer to this problem is:

$$T_{out} = \frac{T_h + T_c}{2}; \quad F_c = F_h \tag{20}$$

Also, in order to control the dynamics of each unit independently, cascade control has been implemented. Essentially, the multi-unit scheme sends a temperature set point u to each temperature control loop, which is controlled using a PI controller. This way that the static curves are identical (steady state error is zero). However, by tuning the temperature controllers differently, the two units will have different dynamics.

4.2 Experiments conditions tested

Four experiments are performed for all cases. In Experiment 1, the system is initialized to a value higher than the optimum with a positive value of Δ. Experiment 2 starts with an initial condition that is less than the optimum. Experiment 3 and Experiment 4 start from the initial conditions of Experiment 1 and 2 respectively, but with negative Δ.

It is always arranged that Unit 1 is slower than Unit 0. So, in Experiments 1 and 4, the optimum is closer to the faster unit, while the optimum is closer to the slower unit in the other two experiments. Experiments 1 and 4 show a configuration that overestimates the gradient, while experiments 2 and 3 present a configuration that underestimates the gradient. The gain K is the same for all experiment and is chosen so that the scheme is always stable.

Exp.	Start Temp	Temp Unit 0 (fast)	Temp Unit 1 (slow)
1	47 C	+1 C	-1 C
2	33 C	+1 C	-1 C
3	47 C	-1 C	+1 C
4	33 C	-1 C	+1 C

Table 1: Experimental plan for all experiments

4.3 Experiments with two real units.

In this set of experiments, the controller of "unit 1" has been tuned so that the internal loop (dynamics between the temperature output and temperature set point) is two times slower than its counterpart "unit 0". The results are presented in Figures 5-8.

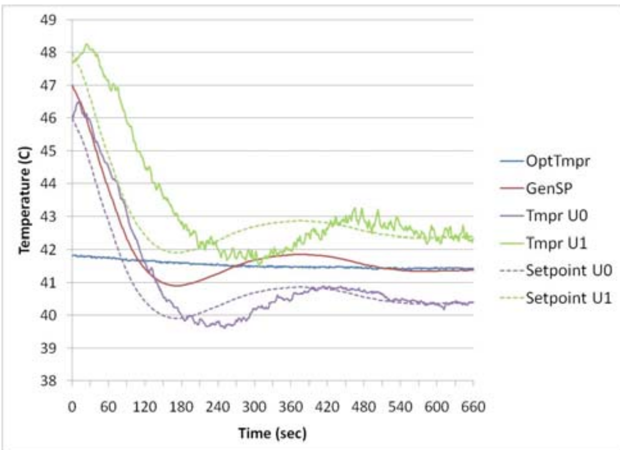

Fig. 5. Evolution for Experiment 1 with real units.

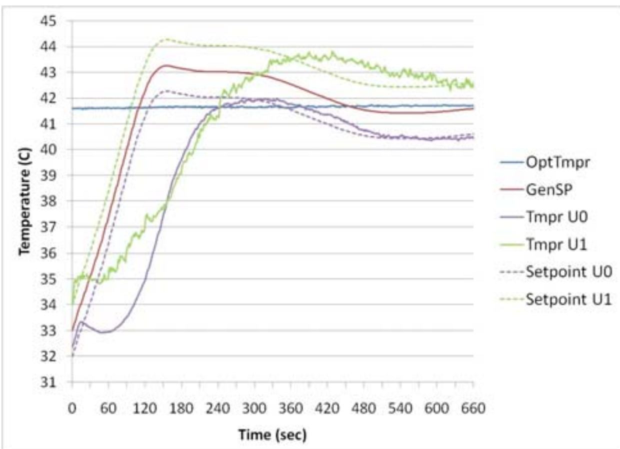

Fig. 6. Evolution for Experiment 2 with real units.

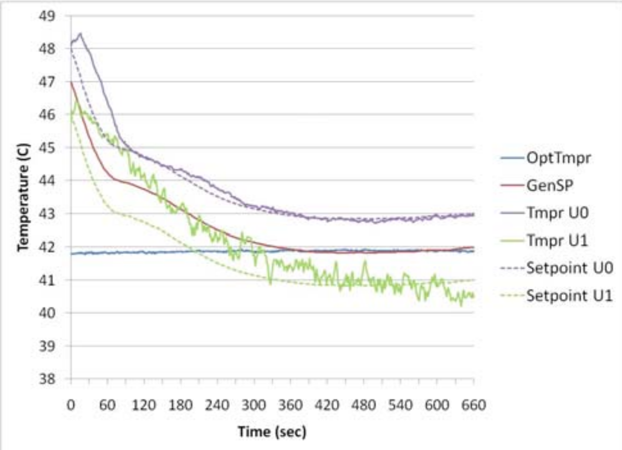

Fig. 7. Evolution of the system for Exp. 3 with real units.

