Identification of an Ill-Conditioned Distillation Column Process using Rotated Signals as Input

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Abstract: The standard uncorrelated test signals estimate the low-gain direction of ill-conditioned multivariable systems poorly. Therefore, the low-gain information needs to be excited more. In this paper, identification of an ill-conditioned distillation column process using rotated signals is proposed. Rotated input signals allow more excitation to be applied in the weak gain direction of the process and less excitation in the strong gain direction. In this approach, the singular value decomposition (SVD) of the steady state gain matrix is used to rotate the input signals along the directions of the right singular vectors. Simulation results show good accuracy of the proposed method in identifying low and high gain directions.

Keywords: Rotated Input Design, Ill-conditioned Process, System Identification, Subspace Method, Distillation Column.

1. INTRODUCTION

Unlike SISO processes, MIMO processes may show ‘‘directions’’ (in the input vector space) in which the (steady-state or dynamic) effect of the inputs on the process outputs is much larger than in other directions (Zhu *et al.*, 2001, 2006). In such situations the process is said to be ill-conditioned. An ill-conditioned problem is a specific problem for multivariable processes.

Control-relevant identification of an ill-conditioned system requires special techniques. The directionality of such systems should be taken into account in the identification test signal design. Traditional uncorrelated open-loop step tests tend to excite the system mostly in high-gain directions. Therefore, the input test signals should be selected correctly (Zhu *et al.*, 2001, 2006).

In MIMO processes, the existing input test signal design methods can be divided in two categories, sequential input testing and simultaneous input testing (Conner *et al.*, 2004).

In sequential input testing, one signal, often PRBS (pseudo random binary sequence) or GBN (generalized binary noise) signal, is applied to each input separately while the other inputs are kept at their nominal values (Conner *et al.*, 2004). This input excitation usually takes a long time because the inputs are perturbed one at a time (Li *et al.*, 2008).

Simultaneously input testing excites more than one input at a time (Conner *et al.*, 2004). This method leads to more efficient use of the plant testing time (Conner *et al.*, 2004). Gevers *et al.* (Gevers *et al.*, 2006) using variance analysis shows that it is better to excite all inputs simultaneously. However, simultaneous uncorrelated open-loop tests cannot usually excite the ill-conditioned processes in low-gain direction (Zhu *et al.*, 2001, 2006). In these systems, the information of low gain direction is dominated by the noise (low SNR) and no good identification results can be achieved. This problem is caused by poor data not related to identification methods or model structure (Zhu *et al.*, 2001, 2006).

In order to increase the SNR in the low gain direction, one can replace the standard uncorrelated PRBS or GBN inputs with highly correlated signals as inputs. Koung and MacGregor (Koung *et al.*, 1993) proposed rotated inputs. These signals allow more excitation to be applied in the weak gain direction of the process and less excitation in the strong gain direction (Conner *et al.*, 2004). In their approach, the singular value decomposition (SVD) of the steady state gain matrix is used to rotate the input signals along the directions of the right singular vectors (Li *et al.*, 2008). Therefore, for constructing a rotated input signal, preliminary knowledge of the steady-state gain matrix is needed (Conner *et al.*, 2004).

In this paper, MIMO rotated input design for an ill-conditioned distillation column process identification is proposed.

The paper is organized as follows: In Section 2 and 3, ill-conditioned processes and rotated input design are respectively described. In Section 4, the application of the proposed method is carried out on high-purity distillation column as an ill-conditioned process. Finally, section 5 concludes the paper.

