New tuning rules for PI and fractional PI controllers

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Abstract: This paper presents new tuning rules for PI and fractional PI control of processes that are typically found in process control. The rules are based on characterization of process dynamics by three parameters that can be obtained from a simple step response experiment. The rules are obtained by minimizing a frequency objective function subject to a constraint on the maximum sensitivity. Comparisons with classical tuning rules show that they are very simple but give substantially better performance.

Keywords: Fractional control, PID control, design, tuning methods, optimization, process control.

1. INTRODUCTION

In spite of all the advances in process control over the past several decades, the proportional integral (PI) and the proportional integral derivative (PID) controller remains to be certainly the most extensive option that can be found on industrial control applications, see Åström and Hägglund (2001). The transparency of the PID control mechanism, the availability of a large number of reliable and cost-effective commercial PID modules, and their widespread acceptance by operators are among the reasons of its success, see Gude and Kahoraho (2007).

Over the last half-century, a great deal of academic and industrial effort has focused on improving PID control, primarily in the area of tuning rules. In fact, since Ziegler and Nichols proposed their popular tuning rules, Ziegler and Nichols (1942), an intensive research has been done. Works include from modifications of the original tuning rules, see Chien *et al.* (1952), Hang *et al*. (1991), and Åström and Hägglund (2004), to a variety of new techniques, see Åström and Hägglund (1995).

Fractional calculus, which is the expansion to fractional orders, has been known since the development of the regular calculus. However, fractional-order control was not incorporated into control engineering mainly due to the lack of sufficient mathematical knowledge and the limited computational power available at that time.

More recently, Podlubny (1999) has proposed a generalization of the PI and PID controllers, namely the PI^{λ} and $PI^{\lambda}D^{\mu}$ controllers, involving an integrator of order λ and a differentiator of order μ (the orders λ and μ may assume real non-integer values). Podlubny has also demonstrated the better response of these types of controllers, in comparison with the classical PI and PID controllers, when used for the control of fractional-order systems. A frequency domain approach by using fractional PID controllers has also been studied in Vinagre et al. (2000). However, the design methods for fractional controllers are a recent research area, see Capponetto *et al.* (2002) and Monje *et al.* (2005).

Given that the most common control structure used in the process industry is the PI controller, Åström and Hägglund (2001), an immediate approach that should be taken into account is to use the fractional PI^{λ} controller. Because of the widespread use of PI controllers and the potentials of fractional PI^{λ} controllers, see Gude and Kahoraho (2009), it is interesting to have simple but efficient methods for tuning these kind of controllers.

In this paper, we have developed new simple tuning methods for PI and PI^{λ} controllers that give significantly better performance for a wide range of processes.

The layout of this paper is the following. The different controllers and the test batch considered in this paper are presented in Section 2. The design method is treated in Section 3. This is followed by the main results obtained in this paper: new tuning rules for PI and PI^{λ} controllers in Section 4. In Section 5 the developed tuning rules are applied to a process and a comparison between different tuning rules is made. Finally conclusions and final remarks are drawn in Section 6.

2. CONTROLLERS AND TEST BATCH

2.1 Plant knowledge

To be accepted in industrial applications controller tuning rules must be based on a limited amount of plant knowledge that is easy to obtain. The plant can the be characterized by its τ value:

$$\tau = \frac{L}{L+T} \quad (1)$$

This parameter is usually called the *normalized dead time*. It is essentially the classical *controllability ratio* L/T, but the parameter τ has the advantage that it is in the range from 0 to 1. The *controllability ratio* was often mentioned in the early process control literature, see Cohen and Coon (1953). This parameter can be used to characterize the difficulty of controlling a process. Roughly speaking, processes with small τ can be considered easy to control and the difficulty in controlling the system increases as τ increases.

