Control Loop Performance Monitoring using the Permutation Entropy of Error Residuals

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Abstract: The predictability of a control-loop behavior beyond its control horizon is an unambiguous indication of loop malfunctioning. Based on the dynamic complexity of the error residual time series the permutation entropy is proposed to define a sensitive index for performance monitoring using data from close-loop operation. A generic framework to understand and quantify the distinctive increase in predictability of the controller error resulting from ill-tuning, sensor errors and actuator faults using a entropy-like index is presented. The dynamic complexity of a well-performing control loop should correspond to the maximum entropy. As loop performance degrades the entropy of its residual time series decreases and any loss of dynamic complexity in the control system gives rise to an increase of the predictability of the control error time series. Results obtained using the proposed performance index along with its confidence interval for industrial data sets are presented to discuss the influence of the sample size, control horizon, and variance estimation in the assessment of close-loop performance.

Keywords: Control-loop performance; Monitoring; Permutation entropy; Fault detection and diagnosis; Error predictability; Ordinal patterns; Time-series analysis.

1. INTRODUCTION

Control loops implementing a hierarchy of functions for process regulation and optimization are the cornerstone of safety and economy in process plants (Thornhill et al., 1999). Many loops are just PID controllers whilst other may be more advanced ones, such as inferential loops, MPCs and real-time optimizers working on top of the regulation layer. It is well known that in most industrial environments the behavior of control loops deteriorate with time due to a number of reasons, e.g. plant-wide perturbations, fouling, utility constraints and raw material variability. Accordingly, process dynamic characteristics change over time and, if not properly maintained, most control loops will perform poorly after some time, which can lead to degraded process operation. In particular, ill-functioning of the regulation layer can easily cancel the benefits of advanced control systems and real-time optimization (Jelali, 2006; AlGhazzawi and Lennox, 2009). With the increasing complexity of control structures and the sheer number of controllers in modern process plants, the automation of performance monitoring tasks is mandatory.

Systematic assessment of SISO control loops can be traced back to the seminal work of Harris (1989) who related the performance of a single-loop control system to the controller errors of a minimum variance controller. The latter, even though it is rather impractical to be implemented, serves as a performance benchmark to provide a lower bound for the variance of the controlled variable. On this basis, the well-known Harris index is defined as the ratio of the variance achievable using minimum variance controller to the variance measured under the current control law (Desborough and Harris, 1992, 1993). As the value of this statistic is reduced then so too does the measured performance of the control system. The key advantage of the Harris approach to control loop monitoring is that only routine close-loop operating data are required to determine the performance of the control system. This fact has made the approach very attractive to industry and it is now applied as a matter of routine by many companies. However, a disadvantage of the Harris index is that it is based on a rather extreme (in terms of cost and energy involved) behaviour and no hints are provided for characterizing the behaviour a well-performing realistic controller based on the control task for which it was designed. Also, it is difficult to pinpoint an informative threshold for the Harris index to differentiate between normal and faulty operation of a control loop.

Based on the insightful concept of control horizon, Thornhill *et al.* (1999) proposed the predictability of the error time series to characterize the performance of a SISO controller. The predictability of a control-loop behaviour beyond its control horizon is an unambiguous indication of loop malfunctioning in biological systems (Li, Ouyang and Richards, 2007). Along this research avenue, Ghraizi *et al*. (2007), proposed a practical index for performance monitoring of a control loop based on the analysis of the predictability of the error time series and emphasizes proper selection of the control horizon using engineering judgment and the amplitude and frequency of disturbances to which the loop is designed for. To develop ideas further, Martínez and de Prada (2007) resort to ordinal analysis methods of the error time series to define a performance index for performance monitoring based on the permutation entropy.

In this work, the interplay between predictability of controller behaviour and its dynamic complexity for performance monitoring is highlighted by resorting to the residual error time series, which is obtained using a regression model. A generic framework to understand and quantify the distinctive increase in predictability of the controller error resulting from ill-tuning, sensor errors and actuator faults using an entropy-like index is proposed. A well-performing controller should behave so that the sequence of residuals in the error series looks like one generated using *i.i.d.* samples from a random walk and the corresponding dynamic complexity is thus maximum. Accordingly, ordinal patterns in the error residuals will all be equally probable and the corresponding permutation entropy will be then the highest possible.

