Data-driven Control Loop Diagnosis: Dealing with Temporal Dependency in Bayesian Methods

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Abstract: Conventional Bayesian methods commonly assume that the evidences are temporally independent. This condition does not hold for most practical engineering problems. With evidence transition information being considered, the temporal domain information can be synthesized within the Bayesian framework to improve the diagnosis performance. A data-driven algorithm is developed to estimate the evidence transition probabilities. The application in a pilot scale process is presented to demonstrate the data dependency handling ability of the proposed approach.

Keywords: Performance monitoring, Performance assessment, Bayesian diagnosis, Evidence dependency

1. INTRODUCTION

Control loop performance assessment and diagnosis has been an active area of research in the process control community. A number of control performance methods are available, including the ones based on minimum variance control (MVC), linear quadratic Gaussian control (LQG), historical data trajectories, and user-specified control, etc (Huang and Shah, 1999; Harris et al., 1999; Qin, 1998; Jelali, 2006; Schafer and Cinar, 2004; Patwardhan and Shah, 2002). Several surveys on the control performance assessment research are available (Harris et al., 1999; Qin, 1998; Hoo et al., 2003; Hugo, 2006; Jelali, 2006). Besides performance evaluation of control loops, significant progress has also been made in the development of monitoring algorithms for process and instrument components within the control loops, such as sensor monitor, valve stiction monitor, process model validation monitor (Qin and Li, 2001; Ahmed et al., 2009; Choudhury et al., 2008; Mehranbod et al., 2005). A number of successful industry applications of the process monitors have been reported. However, many practical problems remain. One of the outstanding problems is that the monitoring algorithms are often designed for one specific problem. An implicit assumption that other unattended components are in good shape is made. Clearly this assumption does not always hold, and thus it may lead to misleading results. It is desirable to develop approaches that not only monitor the performances of single components, but also are capable of synthesizing the information from different monitor outputs to isolate underlying source of problematic control performance.

According to Huang (2008), several challenging issues exist for the process monitor synthesizing problem. The first one is the similar symptoms among different problem sources. For instance, oscillations can either be invoked by a sticky valve or an improperly tuned controller. Another problem is that no process monitor has 100% detection rate and 0% false alarm rate, and thus a probabilistic framework should be built to represent the uncertainties. Third, a large number of the developed monitoring algorithms are purely data based without any a priori process information. Incorporating a priori process knowledge into the diagnosis framework is challenging, but better diagnosis performance can be expected by doing so.

The Bayesian method sheds lights on the problem solutions by providing a probabilistic information synthesizing framework. Applications of the Bayesian methods have been reported in medical science, image processing, target recognition, pattern matching, information retrieval, reliability analysis, and engineering diagnosis (Dey and Stori, 2005; Mehranbod et al., 2005; Steinder and Sethi, 2004; Chien et al., 2002). It is one of the most widely applied techniques in probabilistic inferencing. Built upon previous work in Bayesian fault diagnosis by Pernestal (2007) and a framework laid out by Huang (2008), Qi and Huang (2008) developed a data-driven Bayesian algorithm for control loop diagnosis with consideration of missing data. The algorithm is tested through simulation, where the information synthesizing ability of the proposed approach is demonstrated. However, the existing Bayesian methods have not considered temporal dependency problem. In this paper, a new algorithm is developed with consideration of temporal dependency, so as to achieve more reliable and better diagnosis performance.

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The remainder of this paper is organized as follows. In Section 2, the control loop diagnosis problem and related preliminaries are described, and the data-driven Bayesian approach developed in (Qi and Huang, 2008) is briefly revisited. The rationale to consider evidence temporal dependency is detailed in Section 3. The estimation algorithm for the evidence transition probability is developed in Section 4. Section 5 presents application of the proposed approach to a pilot scale process. Finally the Section 6 concludes this paper.

2. DATA-DRIVEN BAYESIAN DIAGNOSIS
METHOD REVISIT

2.1 Control Loop Diagnosis Problem

Generally a control loop consists of the following components: controller, actuator, process, and sensor. These components may all suffer from malfunctions. In this work, monitors are assumed to be available for some or all of the components of interest. These monitors, however, are all subject to disturbances and thus can produce false alarms, and each monitor can be sensitive to abnormalities of other problem sources. Our target is to pinpoint the source of problematic control performance based on the collected monitor output data.