2.2 The test batch

The design method presented in the next section requires the transfer function of the process to be known. The results of this investigation depend critically on the chosen test batch. To apply the method we therefore have to choose process models that are representative for the dynamics of typical industrial processes. Processes with the following transfer functions have been used:

$$G_1(s) = \frac{e^{-s}}{(1+sT)^2}$$

T = 0.01, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 1, 2, 4, 6, 8, 10

$$G_2(s) = \frac{1}{(s+1)^n}$$

n = 3, 4, 5, 6, 7, 8

$$G_3(s) = \frac{1}{(1+s)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)} \quad (2)$$

α = 0.1, 0.2, 0.5, 0.7

$$G_4(s) = \frac{1-\alpha s}{(s+1)^3}$$

α = 0.1, 0.2, 0.5, 1, 2

$$G_5(s) = \frac{1}{(1+s)(1+sT)}$$

T = 0.02, 0.05, 0.1, 0.2, 0.5

The process (3) is the standard model that has been used in many investigations of PID tuning.

$$G(s) = K_P \frac{e^{-Ls}}{(1+sT)} \quad (3)$$

The test batch (2) does, however, not include this transfer function because this model is not representative for typical industrial processes, see Åström and Hägglund (1995). Tuning based on the model (3) typically gives controller gains that have a different behaviour from the other processes in the test batch, see Hang *et al.* (1991). This is remarkable because tuning rules have traditionally been based on this model.

The processes selected in the test batch (2) are representative for many of the processes typically found in process control, see for example Åström and Hägglund (2000) and Gorez (2003), suggested as standard benchmark models for testing PID controllers. The test batch includes processes that range from delay-dominated to lag-dominated processes. They include all kinds of plants with poles strictly on the negative real axis, such as plants with time delay or non-minimum phase zeros, plants of high and low orders, plants with multiple and spread poles, etc. All processes are normalized to have unit steady state gain and have a parameter that can be changed to influence the response of the process. The parameter ranges have been chosen to give a wide variety of responses. The normalized time delay ranges from 0.17 to 1 for G_1. The rest of the processes have values of τ in the range $0 < \tau < 0.5$

2.3 PI and PI$^\lambda$ controllers

In this paper, two different controllers are considered: the PI and the fractional PI$^\lambda$ controller, which is a generalisation of the PI controller. It is a non-integer order controller of the form:

$$C(s) = K + \frac{k_i}{s^\lambda} \quad (4)$$

where K is the proportional gain, k_i the integral gain, and λ the fractional order of the integral part.

The interest of this kind of controller is justified by a better flexibility, since it exhibits a fractional integral part of order λ. Thus, three parameters can be tuned in this structure (K, k_i, and λ), that is, one more parameter than in the case of conventional PI controller ($\lambda = 1$). We can take advantage of the fractional order λ to improve the performance.

3. THE DESIGN METHOD

Within the process industry, regulation performance is often of primary importance since most controllers operate as regulators, see Shinskey (1996). Regulation performance is often expressed in terms of the control error obtained for certain disturbances. A load disturbance is typically applied at the process input. Typical criteria are to minimize a loss function of the form:

$$I = \int_0^\infty t^n |e(t)|^m dt \quad (5)$$

where the error is defined as e(t) = r(t) – y(t). Common cases are IAE (n = 0, m = 1), ISE (n = 0, m = 2), or ITSE (n = 1, m = 2).

However, Kristiansson and Lennartson (2002) defined another performance criterion in the frequency domain as an alternative to the above criteria based on a function of the error signal. It is formulated as:

$$J_v = \left\| \frac{1}{s} G(s) S(s) \right\|_\infty = \max_\omega \left| \frac{1}{j\omega} \cdot \frac{G(j\omega)}{1+L(j\omega)} \right| \quad (6)$$

The proposed performance criterion is mainly a measure of the system ability to handle low-frequency load disturbances.