2. MONITORING METHODOLOGY

2.1 *Predictability analysis*

The performance-monitoring concept revolves around the idea of predictability of controller behaviour beyond a chosen horizon *b*. If a control loop exhibits “good” performance, it should be able to cancel any disturbance entering the loop up to present time *t*, or follow a set point change correctly, after some sensible time interval *b* (expressed in terms of sampling periods). Then, it can be argue that, as from time *t+b* onwards, the error time series cannot be distinguished from a random walk stochastic process so that it cannot be predicted at all using information up to time instant *t* (see Fig. 1 for details). Nevertheless, over the control horizon *b*, the controller behaviour is fully predictable since it corresponds to its own control policy built-in by design. By contrast, error time series of a control loop exhibiting “poor” performance, will show patterns of behaviour (oscillations, steady-state errors, etc.) which can be predicted after time instant *t+b* using present and past measurements.

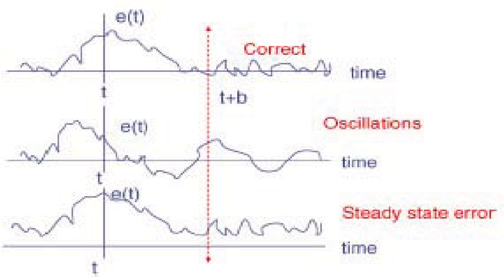

Fig. 1. Error patterns and their predictability

The most sensitive approach to detect patterns of predictability in the time series is analyzing the time series of error residuals $r(t)$ which are obtained using an inductive model to predict future errors.

Let’s denote by $e(t)$ the controller error defined as

$$e(t) = \omega(t) - y(t) \tag{1}$$

Where $\omega(t)$ stands for the desired set-point at any time t, and $\hat{e}(t)$ stands for the prediction of such error based on past values of the controller error. The difference between the actual and predicted controller errors is the residue *r(t)* whose time series has a dynamic complexity closely related to the predictability patterns in the controller error time series

$$r(t) = e(t) - \hat{e}(t) \tag{2}$$

The error prediction $\hat{e}(t)$ can be obtained in different ways, but the easiest alternative is using the regression model

$$\begin{aligned}\hat{e}(t+b) = e_o + e_1(t) + e_2(t-2) + e_3(t-3)....\\ + e_m(t-m+1)\end{aligned} \tag{3}$$

Where time indices refer to sampling periods, *m* is the model order and a_i is the unknown parameters, which are fitted using a dataset of size *n* by means of the least-square regression:

$$[a_0, a_1, ..., a_m]^T = (X^T X)^{-1} X^T Y \tag{4}$$

Where

$$X = \begin{bmatrix} 1 & e(1) & e(2) & \cdots & e(m) \\ 1 & e(2) & e(3) & \dots & e(m+1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & e(n-b-m+1) & \cdots & \cdots & e(n-b) \end{bmatrix} \tag{5}$$

And

$$Y = \left[e(m+b) \;\; e(m+b+1) \cdots e(n) \right]^T \tag{6}$$

It is worth noting that for a well-performing controller in a given time interval the sequence of error residuals is a chaotic, completely random, and non-stationary stochastic process exhibiting maximum dynamic complexity. The reader is referred to the work of Peng et al. (2009) for an interesting discussion on the meaning of regularity and dynamic complexity in physiologic time series from highly controlled biological systems. To quantify the dynamic complexity of residuals there are several options.