2. ILL-CONDITIONED PROCESS

Consider the multivariable (MIMO) system with n inputs and n outputs as follows

$$y(j\omega) = G(j\omega)u(j\omega) \tag{1}$$

The singular value decomposition (SVD) of G can be written (Skogestad *et al.*, 2001) as:

$$G(j\omega) = U(j\omega)\Sigma(j\omega)V^H(j\omega) \quad (2)$$

where U and V are the left and the right singular unitary matrices, respectively. Matrix Σ is the singular value matrix which is diagonal containing the singular values σ_i in decreasing order. The complex frequency $j\omega$ denotes that the SVD in general is a frequency dependent measure. For 2×2 processes, we can write (Jacobsen, 1994)

$$U = [\bar{u} \;\; \underline{u}], \;\; \Sigma = diag(\bar{\sigma}, \underline{\sigma}), \;\; V = [\bar{v} \;\; \underline{v}] \quad (3)$$

and

$$G\bar{v} = \bar{\sigma}\bar{u}, \; G\underline{v} = \underline{\sigma}\underline{u} \quad (4)$$

where $\bar{\sigma}$ denotes the maximum gain of G (in terms of 2-norm), $\bar{v}$ and $\bar{u}$ are the corresponding input and output directions, respectively. Similarly, $\underline{\sigma}$ is the minimum gain of G with corresponding input direction $\underline{v}$ and output direction $\underline{u}$. Note that the singular values and the corresponding input and output directions are frequency dependent; however, for simplicity $j\omega$ is omitted.

The condition number of gain matrix G is given by the ratio of the upper and lower singular values as follows (Jacobsen, 1994)

$$\gamma(G) = \bar{\sigma} / \underline{\sigma} \quad (5)$$

A process is said to be ill-conditioned if $\gamma(G) >> 1$ in some frequency range (Jacobsen, 1994).

In these processes, the process gain is strongly dependent on the direction of the input vector (Jacobsen, 1994). Therefore, the response of the plant is much stronger if input vector is in the high gain direction than if it lies along the low gain direction (Jacobsen, 1994). This can cause difficulties in the identification of ill-conditioned processes. In other words, ill-conditioned processes represent one of the most difficult kinds of linear processes to be identified (Micchi *et al.* 2008).

The ill-conditioned processes also have strongly interactions. The relative gain array (RGA), proposed by Bristol (Bristol, 1966), is a valuable criterion for evaluating the degree of interactions or directionality. The elements of the RGA is defined as follows (Zhu *et al.*, 2006)

$$\lambda_{ij}(j\omega) = g_{ij}(j\omega)[G^{-1}(j\omega)]_{ji} \quad (6)$$

where g_{ij} is the i,j element of G. As the elements in each row and column in the RGA adds up to unity, it is sufficient to consider the 1,1 element for the 2×2 case (Jacobsen, 1994). When one refer to the RGA, it means the 1,1 element of the RGA, i.e., λ_{11}. Large value of λ_{11} denotes that the process is strongly interactive (Jacobsen, 1994).

Note that there are differences between strongly interactive and ill-conditioned processes. A strongly interactive process is always ill-conditioned while the opposite is not always true (Jacobsen, 1994).

3. ROTATED INPUT DESIGN

Consider the singular value decomposition (SVD) of the steady state gain matrix $\bar{G}$.

$$\bar{G} = \bar{U}\bar{\Sigma}\bar{V}^H \quad (7)$$

Using the above equation, the steady state output of process, with N input-output data, can be written as (Conner *et al.*, 2004)

$$\bar{Y}^T = \bar{G}\hat{U}^T = \bar{U}\bar{\Sigma}\bar{V}^H\hat{U}^T \quad (8)$$

where

$$\bar{Y} = [\bar{Y}_1 \; \bar{Y}_2 \ldots \bar{Y}_n] \quad (9a)$$

$$\hat{U} = [U_1 \; U_2 \ldots U_n] \quad (9b)$$

and

$$\bar{Y}_i = [\bar{y}_i(1) \; \bar{y}_i(2) \ldots \bar{y}_i(N)]^T \quad (10a)$$

$$U_j = [u_j(1) \; u_j(2) \ldots u_j(N)]^T \quad (10b)$$

In theses equations, an overbar indicates steady state of a variable if inputs are held constant from the current time forward.