Robustness is an important consideration in control design. There are many different criteria for robustness. Many of them can be expressed as restrictions on the Nyquist curve of the loop transfer function L(s) = G(s)C(s). Åström and Hägglund (1995) introduced the maximum sensitivity function of the closed-loop system, M_S, as a tuning parameter for PID controllers. The constraint (7) that sensitivity function S(jω) is less than a given value M_S implies that the loop transfer function should be outside a circle with radius $1/M_S$ and center at –1.

$$\|S(s)\|_\infty = \max_\omega |S(j\omega)| = \max_\omega \left| \frac{1}{1+L(j\omega)} \right| \le M_S \tag{7}$$

The design problem discussed in this paper can be formulated as an optimisation problem: *Find parameters of the different controllers that minimize performance criterion (6) subject to the robustness constraint (7).*

A reasonable ambition in all control design is to keep the control signal as small as possible. Control system design very often deals with the trade-off between performance and control effort, provided that a reasonable mid-frequency robustness is guaranteed, see for example Gude and Kahoraho (2009). Therefore, introduce the control effort criterion:

$$J_u = \|C(s)S(s)\|_\infty = \max_\omega \left| \frac{C(j\omega)}{1+L(j\omega)} \right| \tag{8}$$

4. RESULTS

An empirical method is used to develop the new tuning rules. The design method proposed in Section 3 with M_S = 1.4 was applied to all processes in the test batch (2). This value of M_S provides a good compromise between performance and robustness. This gave the corresponding parameters K, T_i, for the PI, and K, T_i, and λ, for the fractional PI controller. The process parameters K_P, L and T were also computed from the step response experiment. The controller gain is normalized by multiplying it either with the static process gain K_P or with the parameter a = $K_P L/T$. Integration time is normalized by dividing by T or by L. We will represent normalized controller parameters as functions of τ. Data obtained can be well approximated by functions having the form:

$$f(\tau) = a\tau^b + c \tag{9}$$

4.1 PI controller

Simplified tuning rules for PI controllers will be first obtained. Figures 1 and 2 show the normalized proportional gains and integration times, respectively, as a function of normalized time delay τ when the design procedure is applied to all processes in the test batch (2). The curves drawn correspond to the results obtained by curve fitting. Both figures show that there appears to be a good correlation between the normalized controller parameters and the normalized time delay τ. This indicates that it is possible to develop good tuning rules based on the *KLT*-model. However, Figures 1 and 2 also show that parameters KK_P, aK, T_i/L, and T_i/T range from 0.16 to 23.8, from 0.21 to 3.15, from 0.34 to 8.2, and from 0.1 to 6.8, respectively.

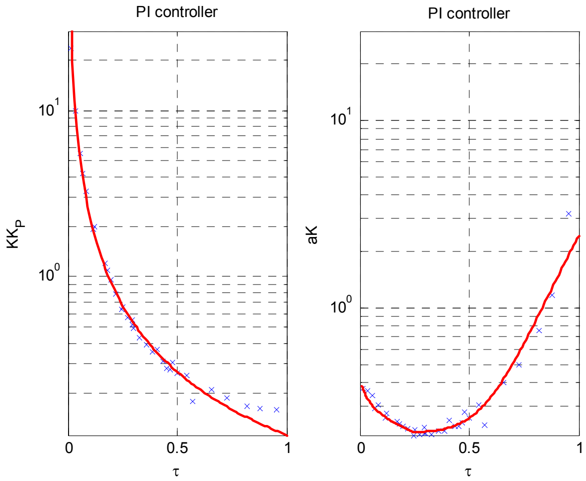

Fig. 1. Normalized PI controller proportional gains plotted versus normalized time delay τ for the test batch. The solid lines correspond to the tuning rules obtained in Table 1.

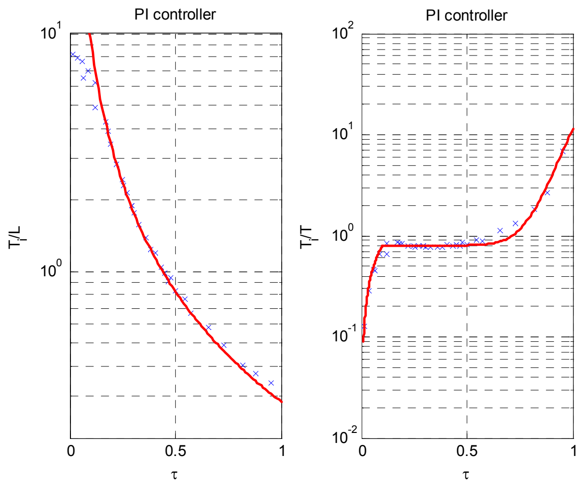

Fig. 2. Normalized PI controller integration times plotted versus normalized time delay τ for the test batch. The solid lines correspond to the tuning rules obtained in Table 1.