In a previous work, the authors, Ghraizi *et al*. (2007), used a performance index based on the ratio between the variance of the residuals and the variance of the errors:

$$PI = \frac{\sigma_r^2}{\sigma_e^2} \tag{7}$$

Assuming that in a perfectly predictable loop the variance of

the residual would be zero, while non-predictable random walk would give a variance similar to the loop error, this expression would provide an index ranging from zero to one that will measure the performance of the controller. In order to obtain a confidence interval of the index the following analysis can be performed:
It is known that the following ratio

$$\frac{(n-1)\hat{\sigma}^2}{\sigma^2} \tag{8}$$

between the estimated and real variance for an stochastic process must follow a χ^2 distribution with n-1 degrees of freedom. Applying this line of reasoning to the residuals and the controller errors and dividing them, we can obtain:

$$\frac{\sigma_r^2 \hat{\sigma}_e^2}{\sigma_e^2 \hat{\sigma}_r^2} \tag{9}$$

Which will follow a F-distribution with n-1,n-1 degrees of freedom. Hence:

$$\begin{gathered} P(F_{1-0.5\alpha,n-1} \le \frac{\hat{\sigma}_e^2 \sigma_r^2}{\sigma_e^2 \hat{\sigma}_r^2} \le F_{0.5\alpha,n-1}) = 1-\alpha \\ P(\frac{\hat{\sigma}_r^2}{\hat{\sigma}_e^2} F_{0.5\alpha,n-1}^{-1} \le \frac{\sigma_r^2}{\sigma_e^2} \le \frac{\hat{\sigma}_r^2}{\hat{\sigma}_e^2} F_{0.5\alpha,n-1}) = 1-\alpha \end{gathered} \tag{10}$$

Will can be used to compute the 100(1-α) % interval of confidence for the *PI* index defined in (7).

In practice, when this index is computed, large confidence interval appears sometimes, mainly when loop performance degrades, which reduces the interest in the above method. An alternative not based on statistical assumptions, which are always difficult to verify, would be desirable. In this regard, an appealing and sound choice is resorting to an entropy-like index based on ordinal patterns of the residual time series.

2.2 *Residual order patterns*

The complexity of a residual time series can be quantified by means of its symbolic dynamics. A new permutation method was proposed by (Bandt and Pompe, 2002; Bandt, 2005) to map a continuous time series onto a symbolic sequence; the statistics descriptive of the dynamic complexity of the symbolic time series is called permutation entropy. Given a data set for the scalar residual time series $r(t), t = 1,....,n$, the local order of the series can be characterized by patterns in vectors $\Re(t)$ ensembled as follows

$$\Re(t) = [r(t), r(t+\ell),...r(t+(\kappa-1)\ell))] \tag{11}$$

where κ is the embedded dimension parameter and ℓ is the lag parameter (here $\ell = 1$). Then entries in each $\Re(t)$ are arranged in increasing order which allows assigning to it one out of the possible order patterns. For κ different numbers, there will be $\kappa!$ possible order patterns π, which are also called permutations. In Fig. 2 the six order patterns for $\kappa = 3$ are shown. Let $f(\pi)$ denote the frequency of permutation π in the data set whereas $\rho(\pi) = f(\pi)/(n-(\kappa-1)\ell)$ is the relative frequency. For a perfectly working controller the relative frequencies should all be close to $1/\kappa!$

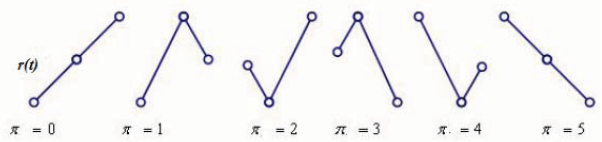

Fig. 2. Ordinal patterns in $\Re(t)$ *for* $\kappa = 3$

2.3 *Permutation entropy and performance monitoring*

The local permutation entropy of order κ for the error residual time series is defined as

$$H_\kappa = -\sum_{\pi=1}^{\kappa!} \rho(\pi) \ln \rho(\pi) \tag{12}$$

The largest possible value for the permutation entropy will correspond to the perfectly working controller where all $\kappa!$ permutations are equally probable which coincides with a residual time series of maximum complexity where the permutation entropy is ln $\kappa!$

Permutation entropy depends on the selection of κ. When κ is too small a value (say less than 3), the scheme will not work, since there are only very few distinct states for characterizing the control system behavior. For too large values of κ (greater than 6), the number $\kappa!$ of permutations which can appear in the time series can result in computer memory problems, due to the large number of data points that need to be examined. In the present work, only values of $\kappa = 3$, 4 or 5 will be used.