Now, the original inputs, $\{U_i\}$, are scaled by the singular values $\{\sigma_i\}$ to give new inputs $\tilde{U}$ (Conner *et al.*, 2004).

$$\tilde{U} = [U_1 \;\; U_2(\frac{\sigma_1}{\sigma_2}) \;\; U_3(\frac{\sigma_1}{\sigma_3}) \; \ldots \; U_n(\frac{\sigma_1}{\sigma_n})] \quad (11)$$

Then the rotated inputs are produced as follows (Conner *et al.*, 2004)

$$\Xi = \alpha\tilde{U}V^H \quad (12)$$

where α is a factor that should be adjusted so that the outputs do not exceed prespecified limits.

By using the rotated inputs, Ξ, instead of the original inputs, $\hat{U}$, equation (8) can be rewritten as (Conner *et al.*, 2004)

$$\bar{Y}^T = \alpha\bar{U}\bar{\Sigma}\tilde{U}^T \quad (13)$$

Therefore, the modes of the steady-state gain matrix are individually excited by scaled, uncorrelated PRBS or GBN signals (Conner *et al.*, 2004).

A generalization of the rotated inputs design procedure to non-square multivariable systems of arbitrary dimensions is presented at Micchi *et al.*, 2008.

4. CASE STUDY: HIGH-PURITY DISTILLATION COLUMN

A binary distillation column as in Fig. 1 is considered as an ill-conditioned process. The column is running in LV-configuration.

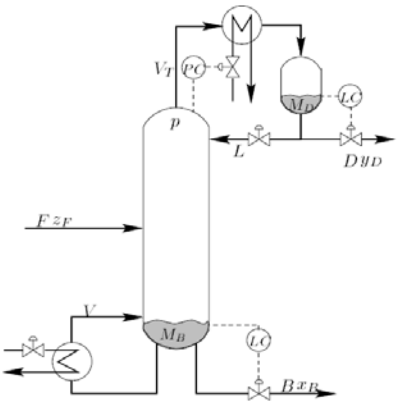

Fig. 1. High-purity distillation column

In this study, reflex (L) and boilup (V) flow rates are considered as the inputs and distillate (y_d) and bottom (x_b) compositions are considered as the outputs of the distillation column. For more detailed description, one can refer to Skogestad, 1997.

High-purity distillation is a challenging process application for system identification because of its nonlinear and strongly interactive dynamics (Rivera *et al*., 2007). Despite their nonlinear behavior, the ability to control high-purity distillation columns using linear controllers is desirable in practice for reasons of simplicity (Rivera *et al*., 2007). Thus, in many studies like this study, a linearized model is used.

The linear model of a distillation column can be described as (Jacobsen, 1994)

$$\begin{bmatrix} y_1(j\omega) \\ y_2(j\omega) \end{bmatrix} = G(j\omega) \begin{bmatrix} u_1(j\omega) \\ u_2(j\omega) \end{bmatrix} \qquad (14)$$

where the state space model of G is as follows

$$\dot{x} = \begin{bmatrix} -0.0051 & 0 & 0 & 0 & 0 \\ 0 & -0.0737 & 0 & 0 & 0 \\ 0 & 0 & -0.1829 & 0 & 0 \\ 0 & 0 & 0 & -0.4620 & 0.9895 \\ 0 & 0 & 0 & -0.9895 & -0.4620 \end{bmatrix} x + \begin{bmatrix} -0.629 & 0.624 \\ 0.055 & -0.172 \\ 0.030 & -0.108 \\ -0.186 & -0.139 \\ -1.230 & -0.056 \end{bmatrix} u \qquad (15a)$$

$$y = \begin{bmatrix} -0.7223 & -0.5170 & 0.3386 & -0.0163 & 0.1121 \\ -0.8913 & 0.4728 & 0.9876 & 0.8425 & 0.2186 \end{bmatrix} x \qquad (15b)$$

Fig.2 and Fig.3 respectively show the singular values and RGA plotted as functions of frequency for the high-purity distillation column.

It can be seen that the process has large condition number and high RGA in the low frequency range.