To adopt the Bayesian method for control loop diagnosis, several notations need to be introduced (Qi and Huang, 2008).

Mode $M$ Assume that a control loop under diagnosis consists of $P$ components of interest: $C_1, C_2, \ldots, C_P$, among which the problem source may lie in. Each component is said to have a set of discrete operating status. For instance, the sensor might be “biased” or “unbiased”. An assignment of operating status to all the components of interest in the control loop is called a mode, and denoted as $M$; $M$ can take different values and a specific value is denoted by $m$. For example, $m=(C_1=well\ tuned\ controller, C_2=value\ with\ stiction, \ldots)$. Suppose that component $C_i$ has $q_i$ different status. Then the total number of possible modes is

$$Q = \prod_{i=1}^{P} q_i,$$

and the set of all possible modes can be denoted as

$$\mathcal{M} = \{m_1,m_2,\ldots,m_Q\}.$$

Evidence $E$ The monitor readings, called evidence, are the input to the diagnostic system, and are denoted as $E = (\pi_1,\pi_2,\ldots,\pi_L)$, where $\pi_i$ is the output of the $i$-th monitor, and $L$ is the total number of the monitors. Often the continuous monitor readings are discretized according to predefined thresholds. In this work, monitor readings all take discrete values. For example, the control performance monitor may indicate “optimal”, “normal”, or “poor”. A specific value of evidence $E$ is denoted as $e$; for example, $e= (\pi_1=optimal\ control\ performance, \pi_2= no\ sensor\ bias, \ldots)$. Suppose that the single monitor output $\pi_i$ has $k_i$ different discrete values. Then there are totally

$$K = \prod_{i=1}^{L} k_i$$

different evidences, and the set of all possible evidence values can be denoted as

$$\mathcal{E} = \{e_1,e_2,\ldots,e_K\}.$$

Historical evidence data set $\mathcal{D}$ In this paper, process data refer to the readings from physical instruments such as temperature, pressure, etc. The evidence data refer to the readings of monitors which are calculated typically from a section (window) of process data. Historical evidence data are retrieved from the past record where the mode of the control loop, namely, status of the components of interest in the control loop, is available, and the monitor readings are also recorded. Each sample $d^t$ at time $t$ in the historical evidence data set $\mathcal{D}$ consists of the evidence $E^t$ and the underlying mode $M^t$. This can be denoted as $d^t=(E^t,M^t)$, and the set of historical evidence data is denoted as

$$\mathcal{D} = \{d^1,d^2,\ldots,d^N\},$$

where $N$ is the number of historical evidence data samples. In (Qi and Huang, 2008), all the historical evidence data samples are assumed to be independent as commonly assumed in the data-driven Bayesian approaches.

2.2 Data-driven Bayesian Diagnosis Approach

This section will give a brief review of the data-driven Bayesian approach proposed by Qi and Huang (2008). Given current evidence $E$, historical evidence data set $\mathcal{D}$, the posterior probability of each possible operating mode can be calculated according to Bayes’ rule:

$$p(M|E,\mathcal{D}) \propto p(E|M,\mathcal{D})p(M),\quad (1)$$

where $p(E|M,\mathcal{D})$ is the likelihood probability; $p(M)$ is the prior probability of mode $M$. Among all the possible modes, generally the one with the largest posterior probability is taken as the underlying mode based on the maximum a posterior (MAP) principle, and the abnormality associated with this mode is normally diagnosed as the problem source.

Since prior probabilities are determined by a priori information, the main task of building a Bayesian diagnostic system is the estimation of the likelihood probabilities with historical evidence data $\mathcal{D}$. In (Qi and Huang, 2008), a data-driven Bayesian algorithm for estimation of the likelihood probability is proposed based on the work by Pernestal (2007) and Huang (2008).