For loop monitoring, the permutation entropy of the residual time series is obtained from a sample where the total number of patterns counted is n and the tally number for the *ith* pattern in the sample is denoted by f_i

$$\hat{H}_\kappa^n = -\sum_{i=1}^{\kappa!} \left(\frac{f_i}{n} \ln \frac{f_i}{n} \right) \tag{13}$$

The corresponding variance for this sample estimation of the permutation entropy is (see Moddemeijer, 1989, for details)

$$Var\left(\hat{H}_\kappa^n\right) = \frac{1}{n}\left(\sum_{i=1}^{\kappa!} \frac{f_i}{n} \ln^2 \frac{f_i}{n} - \left(\frac{f_i}{n} \ln \frac{f_i}{n} \right)^2 \right) \tag{14}$$

Based on equations (13) and (14) and a sample of size n, the following performance index is proposed

$$PI = \frac{\hat{H}_\kappa^n}{\ln \kappa!}, \kappa = 3,4,... \tag{15}$$

Since $\ln \kappa!$ is a constant, the variance for the sample estimation of the performance index can be written as

$$Var(PI) = \frac{Var\left(\hat{H}_\kappa^n\right)}{\ln \kappa!}, \kappa = 3,4,... \tag{16}$$

The highest value of *PI* is one, which means the error residual time series is complete random and is its dynamics is very complex; the smallest possible value of *PI* is zero, which means the error residual time series is highly regular.

Eq. (14) is very important for the following reasons: it is possible to make a very reliable characterization of the variance of a sample-based estimation of the performance index PI in Eq. (15). For a well-performing control loop, since the probability for ordinal patterns are all the same, after elementary algebra steps in Eq. (14) an *exact* measure of the variance for the performance index is obtained

$$\frac{\left[\ln\left(\frac{1}{\kappa!}\right)\right]^2(\kappa!-1)}{\ln\kappa!\; n\; \kappa!} \tag{17}$$

As can be readily calculated, this variance for a properly working loop is very small even for small sample sizes. For example, for n=1000 and κ=3, the variance of PI is $\cong$0.00149 for a perfectly working controller. For a much smaller sample size such n=100 is still rather small ($\cong$0.0149).

To compute a 100(1-α) % confidence interval for the PI index estimated through Eq. (15), the Student's t-distribution is assumed so that the sample variance in Eq. (16) is used to define upper/lower limits in the usual way as follows

$$\pm t_{1-\alpha/2,n-1}\sqrt{\frac{Var(PI)}{n}} \tag{18}$$

where α defines the chosen level of confidence.

3. RESULTS

In order to show the applicability of the proposed method, several data sets from an industrial site have been considered. They correspond to routine plant data of a set of typical control loops for pressure, flow, etc. The first one is displayed in Fig.3 and contains 9000 samples of the controlled and manipulated variables as well as the set point of a pressure control loop.

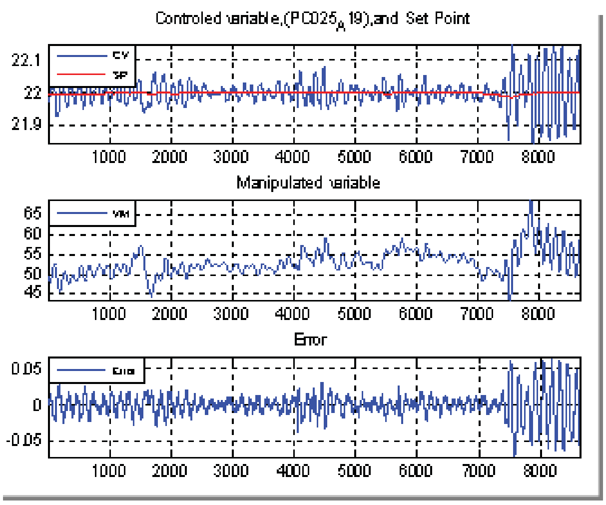

Fig.3 Data from a pressure control loop. Upper graph: Set point (in red) and gauge pressure. Middle graph: control valve signal. Bottom graph: error between set point and pressure readings.