This indicates clearly that it is not possible to obtain good tuning rules that do not depend on τ. The deviations from the solid lines in the figure is about ± 15%

Table 1. Tuning formulae for the PI controller. The table gives the parameters of the functions of the form (9) for the normalized controller parameters and M_S = 1.4.

f(τ)	a	b	c	τ
KK_P	0.09793	–1.3676	0.01378	0 < τ < 1
aK	–0.6473	0.1128	0.77	0 < τ < 0.25
	2.212	5.7	0.2163	0.25 < τ < 1
T_i/L	0.2967	–1.497	–0.01252	0 < τ < 1
T_i/T	5.479	0.8154	–0.03853	0 < τ < 0.1
	10.7	11.79	0.8028	0.1 < τ < 1

Table 1 gives the coefficients for functions of the form (9) fitted to the data available in Figures 1 and 2. The corresponding graphs are shown in solid lines in figures.

4.2 PI^{λ} controller

Simplified tuning rules for fractional PI controllers will be now obtained. Figures 3, 4, and 5 show the normalized proportional gains, the normalized integration times, and the controller fractional order, respectively, as a function of normalized time delay τ when the design procedure is applied to all processes in the test batch (2). The curves drawn correspond to the results obtained by curve fitting. Both figures show that there appears to be a good correlation between the normalized controller parameters and the normalized time delay τ. This indicates that it is possible to develop good tuning rules for fractional PI controllers based on the *KLT*-model.

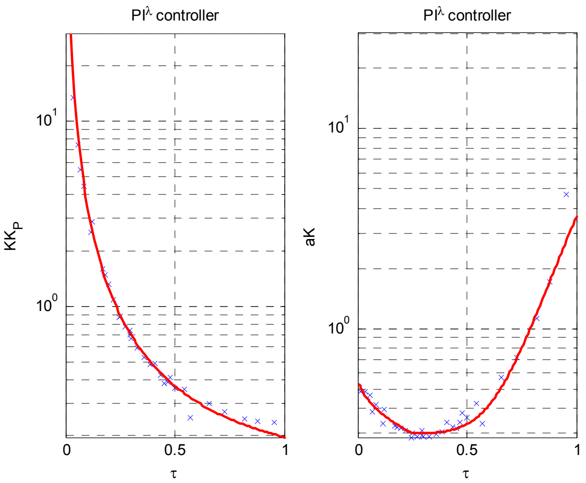

Fig. 3. Normalized PI^{λ} controller proportional gains plotted versus normalized time delay τ for the test batch. The solid lines correspond to the tuning rules obtained in Table 2.

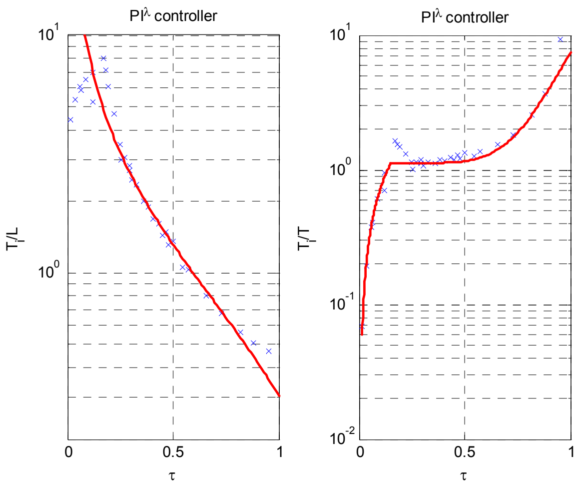

Fig. 4. Normalized PI^{λ} controller integration times plotted versus normalized time delay τ for the test batch. The solid lines correspond to the tuning rules obtained in Table 2.