At steady state, the high-gain singular value and the low-gain singular value are equal to $\bar{\sigma} = 198.2$ and $\underline{\sigma} = 1.36$, respectively. Steady state condition number is $\gamma \approx 146$.

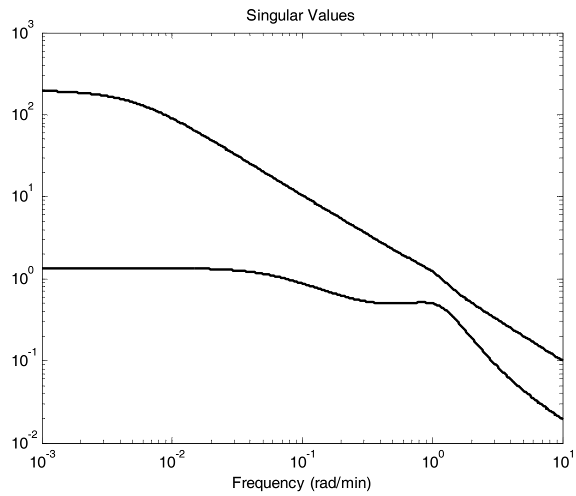

Fig. 2. Singular values of the distillation column

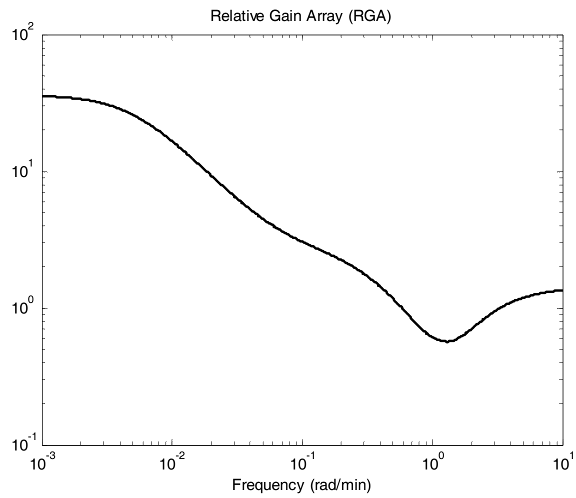

Fig. 3. RGA of the distillation column

Therefore, the largest effect on the outputs is obtained by moving the inputs in opposite directions which causes the two outputs to move in the same direction. The smallest effect is obtained by moving the inputs in the same direction which moves the two outputs in opposite directions.

4.1 Input Test Signals

For identification of the high purity distillation column, two open-loop test signals are considered: uncorrelated signals and rotated signals as outlined in Section 3.

Identification data are constructed by using generalized binary noise (GBN) signals as inputs, both uncorrelated and correlated in the case of rotated inputs. GBN signals, proposed by Tulleken (Tulleken, 1990), have many favorable features, in particular in terms of frequency content, which is typically superior to that of pseudo-random binary noise (PRBS) and of step signals (Zhu, 2001).