Despite the fact that pressure readings stand close to the set point, the loop experiments an oscillatory behavior, likely as result of a too tight tuning. This can be better observed in a close-up to the data, as the one shown in Fig.4.

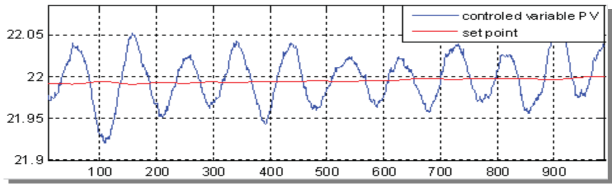

Fig.4. A detailed view of the first 1000 data of the time evolution of pressure and its set point for the example data set shown in Fig.3.

Results obtained from the application of the proposed method to pressure loop are discussed next. Fig.5. displays the actual error and its predictions computed from the expression (3). As can be seen, predictatility is significantly high, as one could expect in a badly tuned control loop whereas residuals are small. The residual time series is displayed in the upper part of Fig. 6, exhibiting a certain regularity, whereas in the bottom part the values of the proposed performance index (15) computed at regular time intervals every 120 data are shown for the case $\kappa = 3$ and b = 12.

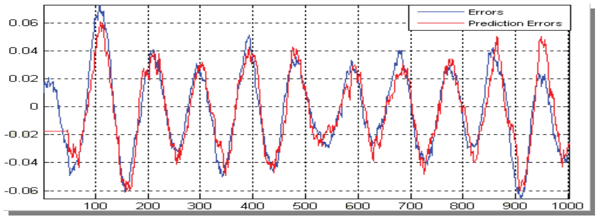

Fig.5. A detailed view of the errors and its predictions (in red) for the first 1000 data using prediction formula (3).

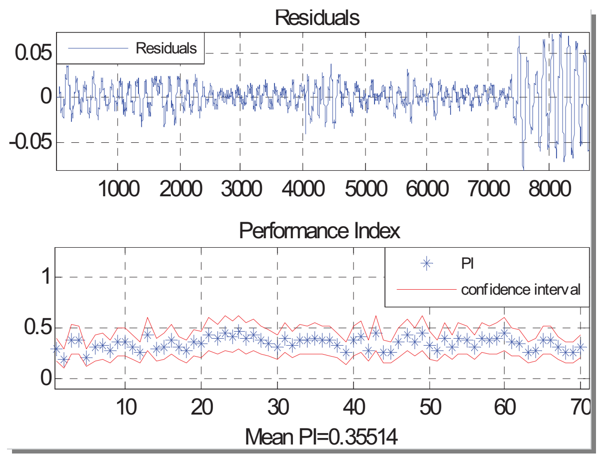

Fig.6. Upper graph: Residuals of the predictions computed with expression (2) for the example of Fig.3. Lower graph: Performance index computed from (16).

As can be seen, in this example, sample estimation the performance index have an average value of 0.35, which is an indication of poor performance of the pressure loop. For the sake of comparison, the PI computed from (7) is displayed in Fig.7. Even though consistent results are obtained, but the confident limits rise up to 1 which creates a great deal of uncertainty in the estimation of this performance index.

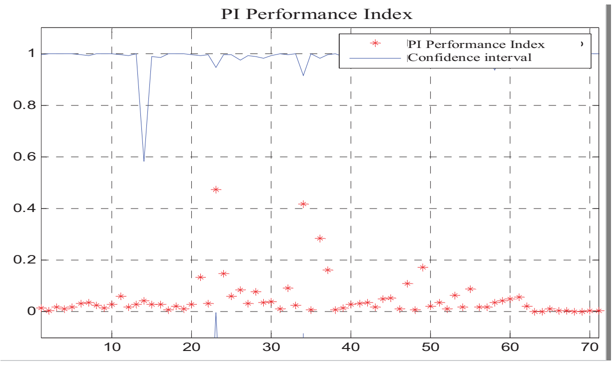

Fig.7. Performance index (in red) computing with expression (7) for the example of Fig.3. Blue line gives the upper confidence limit based on Eqs. (9) and (10).