Suppose that the likelihood of evidence $E = e_i$ under mode $M = m_j$ is to be calculated, where

$$e_i \in \mathcal{E} = \{e_1,\ldots,e_L\},$$

and

$$m_j \in \mathcal{M} = \{m_1,\ldots,m_Q\}.$$

The following result can be obtained for calculating the likelihood (Pernestal, 2007):

$$p(E=e_i|M=m_j,\mathcal{D}) = \frac{n_{e_i|m_j} + a_{e_i|m_j}}{N_{m_j} + A_{m_j}},\quad (2)$$

where $n_{e_i|m_j}$ is the number of historical evidence samples with the evidence $E = e_i$, and mode $M = m_j$; $a_{e_i|m_j}$ is the number of prior samples that is assigned to
evidence $e_i$ under mode $m_j$; $N_{m_j} = \sum_i n_{i|m_j}$, and $A_{m_j} = \sum_j a_{i|m_j}$.

3. DEPENDENCY IN HISTORICAL EVIDENCE DATA

Note that in the approach described in Section 2, an assumption is that the current evidence only depends on current mode, and is independent on the previous samples. This assumption is true for appropriate designed monitors, as explained below.

The independency among evidences relies on how the evidence data are sampled, and how the disturbance affects the monitor outputs. If the evidence samples are collected with sufficiently large intervals, or if the disturbance has no or weak correlation among the evidence samples, the evidences may be considered as independent. Generally the first requirement regarding the sampling interval can be easily satisfied by leaving sufficient gap between consecutive monitor readings. However, there is no guarantee that the disturbance is uncorrelated in practical applications. If disturbance has long-term autocorrelation and the gap between consecutive monitor readings is not large enough, then the temporal independency assumption of monitor readings cannot apply. A simple practical example of long-term autocorrelation of the disturbance is the ambient temperature change. Consider that each monitor reading is calculated based on 1-hour data and there is no overlap in the use of data. Assume that some of the monitor outputs are affected by the ambient temperature. Due to the cyclic change of temperature within 24 hours, the evidence samples should follow a predictable pattern. Apparently it is more justifiable to consider the dependency between those evidence samples than ignoring it in this example.

Besides the practical issues, another limitation with the conventional Bayesian approach ignoring evidence dependency is its inability to capture all time domain information. An illustrative problem is presented in the following. Suppose that the system under diagnosis has two modes $m_1$ and $m_2$. One monitor $\pi$, with two discrete outcomes, 0 and 1, is available. A set of 100 samples of the monitor outputs is shown in Figure 1. The title in each plot indicates the underlying operating mode under which the data are collected.

![Fig. 1. Monitor outputs of the illustrative problem](image)

The likelihood probability of evidence being 0 or 1 under the two modes is almost identical. This may invoke confusion in the diagnosis, which will lead to higher false diagnosis rate. By looking at the data plot in Figure 1, one can argue that distinguishing the two modes should not be such a difficult task. Although the evidences under $m_1$ and $m_2$ share similar likelihood, the frequencies of the evidence change apparently differ far from each other. The limitation with the conventional Bayesian method without considering evidence dependency is that the temporal information has not been completely used, leading to less efficient diagnosis performance. In summary it is desirable to take the evidence dependency into consideration when building the diagnostic model.

With the consideration of evidence dependency, the mode posterior probability is calculated as

$$p(M^t|M, E^{t-1}, D) \propto p(E^t|M, E^{t-1}, D)p(M).$$

(3)

Comparing the difference between Equation 1 and Equation 3, the main task of building a Bayesian diagnostic system boils down to the estimation of the evidence transition likelihood probability with historical evidence data $D$, $p(E^t|M, E^{t-1}, D)$.

4. EVIDENCE TRANSITION PROBABILITY ESTIMATION

The intention of the estimation of evidence transition probability is to make the estimated likelihood probabilities be consistent with historical evidence data set $D$ in which the evidence dependency exists. Our goal is to calculate the likelihood probability of an evidence $E^t$ given current underlying mode $M^t$ and previous evidence $E^{t-1}$ to reflect the dependency with the Markov property, so every composite evidence sample for evidence transition probability estimation purpose should include three elements,

$$d_E^{t-1} = \{M^t, E^{t-1}, E^t\}.$$ 

(4)

Accordingly, the new composite evidence data set $D_E$, which is assembled from historical evidence data set $D$ to estimate transition probability, is defined as

$$D_E = \{d_E^1, \cdots, d_E^{t-1}\} = \{(M^1, E^1, E^2), \cdots (M^t, E^{t-1}, E^t)\},$$

(5)

Figure 2 depicts how the original collected historical evidence data are divided to form composite evidence samples. In Figure 2, the part highlighted with shadows or gray and enclosed by the dash-lined or solid-lined frame is a single composite evidence sample described by Equation 4.