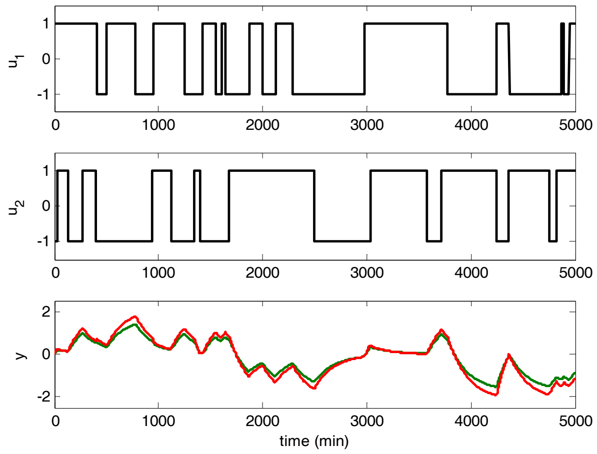

Fig. 4. Data collected using uncorrelated GBN signals as inputs

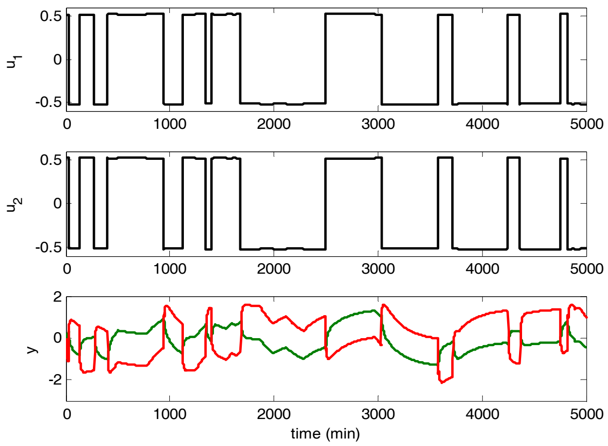

Fig. 5. Data collected using rotated GBN signals as inputs

Hence, two independent GBN signals with amplitude ± 1, switching time $T_{sw} = 300\,\text{min}$, and final time $T_f = 5000\,\text{min}$ are applied on the inputs of the plant simultaneously. The sampling time is selected to be 1 min.

Normally, distributed output noise with a signal-to-noise ratio (SNR) of 10 is added to both outputs. The input-output data from the two experiments are shown in Fig. 4 and Fig. 5.

Fig. 6 and Fig. 7 show the excitations of output directions in the uncorrelated and the correlated tests.

It can be seen that the uncorrelated test inputs only excite the high gain direction. In other words, in this case, the outputs of the process have no information about the low-gain direction of the model. It is clear that the low-gain direction information needs to be excited more in order to obtain good estimates of model. Therefore, strongly correlated test inputs with larger amplitudes are needed (see Fig. 7).

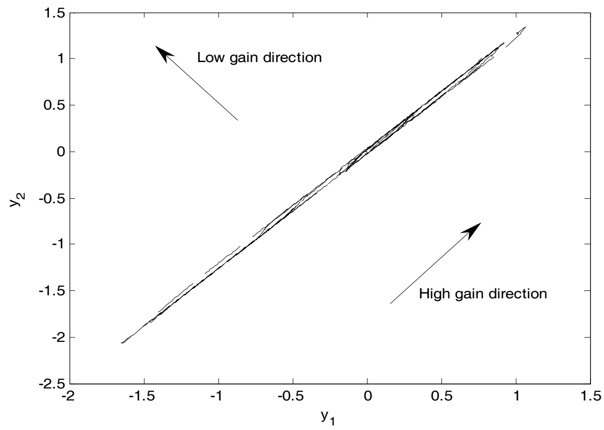

Fig. 6. Excitation of output directions in the uncorrelated test

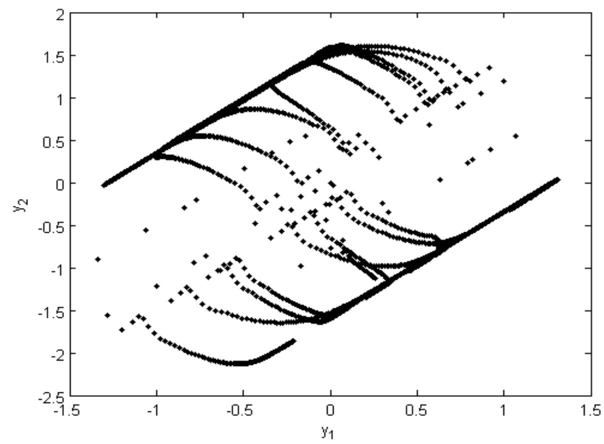

Fig. 7. Excitation of output directions in the rotated test

4.2 Subspace Identification

In this paper, a MIMO structure for the model is considered. In other words, there is a common model for all outputs. System identification is performed using subspace identification (SID) method. This method involves particular matrices obtained from output and input data and performs projection operations to cancel out the noise contributions. Thus, the system model is obtained in state-space form using these projected data matrices. A detailed treatment of this method can be found in Van Overschee *et al.*, 1996.