As in the case of the PI controller, it is not possible to obtain good tuning rules that do not depend on τ. The deviations from the solid lines in the figure is about ± 15%. Table 2 gives the coefficients of functions of the form (9) fitted to the data available in Figures 3, 4, and 5. The corresponding graphs are shown in solid lines in these figures.

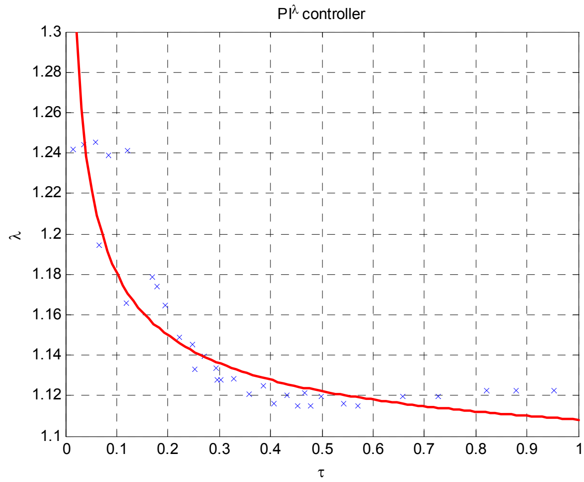

Fig. 5. Fractional order λ plotted versus normalized time delay τ for the test batch. The solid lines correspond to the tuning rules obtained in Table 2.

Table 2. Tuning formulae for the fractional PI^{λ}. The table gives the parameters of the functions of the form (9) for the normalized controller parameters and M_S = 1.4.

f(τ)	a	b	c	τ
KK_P	0.08621	–1.594	0.1096	0 < τ < 1
aK	–0.5643	0.2715	0.6866	0 < τ < 0.25
	3.327	6.593	0.2983	0.25 < τ < 1
T_i/L	1.17	–0.8997	–0.8666	0 < τ < 1
T_i/T	8.549	1.052	–0.04380	0 < τ < 0.15
	6.271	7.304	1.12	0.15 < τ < 1
λ	0.03512	–0.4862	1.073	0 < τ < 1

Figure 6 shows the ratio between the optimal J_v–values obtained with a PI^{λ} and PI controller applied to the processes in the test batch. It shows that the benefit in using a PI^{λ} instead a PI controller is more than 12% for delay–dominated processes, about 11% for balanced lag and delay processes, and tends to 18% for lag–dominated processes.

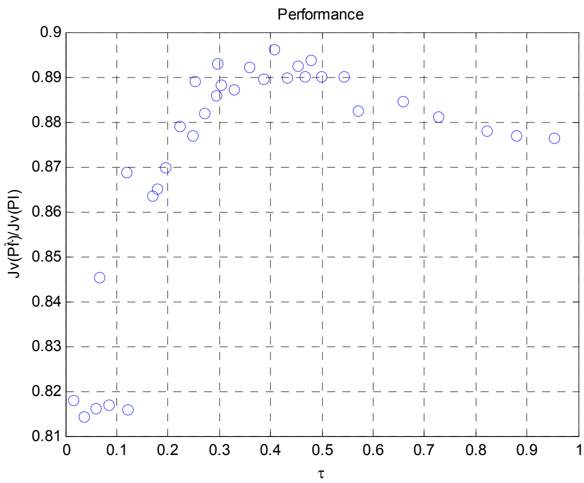

Fig. 6. Ratio of the J_v–values obtained for a PI^{λ} and a PI controller for different values of τ applied to the processes of the test batch.