Suppose that the evidence transition probability from $E^{t-1} = e_s$ to $E^t = e_t$ under mode $M^t = m_k$ is to be estimated from the composite evidence data set,

$$p(E^t|E^{t-1}, M^t, D_E) = p(e_t|e_s, m_k, D_E)$$

(6)
In Equation 11, the first term, and possible evidence transition likelihood parameters, the transition probability $p(e_t|e_s, m_k, D_E)$ can only be estimated from the composite evidence data subset $D_{E|mk}$ where the mode $M'$ is $m_k$,

$$p(e_t|e_s, m_k, D_E) = p(e_t|e_s, m_k, D_{E|mk}, D_{E|m_k})$$

(9)

where $D_{E|m_k}$ is the composite evidence data set whose underlying mode $M'$ is not $m_k$. To simplify notations, the subscript $m_k$ will be omitted when it is clear from the context.

Define $\Phi_s = \{\phi_{s,1}, \phi_{s,2}, \ldots, \phi_{s,K}\}$ as the likelihood parameters for all possible evidence transition from evidence $e_s$ under mode $m_k$, where $\phi_{ij} = p(e_j|e_s, m_k)$ is the transition probability from evidence $e_s$ to $e_j$, and $K$ is the total number of possible evidences. The likelihood probability can be computed by marginalization over all possible evidence transition likelihood parameters,

$$p(e_t|e_s, m_k, D_E) = \int_{\Phi_1,\ldots,\Phi_K} p(e_t|\Phi_1,\ldots,\Phi_K, e_s, m_k, D_E) \cdot f(\Phi_1,\ldots,\Phi_K|e_s, m_k, D_E) d\Phi_1 \cdots d\Phi_K$$

(10)

where $\Psi_i$ is the space of all the likelihood parameters $\Phi_i$.

$$f(\Phi_1,\ldots,\Phi_K|e_s, m_k, D_E)$$

can be calculated according to Bayes’ rule,

$$f(\Phi_1,\ldots,\Phi_K|e_s, m_k, D_E) \propto p(D_E|e_s, m_k, \Phi_1,\ldots,\Phi_K)f(\Phi_1,\ldots,\Phi_K|e_s, m_k).$$

(11)

In Equation 11, the first term, $p(D_E|e_i, m_k, \Phi_1,\ldots,\Phi_K)$ is the composite evidence data likelihood given parameter sets $\{\Phi_1,\ldots,\Phi_K\}$. It should be noted that likelihood of composite evidence data $D_E$ is solely determined by the mode and parameter sets $\{\Phi_1,\ldots,\Phi_K\}$, and thereby is independent of $e_s$ given the mode and the likelihood parameters, i.e.,

$$p(D_E|e_s, m_k, \Phi_1,\ldots,\Phi_K) = p(D_E|m_k, \Phi_1,\ldots,\Phi_K)$$

(12)

where $\tilde{n}_{i,j}$ is the number of evidence transition from $e_i$ to $e_j$ in the composite evidence data set.

Assume that the priors for different parameter sets $\Phi_i$ and $\Phi_j$, for $i \neq j$, are independent (Pernestal, 2007),

$$f(\Phi_1,\ldots,\Phi_K|e_s, m_k) = f(\Phi_1|e_s, m_k) \cdots f(\Phi_K|e_s, m_k).$$

(13)

Dirichlet distribution is commonly used to model priors of the likelihood parameters with parameters $b_{11},\ldots, b_{1K}$,

$$f(\Phi_1|e_s, m_k) = \frac{\Gamma(\sum_{i=1}^{K} b_{1i})}{\prod_{j=1}^{K} \Gamma(b_{1j})} \prod_{i,j} b_{ij}^{-1}$$

(14)

where $b_{ij}$ can be interpreted as the number of prior samples for evidence transition from $e_i$ to $e_j$. $\Gamma(\cdot)$ is the gamma function,

$$\Gamma(x) = (x - 1)!,$$

(15)

where $x$ is positive integer.