For identifying the system using subspace method, model order should be selected. In this paper, the model order is determined using number of nonzero singular values of matrix M given by (Misra *et al.*, 2003)

$$M = Y_f \,/_{U_f}\, W_p \prod\nolimits_{U_f^T}^{\perp} = Y_f \prod\nolimits_{U_f^T}^{\perp} \left[[W_p \prod\nolimits_{U_f^T}^{\perp}]^{+} [W_p \prod\nolimits_{U_f^T}^{\perp}] \right] \tag{16}$$

where

$$Y_f \underset{U_f}{/} W_p = [Y_f \prod\nolimits_{U_f^T}^{\perp}][W_p \prod\nolimits_{U_f^T}^{\perp}]^{+} W_p; \; W_p = \begin{bmatrix} U_p \\ Y_p \end{bmatrix} \tag{17a}$$

$$\prod\nolimits_{U_f^T}^{\perp} = I - \prod\nolimits_{U_f^T}^{\perp} \tag{17b}$$
$$= I - U_f (U_f^T U_f)^{+} U_f^T$$

(Y_f, U_f) and (Y_p, U_p) are future and past output-input data, respectively, that is:

$$Y_f = \begin{bmatrix} y_r & y_{r+1} & \dots & y_{r+M-1} \end{bmatrix} \tag{18a}$$
$$U_f = \begin{bmatrix} u_r & u_{r+1} & \dots & u_{r+M-1} \end{bmatrix}$$

$$Y_p = \begin{bmatrix} y_0 \; y_1 \dots y_{M-1} \end{bmatrix} \tag{18b}$$
$$U_f = \begin{bmatrix} u_0 \; u_1 \dots u_{M-1} \end{bmatrix}$$

where r is greater than the system order n ($r > n$) and $M = N - 2r + 1$. The order of system is determined using singular value decomposition of matrix M in equation (16) as follows:

$$M = \begin{bmatrix} \hat{Q}_s & \hat{Q}_n \end{bmatrix} \begin{bmatrix} \hat{S}_s & 0 \\ 0 & \hat{S}_n \end{bmatrix} \begin{bmatrix} \hat{V}_s^T \\ \hat{V}_n^T \end{bmatrix} \tag{19}$$

In the absence of noise, the rank of this matrix is exactly n. Thus there are exactly n nonzero singular values in the SVD in equation (19). In the presence of noise, however, the data matrix on the left hand side of (19) becomes a full rank matrix. The selection of the sizes of $\hat{S}_s$ and $\hat{S}_n$ then requires determining which singular values can be considered small, hence essentially zero, and which ones large. If the noise level is not too high, there is usually a significant difference between noise and signal singular values, and their separation is easily achieved (Misra *et al.*, 2003). However, for ill-conditioned systems, even a small magnitude of noise can make it very difficult to determine the system order correctly (Misra *et al.*, 2003).

For solving this problem, additional requirements must be posed on the input signals. In other words, input signals must excite ill-conditioned system in order to produce output signals as uncorrelated as possible. Therefore, rotated input signal is one of the most appropriate input signals to be used in subspace identification of ill-conditioned multivariable systems.

Fig.8 shows the singular values of the matrix M (for $r = 6$) in the rotated inputs case. As can be observed in this figure, the system order is 5.

After model order determination, subspace method MOESP is used to identify the system. Identification for both types of input test is performed over 20 simulation runs. For estimated model validation, the singular values and RGA are checked. Fig. 9-12 show the singular values and RGA of 20 simulation runs.