4.3 *A simpler tuning rule for PI$^\lambda$ controllers*

As can be seen in Figure 5, optimal λ–value is approximately equal to 1.12 for $0.3 < \tau < 1$. Provided that the maximum difference between the optimal value of λ and 1.12 is, in the worst case, equal to 0.12, i.e. 10%, we will try to develop simple tuning rules for PI$^\lambda$ controllers, fixing the value of λ to 1.12. Figures 7 and 8 show the optimal normalized proportional gains and integration, for a constant value of $\lambda = 1.12$, as a function of the normalized time delay τ. The curves drawn correspond to the results obtained by curve fitting in Table 3.

Figure 9 shows the ratio between the optimal J_v–values obtained with a PI$^\lambda$ with all its parameters free and a PI$^\lambda$ with $\lambda = 1.12$ applied to the test batch. It shows that the J_v–values obtained in both cases are nearly the same for $0.3 < \tau < 1$, and the loss for lag–dominated processes increases but it is, in all cases, less than 5%.

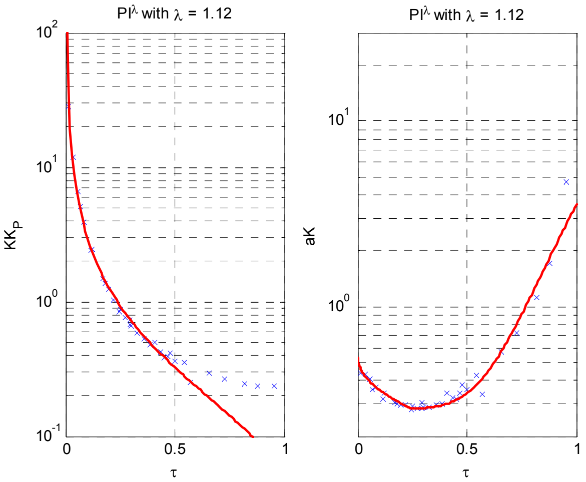

Fig. 7. Normalized PI$^\lambda$ controller proportional gains plotted versus normalized time delay τ for the test batch. The solid lines correspond to the tuning rules obtained in Table 3.

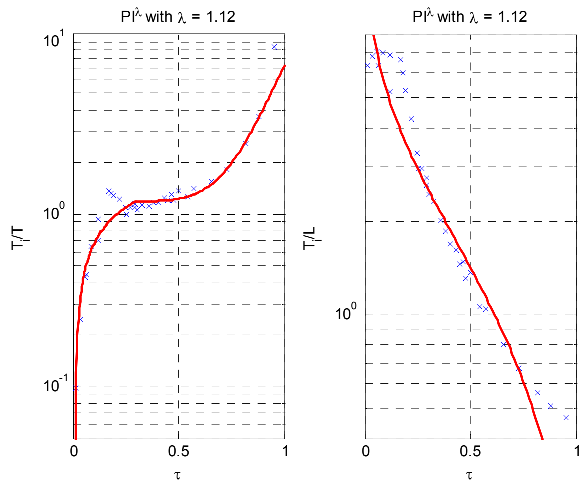

Fig. 8. Normalized PI$^\lambda$ controller integration times plotted versus normalized time delay τ for the test batch. The solid lines correspond to the tuning rules obtained in Table 3.

Table 3. Tuning formulae for fractional PI$^\lambda$ controllers. The table gives the parameters of the functions of the form (9) for the normalized controller parameters, $\lambda = 1.12$ and $M_S = 1.4$.

f(τ)	a	b	c	τ
KK_P	0.2154	–1.169	–0.1592	$0 < \tau < 1$
aK	–0.4645	0.3182	0.5795	$0 < \tau < 0.25$
	3.271	5.75	0.28	$0.25 < \tau < 1$
T_i/L	9.242	–0.1966	–9.171	$0 < \tau < 1$
T_i/T	5.479	0.8154	–0.03853	$0 < \tau < 0.3$
	6.06	7.066	1.18	$0.3 < \tau < 1$
λ	λ = 1.12			$0 < \tau < 1$