Data from a second case study are given in Fig. 8. This time data correspond to a flow control loop that is the slave in a cascade configuration. 16,000 data poins are collected and, as can be seen, the loop behaviour is quite good, with fine set point tracking and moderate control signal changes, except in the range for data poins from 12,000 to 13,000, where extreme values of the set point from the master loop lead to saturation of the manipulated variable.

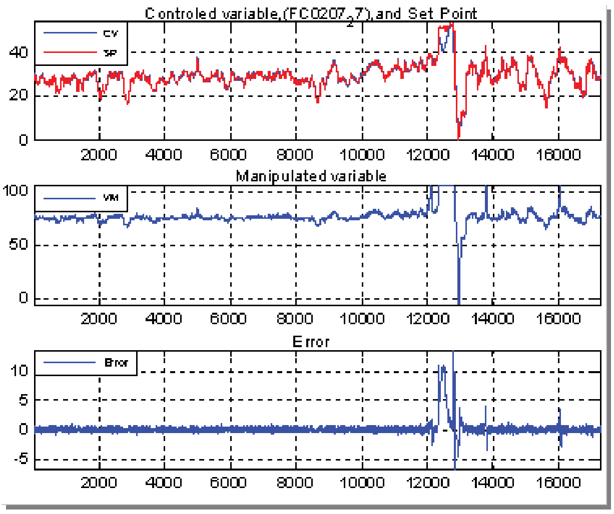

Fig.8. Data from a slave flow control loop. Upper graph: Set point (in red) and flow readins. Middle graph: control valve signal. Bottom graph: error between set point and actual flow.

As can be seen in the close-up displayed in Fig.9, there is a good following of the set point and the predictions of the errors computed with the expression (3) differ from the actual errors, so that the residuals (2), displayed in Fig.10, approach to a random walk, as expected in a loop with good behaviour.

The performance index (15) has been computed using the values $\kappa = 3$ and $b = 12$, at regular time intervals that include 120 data points. The numerical values displayed on the bottom of Fig.10 give regular values around 0.82 that is a reasonable value for this entropy-like index when all patterns are almost equiprobable.

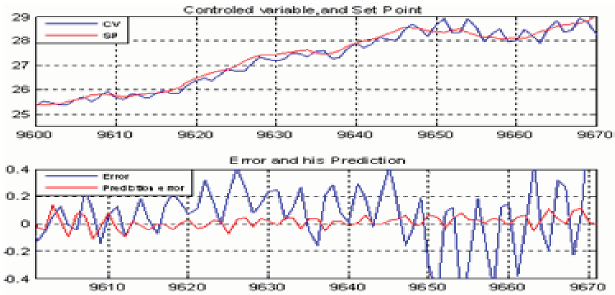

Fig.9. Upper graph: A close-up of data of the time evolution of flow and its set point (in red) for the example of Fig.8. Lower graph: A detailed view of the errors and its predictions (in red) for this range of data using prediction formula (3).

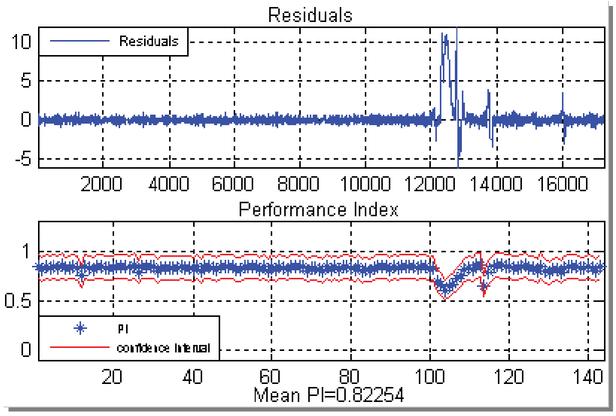

Fig.10. Upper graph: Residuals of the predictions computed with expression (2) for the example of Fig.8. Lower graph: Performance index computed from (16).

Nevertheless, in a set of intervals from data 12,000 on, where the manipulated variable is saturated, the predictions of the errors can be made very well, as can be observed in Fig.11, where the errors and its predictions computed with formula (3) are given.