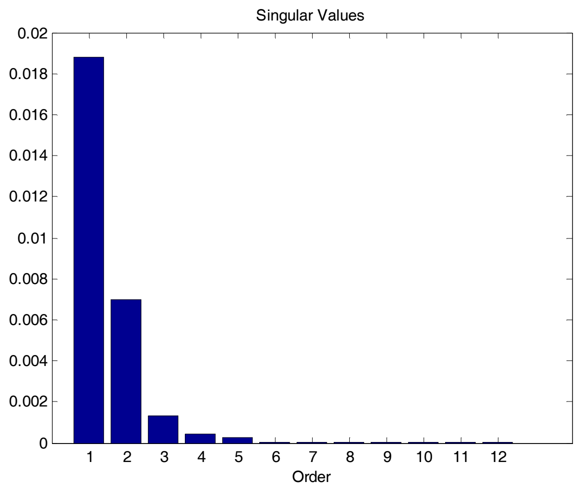

Fig. 8. Singular-value plot for distillation column using rotated input test

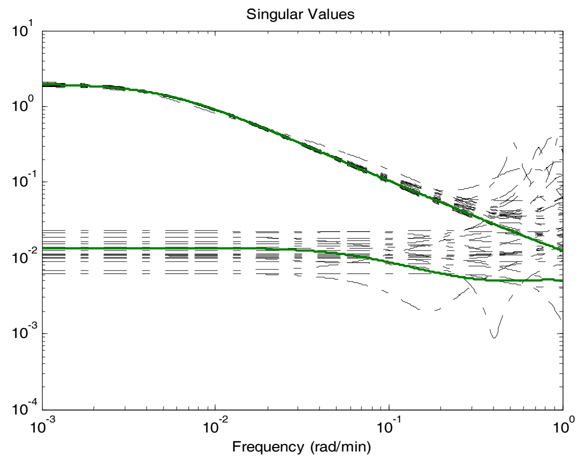

Fig. 9. Singular values of MIMO MOESP models from 20 simulations using uncorrelated test

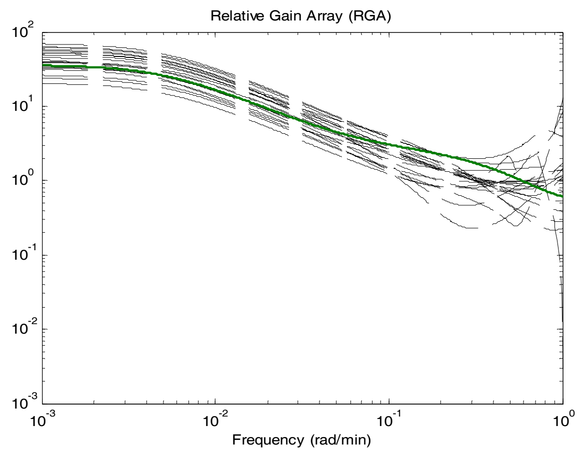

Fig. 10. RGA of MIMO MOESP models from 20 simulations using uncorrelated test

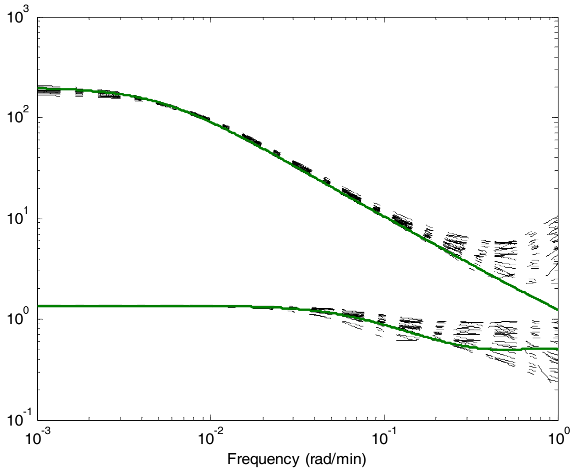

Fig. 11. Singular values of MIMO MOESP models from 20 simulations using rotated input test