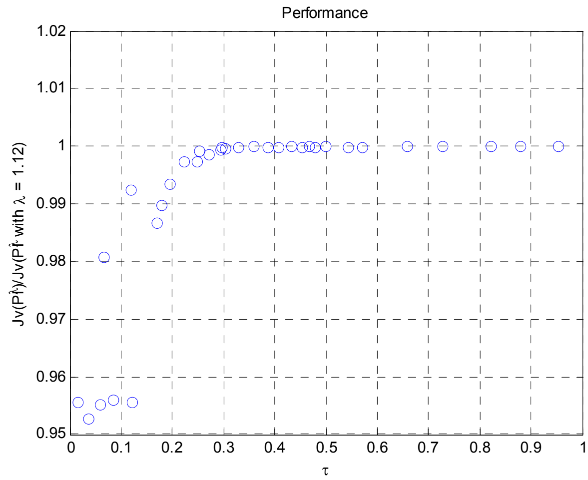

Fig. 9. Ratio of the J_v–values obtained for a PI$^\lambda$ and a PI$^\lambda$ with $\lambda = 1.12$ for different values of τ applied to the processes of the test batch.

5. COMPARISON WITH OTHER DESIGN METHODS

Extensive simulations have been done. Comparisons with classical tuning rules show that proposed tuning rules are very simple but give substantially better performance. However, due to page limitations, only one simulation has been included in this paper. There are many methods for tuning PI controllers. In this Section, the proposed methods for PI and PI$^\lambda$ controllers are compared with the Ziegler-Nichols step response method, Ziegler and Nichols (1942), the Cohen-Coon method, Cohen and Coon (1953), and optimal controllers in terms of J_v, IAE, J_u, and M_S.

For simplicity we will denote the Ziegler Nichols step response method by ZN, the Cohen-Coon method by CC, the optimal PI and PI$^\lambda$ controller obtained using the design method by opt-PI and opt-PI$^\lambda$, respectively, the proposed method for PI controllers by GK, the one for PI$^\lambda$ controllers by f-GK, and the approximation for PI$^\lambda$ controllers with $\lambda = 1.12$ by af-GK.

Consider the process with the following transfer function: $G(s) = 1/(1+s)(1+0.5s)$. We find that the apparent time delay and time constants are $L = 0.193$ and $T = 1.407$. Hence, the controllability index is $L/T = 0.1372$ and $\tau = 0.12$ for this process.

Table 4 contains the values of the different controller parameters obtained with the considered design methods. This table shows that results obtained by GK, f-GK and af-GK are very close to their respective optimal values. The performance obtained for PI^{λ} is substantially better than for PI. ZN and CC give controllers that reduce load disturbances very effectively, however they exhibit a very poor robustness and excessively large control effort.

Table 4. Controller parameters obtained for the different design methods for the considered transfer function.

Method	K	T_i	λ	k_i	M_S	J_u	J_v	IAE
ZN	6.56	0.58	1	11.34	3.11	22.59	0.12	0.16
CC	6.65	0.50	1	13.32	3.62	28.90	0.14	0.18
GK	1.78	1.13	1	1.58	1.38	2.65	0.63	0.63
opt-PI	2.04	1.21	1	1.68	1.40	3.02	0.59	0.59
f-GK	2.62	1.24	1.17	2.12	1.41	3.58	0.45	0.58
opt-PI^{λ}	2.86	1.34	1.24	2.13	1.40	3.79	0.46	0.61
af-GK	2.40	1.32	1.12	1.82	1.39	3.28	0.50	0.60

Parameters obtained by f-GK and optimal PI^{λ} are very close which indicates that little is lost by not using the full transfer function. The improvement in J_v of using a PI^{λ} instead of a PI is about 22%. The value of J_v for GK is about 28% higher compared with f-GK. These improvements are also evaluated in Gude and Kahoraho (2009).

6. CONCLUSIONS

This paper presents new tuning rules for PI and fractional PI control of typical processes found in process control. The rules are based on characterization of the process dynamics by three parameters, i.e. gain K_P, apparent time constant T and apparent time delay L, that can be obtained by a simple step response experiment. The design method consists on minimizing a frequency objective function subject to a constraint on the maximum sensitivity function. Based on these parameters it is possible to develop very simple tuning rules for PI and PI^{λ} controllers that only depend on the normalized time delay τ.