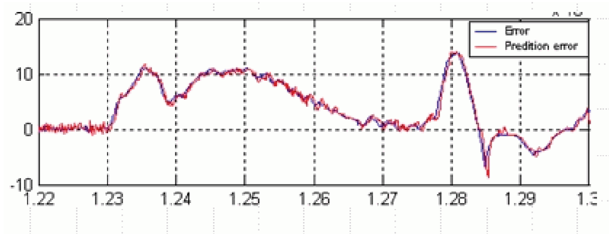

Fig.11. A detailed view of the errors and its predictions (in red) for a range of data above sample 12,000 of the example of Fig.8, using prediction formula (3).

In this case, the residuals are small and do not follow a random walk stochastic process, as can be seen in the corresponding range of variation shown in Fig.10. Accordingly, the *PI* decreases, indicating performance degradation of the loop in this portion of the data set.

For comparison, the *PI* computed with expression (7) and blocks of about 1000 data is given in Fig.12. As we can see, the index is between 0.75 and 0.9 giving consistent indications about the goodness of the loop behaviour, but at the time intervals where the saturation occurs it drops to 0.1, 0.3 as expected. Nevertheless, the confidence bands increases at this precise times, decreasing the certitude of the diagnosis.

A final test was made to compare previous results with the ones provided by the Harris index for this case study, which are displayed in Fig.13. The values for the Harris index range from 0.3 to 0.5 for most samples, indicating that a margin for improvement exists in relation to the best possible linear controller -the minimum variance one- but it is worth noting that this index does not provide a direct measurement of the loop performance. Notice also that the index drops to 0 and 0.1 in the critical range when the saturation of the manipulated variable occurs.

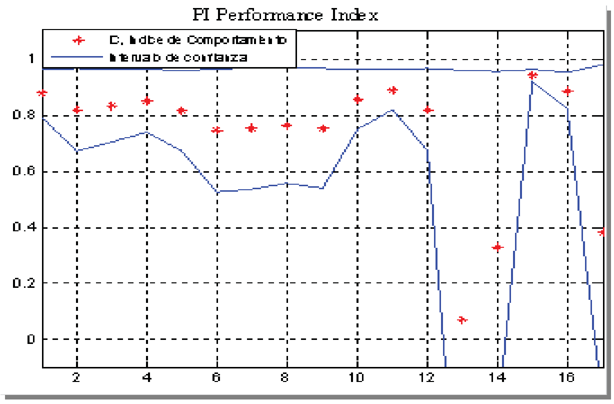

Fig. 12. Performance index (in red) computed with expression (7) for the example of Fig.8. Blue line gives the upper confidence limit.

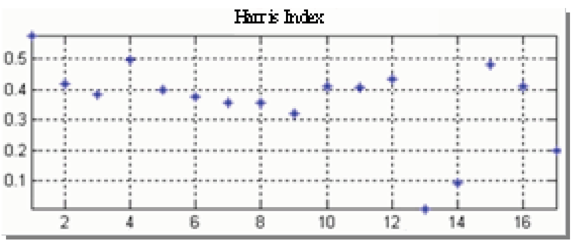

Fig. 13. Sample estimation of the Harris index for the example in Fig. 8 computed from batches of 1,000 data points.

4. CONCLUSIONS

A new index for performance assessment of control loops using normal operating plant data has been proposed. It combines the idea of predictability of the controller error at a point in time beyond the desired settling time of the loop, with an analysis of the corresponding sequence of prediction residuals based on ordinal methods along with the concept of local permutation entropy. The main advantage of the Performance Index defined in this way is the fact that no statistical assumptions are made on the residuals, which allows for a crisp interpretation of sample estimation of the performance index. Moreover, the entropy-like index is easy to compute and can be applied to single isolated loops as well as to cascades or other control configurations including model predictive controllers. For industrial data sets, the proposed *PI* has provides consistent results.

To highlight several advantages some comparisons were made with other indices such the Harris index and a previous *PI* based on the error predictability idea. The proposed method based on the permutation entropy can be applied in real time with minimum computational costs which opens the possibility of automatic supervision of hundreds of control loops of a typical process plant. Also, linking information content with predictability of error residuals is a novel idea for loop monitoring using dynamic complexity.

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