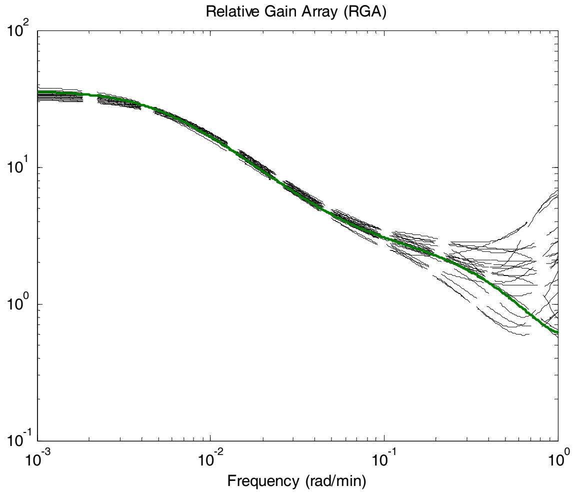

Fig. 12. RGA of MIMO MOESP models from 20 simulations using rotated input test

Note that in these figures, solid lines are the true values and dashed lines are the estimates. It can be seen that the high gain of process is easily estimated whereas the low-gain is very poorly estimated using the uncorrelated test signals. However, by using rotated input signal, good estimates of both low and high gain directions are achieved.

5. CONCLUSIONS

The standard uncorrelated test signals estimate the low-gain direction of ill-conditioned systems poorly. Therefore, the low gain information needs to be excited more in order to obtain good estimates. In this paper, MIMO rotated input design for ill-conditioned process identification was described. Rotated input signals allow more excitation to be applied in the weak gain direction of the process and less excitation in the strong gain direction. In this approach, the singular value decomposition (SVD) of the steady state gain matrix is used to rotate the input signals along the directions of the right singular vectors. The application of the proposed method was carried out on a high-purity distillation column as an ill-conditioned process. Simulation results show good accuracy of the proposed method in identifying both low and high gain directions.

REFERENCES

Bristol, E.H. (1966). On a new measure of interactions for multivariable process control. *IEEE Trans. Automatic Control*, AC-11, 133-134.

Conner, J.S. and Seborg, D.E. (2004). An evaluation of MIMO input design for process identification. *Ind. Eng. Chem. Res.*, 43, 3847-3854.

Gevers, M., Miskovic, L., Bonvin, D., and Karimi, A. (2006). Identification of multi-input systems: variance analysis and input design issues. *Automatica*, 42, 559-572.

Jacobsen, E.W. (1994). Identification for control of strongly interactive plants. *AIChE Annual Meeting*, San Francisco.

Koung, C.W. and MacGregor, J.F. (1993). Design of identification experiments for robust control. A geometric approach for bivariate processes. *Ind. Eng. Chem. Res.*, 32, 1658-1666.

Li, T. and Georgakis, C. (2008). Dynamic input signal design for the identification on constrained systems. *Journal of Process Control*, 18, 332-346.

Micchi, A. and Pannocchia, G. (2008). Comparison of input signals in subspace identification of multivariable ill-conditioned systems. *Journal of Process Control*, 18, 582-593.

Misra, P. and Nikolaou, M. (2003). Input design for model order determination in subspace identification. *AIChE Journal*, 49, 2124-2132.

Rivera, D.E., Lee, H., Mittelmann, H.D., and Braun, M.W. (2007). High-purity distillation: using plant-friendly multisine signals to identify a strongly interactive process. *IEEE Control Systems Magazine*, 28, 72-89.

Skogestad, S. and Postlethwaite, I. (2001). *Multivariable feedback control, Analysis and design*, Second Edition, John Wiley & Sons.

Skogestad, S. (1997). Dynamic and control of distillation columns- A tutorial introduction. *Trans. IChemE*, 75, 539-562.

Tulleken, H.J.A.F. (1990). Generalized binary noise test-signal concept for improved identification-experiment design. *Automatica*, 26, 1, 37-49.

Van Overschee, P. and Demoor, B. (1996). *Subspace identification for linear systems: theory-implementation-applications*, Kluwer, Dordrecht, Netherlands.

Zhu, Y. and Stec, P. (2006). Simple control-relevant identification test methods for a class of ill-conditioned processes. *Journal of Process Control*, 16, 1113-1120.

Zhu, Y. (2001). *Multivariable system identification for process control*, Elsevier Science.