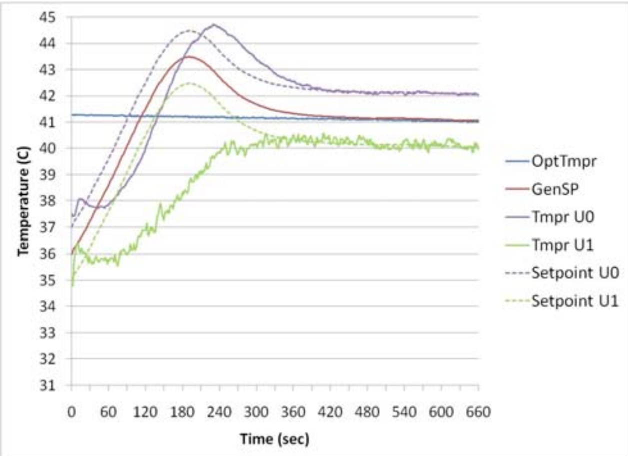

Fig. 8. Evolution of the system for Exp. 4 with real units.

The influence of the sign of Δ can be seen by comparing Fig 5 (Exp1) and Fig 7 (Exp3). The positive value of Δ brings the faster unit closer to the optimum in Exp1 and farther from the optimum in Exp3. Note that in both these experiments, there is a larger difference between the set point and the measured temperature in Unit 1 compared to that of Unit 0. The effect of the transients is more marked when the system is far from the optimum where the set point changes rapidly.

In the case of Exp 1, because of the difference in speed of the respective responses, the gradient is overestimated. This causes the scheme to converge with oscillations. On the contrary, in the case of Exp 3, the gradient is underestimated due the above mentioned speed difference. This underestimation causes the scheme to converge slowly but surely.

The above conclusion can be generalized for the ratio (b/Δ). Comparing Fig 5 (Exp1) and Fig 8 (Exp4), it can be seen that in both these experiments, the faster unit is closer to the optimum (b/Δ > 0). The gradient is overestimated, resulting in an oscillatory response toward the optimum. In contrast, Fig 6 (Exp2) and Fig 7 (Exp3) show slower convergence since the faster unit is farther from the optimum (b/Δ < 0).

Comparing Exp 1 and Exp3, it can be noted that the response for Exp3 is not as smooth as expected. Taking into account that a linear controller was used to control a nonlinear system, such a behavior is expected, The controller parameters are clearly not tuned for all points of operation. The controller settings used favored higher temperatures as can be seen from Figure 3.

4.3 Experiments with a real unit and a virtual unit

The purpose of next series of tests is to check the viability of replacing one of the real units by a mathematical model. This removes one of the major constraints of the scheme, i.e., the availability of two physical identical units in operation. There are two kinds of model that one can use: (i) a first principles/black box dynamic model (ii) a black box static model. Though the dynamic models were tested experimentally, the more interesting and extreme case, i.e., the use of the static model, is presented here.

In this set of experiments, “unit 0” is a static mathematical model whose dynamics is by definition instantaneous and so faster compared to the real unit (unit 1). In practice, it is done by setting the output equal to the set point of the control loop.

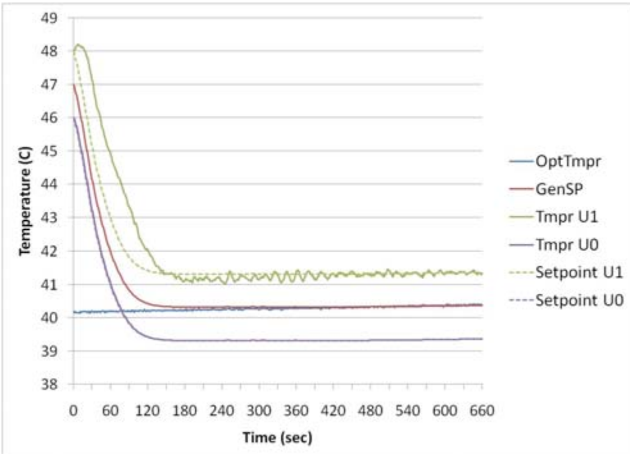

Fig. 9. Evolution for Exp. 1v with real and virtual units.