Substituting Equation 14 and Equation 12 in Equation 11, and then combining it with Equation 10, the following result is obtained,

$$p(e_t|e_s, m_k, D_E) = \frac{\tilde{n}_{s,t} + B_{s,t}}{\tilde{N}_s + B_s}$$

(16)

where $\tilde{N}_s = \sum b_{i,j}$ is the total number of prior samples, and $B_s$ is the corresponding total number of prior samples.

By comparing Equation 2 and Equation 16, we can see that the evidence transition probabilities are also determined by both prior samples and historical samples, similar to the evidence likelihood calculation when the evidences are independent. The difference lies in how the numbers of prior and historical evidence samples are counted. In Equation 2 the prior and historical evidence samples refer to a simple count of the evidence samples corresponding to a certain mode, while in Equation 16 the prior and historical evidence samples refer to the count of composite evidence samples corresponding to a evidence transition under the target mode. Readers are referred to (Qi and Huang, 2008) for detailed explanation of the likelihood calculation.

5. PILOT SCALE EXPERIMENT

5.1 Process Description

The experiment setup is a water tank with one inlet flow and two outlet flows. The schematic diagram of the process is shown in Figure 3. The inlet flow is driven by a
PID controller to provide level control for the tank, and the other one is a manual bypass valve. It is assumed that the bypass valve is closed when the system in its normal operation condition.

Three operating modes are defined, including the normal functioning \((N\text{F})\) mode, and two problematic modes leakage and bias. The problems associated with the two faulty modes are: the tank leakage problem defined as leakage mode, implemented by opening the bypass valve manually, and the sensor bias problem defined as bias mode, implemented by adding a constant bias to the sensor output. The two problems share similar symptoms in terms of shifting the steady state operation point of the process. For instance, when there is a leakage in the tank, the valve adjusted by the PID controller will decrease to maintain the water level; when there is a negative sensor bias, the valve will also decrease. Thus it is not obvious how to distinguish the two faulty modes without any advanced information synthesizing approach. To make things worse, the external disturbance introduced by changing the pump input will also shift the operation point. Thus the operation point may also change during normal operation.

Random binary sequence is introduced into the inlet pump input to simulate temporal dependent disturbances. By defining the high value as 1, and the low value as 0, the disturbance transitions are designed to follow the transition probability matrices presented in Equation 17.

\[
P^{\text{dis}}_{N\text{F}} = \begin{pmatrix} 0 & 1 \\ 1 & 0.2 & 0.8 \end{pmatrix}, \quad P^{\text{dis}}_{\text{leakage}} = \begin{pmatrix} 0 & 1 \\ 1 & 0.8 & 0.2 \end{pmatrix}, \\
P^{\text{dis}}_{\text{bias}} = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 1 & 0.5 & 0.5 \end{pmatrix}. \tag{17}
\]

Two process monitors, process model validation monitor and sensor bias monitor, are designed. Since we mainly focus on the study of the information retrieving and synthesizing abilities of Bayesian approaches with different diagnosis strategies, the selected monitor algorithms are not necessary to have good performances.

The output of process model validation monitor \(\pi_1\) is given by the squared sum of the actual process output residuals, scaled by the magnitude of the process output. Let the simulated output of the nominal model be \(\hat{y}_t\) at each sampling instance \(t\), and the real output be \(y_t\). The output of the model validation monitor \(\pi_1\) is calculated as

\[
\pi_1 = \frac{\sum_{t=1}^{N} (y_t - \hat{y}_t)^2}{\bar{y}}, \tag{18}
\]

where \(\bar{y} = \frac{1}{N} \sum_{t=1}^{N} y_t\) is the mean value of the process output over one monitor reading period, and \(N\) is the length of data segment over the one monitor reading.