In this paper it is also demonstrated that substantially better performance can be obtained using PI^{λ} instead of PI controllers. These tuning rules are shown to give good results compared to a couple of well established classical tuning methods, especially when simplicity, performance and robustness are emphasized.

Future investigation should rely on extending these tuning rules to fractional PID controllers.

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REFERENCES

Åström, K.J. and Hägglund, T. (1995). *PID Controllers: Theory, Design, and Tuning.* Instrument Society of America. Research Triangle Park, NC.

Åström, K.J. and Hägglund, T. (2000). Benchmark systems for PID control. *Proceedings of IFAC workshop on Digital Control*, pp. 165-166. Terrassa, Spain.

Åström, K.J. and Hägglund, T. (2001). The future of PID control. *Control Engineering Practice*, Vol. 9 (11), pp. 1163-1175.

Åström, K.J. and Hägglund, T. (2004). Revisiting the Ziegler-Nichols step response method for PID control. *Journal of Process Control*, Vol. 14(6), pp. 635-650.

Capponetto, R., Fortuna, L., and Porto, D. (2002). Parameter tuning of a non integer order PID controller. *Proceedings of 15th Int. Symp. on Mathematical Theory of Networks and Systems*. Notre Dame, IN.

Chien, K.L., Hrones, J.A. and Reswick, J.B. (1952) On the automatic control of generalized passive systems, *Transactions ASME*, Vol. 74, pp. 175-185.

Cohen, G.H. and Coon, G.A. (1953). Theoretical consideration of retarded control, *Transactions ASME*, Vol. 75, pp. 827-834.

Gorez, R. (2003). New design relations for 2-DOF PID-like control systems. *Automatica*, Vol. 39 (5), pp. 901-908.

Gude, J.J. and Kahoraho, E. (2007). PID control: Current status and alternatives. *In 2nd Seminar for Advanced Industrial Control Applications – SAICA 2007*, pp. 247-253. UNED Ediciones.

Gude, J.J. and Kahoraho, E. (2009). Performance comparison between PI(D) and fractional PI controllers. *In IFAC Workshop on Control Applications and Optimization – CAO'09*. University of Jyvaskyla, Finland.

Hang, C.C., Åström, K.J., and Hägglund, T. (1991). Refinements of the Ziegler-Nichols tuning formula. *IEE Proc., Part D,* Vol. 138 (2), pp. 111-118.

Kristiansson, B. and Lennartson, B. (2002). Robust and optimal tuning of PI and PID controllers. *IEE Proc., Part D*, Vol. 149 (1), pp. 17-25.

Monje, C.A., Vinagre, B.M., Chen, Y.Q., Feliu, V., Lanusse, P., and Sabatier, J. (2005). Optimal tunings for fractional $PI^{\lambda}D^{\mu}$–controllers. *Fractional Differentiation and its Applications*. Le Mehauté, A., Tenreiro Machado, J.A., Trigeassou, J.C., and Sabatier, J. (eds.) Ubooks Verlag, Augsburg, pp. 675-686.

Podlubny, I. (1999). Fractional-order systems and $PI^{\lambda}D^{\mu}$-controllers. *IEEE Trans. Autom. Control*. Vol. 44 (1), pp. 208-214.

Shinskey, F.G. (1996). *Process Control Systems. Application, design, and tuning*. 4th edition. McGraw-Hill

Vinagre, B.M., Podlubny, I., Dorcak, L. and Feliu, V. (2000). On fractional PID controllers: A frequency domain approach. *Proceedings of IFAC workshop on Digital Control,* pp. 51-56. Terrassa, Spain.

Ziegler, J.G. and Nichols N.B. (1942). Optimum settings for automatic controllers. *Transactions ASME,* Vol. 64, pp. 759-768.