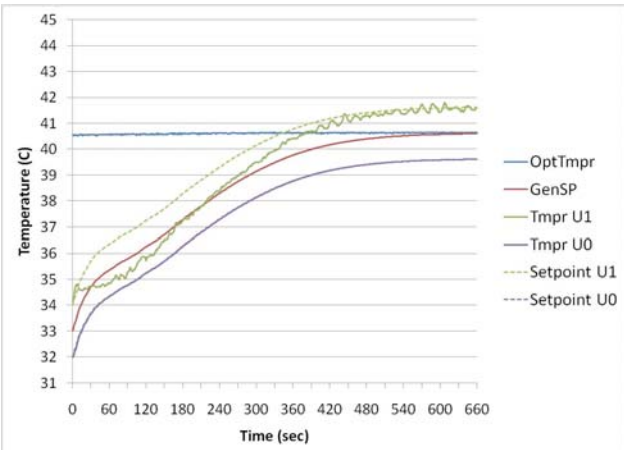

Fig. 10. Evolution for Exp. 2v with real and virtual units.

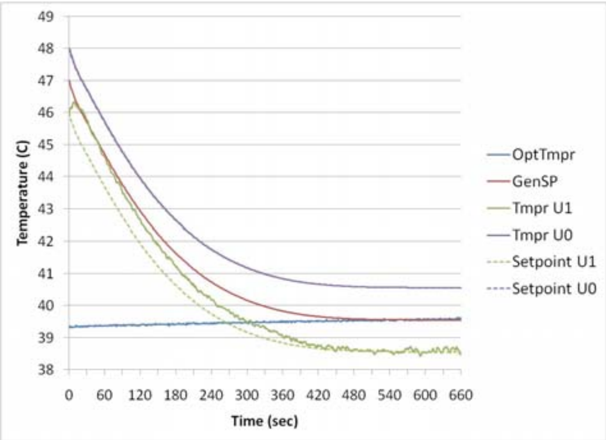

Fig. 11. Evolution for Exp. 3v with real and virtual units.

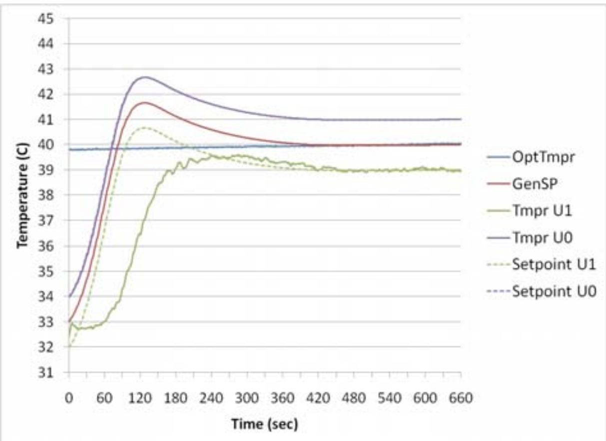

Fig. 12. Evolution for Exp. 4v with real and virtual units.

The results obtained using a system with a real unit and a virtual unit are similar to those obtained with two real units under the same testing conditions. The conditions on stability and convergence remain the same for this series of experiments. Indeed, the comparison of Fig 9 (Exp1v) and Fig 11 (Exp3v) yields the same general conclusion as the comparison of Exp1 and Exp3. It is also true for the comparison of Fig 9 (Exp1v) and Fig 12 (Exp4v) with Exp1 and Exp4 and the comparison of Fig 10 (Exp2v) and Fig 11 (Exp3v) with Exp2 and Exp3. However, for the last comparison, it is easier now to see that the smaller difference between the unit temperatures results in an underestimation of the gradient, thereby leading to slow convergence.

Note that while replacing the real unit by a model, it was assumed that the static characteristics are matched. The only difference between the real and virtual units is at the dynamics level. In this particular example, since the objective function is only dependant on the unit temperature, this assumption of matching the static behavior is easily verified. However, if the objective function is a function of both the output and input (T_{out} and F_c) of the dynamic part, it then becomes mandatory to have a good model of the physical system in order to match the static characteristics.

The conclusion here is that if the static characteristics are matched, then multi-unit optimization can be performed even when the dynamics are not necessarily identical. In extension, a real unit can be replaced by a virtual static model, which has the same static characteristics as the real unit. However, in order to converge to the true optimum, where the static characteristics are different, parameter adaptations are indeed necessary so as to compensate for these differences. In short, the differences in the dynamics can be tolerated, while differences in the static behavior needs to be quantified and compensated.

5. CONCLUSIONS

In this paper, it is shown that it possible to use the multi-unit scheme with differences in dynamics without affect its performance considerably. If the static curves are the same, the equilibrium point and the stability around the optimum are not affected. However, far from the optimum, the choice of the offset plays an important role; it can either make the system converge slowly or make it oscillatory.

Experimentally, it is shown that replacing a real dynamic unit by a simple static mathematical model is indeed viable. This means that the major constraint of having real multiple identical units can be circumvented. A good, not necessarily perfect, approximation of the process is sufficient to this effect.

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