The sensor bias monitor output \(\pi_2\) is obtained by examining the operation point shift. For illustration, consider the scenario when a negative sensor bias occurs. The steady state in terms of the sensor output will not change, since it is controlled by the PID. The steady state output of the controller, i.e., the valve position, however, will decrease. The valve position will reverse in the presence of the positive sensor bias. Thus we can detect the sensor bias by monitoring the deviation of the controller output mean value from the nominal operation point. The output of the sensor bias monitor \(\pi_2\) is calculated as

\[
\pi_2 = \left| u_0 - \frac{1}{N} \sum_{t=1}^{N} u_t \right|, \tag{19}
\]

where \(u_0\) is the nominal operation point of the controller output, \(u_t\) is the controller output at each sampling instance \(t\), and \(N\) is the length of process data segment for a single monitor reading. Note that this monitor will fail for the transition data, thus only steady state data are collected and used in this example.

5.2 Diagnosis Settings and Results

Process data are collected for the three predefined modes. The sampling interval is set to be one second. Every 100 seconds of process data are used for calculation of one monitor reading. Totally 600 monitor readings are calculated from 16.5 hours of process data samples. The collected evidence data of the three modes are divided into two parts for estimation of the likelihood, and for cross-validation respectively. Table 2 summarizes the Bayesian diagnosis parameters.

<table>
<thead>
<tr>
<th>Discretization</th>
<th>(k_1 = 2, K = 2^2 = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical data</td>
<td>120 monitor readings for each mode</td>
</tr>
<tr>
<td>Prior samples</td>
<td>Uniformly distributed with prior sample, for single evidence space, and evidence transition space</td>
</tr>
<tr>
<td>Prior probabilities</td>
<td>(p(N\text{F}) = p(\text{manual}) = 1/3)</td>
</tr>
<tr>
<td>Evaluation data</td>
<td>80 independently generated cross-validation monitor readings for each mode</td>
</tr>
</tbody>
</table>

With the data-driven Bayesian approaches of two different strategies, namely, considering and ignoring the evidence dependency, the diagnosis results in Figure 4 are obtained based on the cross-validation data. In the plot, the gray bars are the numbers of the underlying modes occurred in the validation data set; the light gray and dark bars are the numbers of the diagnosed mode by two diagnostic approaches respectively.

Fig. 4. Numbers assigned to each mode
Owing to the dependent external disturbance, the Bayesian approach ignoring evidence dependency significantly overestimates the number of leakage mode occurrence, and underestimates the number of NF mode. Therefore, its overall correct diagnosis rate is only 51.45%, and is much lower in comparison to the diagnosis rate of the proposed method, which is 73.86%. Not only can better overall performance be obtained by the proposed approach, the diagnosis performance of each single mode, as will be also investigated, is more favorable.

Figure 5 summarizes the diagnosis results in the form of average posterior probabilities. The title of each plot denotes the true underlying mode, and the posterior probability corresponding to the true underlying mode is highlighted with light gray bars. The left panel summarizes the diagnosis results calculated by the approach ignoring evidence dependency; the right panel summarizes the diagnosis results obtained by the approach with consideration of evidence dependency. It is observed that for the three modes, the posterior probabilities assigned to the true underlying modes by the proposed approach are all higher than these assigned by the method ignoring dependency. Thus we can conclude that the proposed approach has better performance for diagnosis of all modes. This conclusion is confirmed by computing the correct diagnosis rate for each mode, as presented in Table 3.

![Diagram](image)

**Fig. 5. Average posterior probability for each mode**

**Table 3. Correct diagnosis rate for each single mode**

<table>
<thead>
<tr>
<th>Evidence Dependency</th>
<th>NF</th>
<th>Leakage</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ignore dependency</td>
<td>6.25%</td>
<td>73.75%</td>
<td>70%</td>
</tr>
<tr>
<td>Consider dependency</td>
<td>55%</td>
<td>78.75%</td>
<td>92.5%</td>
</tr>
</tbody>
</table>

6. CONCLUSION

In this work, a data-driven approach considering evidence dependency is presented. Temporal dependency of monitor outputs is taken into consideration to obtain more accurate diagnosis results. The evidence transition probabilities are estimated from historical data with the developed data-driven algorithm. The method is applied to a pilot scale process, where the performance of the proposed approach is shown superior to that of the method ignoring evidence dependency. In summary, the more information from the time domain is synthesized, the better diagnosis performance is expected.

**REFERENCES**


