

# The Effect of Tuning in Multiple-Model Adaptive Controllers: A Case Study

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**Abstract:** In this paper, two types of multiple-model adaptive controllers are practically evaluated on a laboratory-scale pH neutralization process. The first one is supervisory switching multiple-model adaptive controller (SMMAC) whose model bank is fixed and selected a priori, and another one is a controller based on multiple models, switching, and tuning strategy (MMST) which uses the possibility of model bank tuning. In addition to investigation of the effect of tuning, the advantage of a disturbance rejection supervisor is studied. Various experiments and exhaustive numerical analyses are provided to assess the abilities of the proposed algorithms.

**Key Words:** multiple models, adaptive control, pH control, switching control, pole-placement control

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## 1. INTRODUCTION

Multiple-model adaptive control is a promising approach to control complex, nonlinear, and time-variant systems. On the grounds that a very complicated system is decomposed to simpler and smaller ones in this approach, and therefore, a large set of model uncertainty is converted to smaller sets, this approach results in a robust controller. It is, also, called an intelligent approach if intelligence is defined as rapid and appropriate response to large and sudden variations in a system (Narendra and Balakrishnan, 1997).

Multiple-model approaches are well-known not only in control but also in identification and estimation (Johansen and Murray-Smith, 1997). Multiple-modeling means that a set of models describes a dynamical system instead of one lone model. According to the type of contribution of members of this set to construct the global model of the process, switching or interacting multiple-model approaches are obtained. In switching multiple-model control strategies, at each instant, one model of the bank is selected as the global model, and the controller is designed according to the parameters of the model. This kind of control strategy has been evaluated in many subject areas such as robotics (Czizz and Narendra, 1996), flight control (Boskovic and Mehra, 1999), aerospace applications (Karimi and Landau, 2000), and process control (Pishvaie and Shahrokhi, 2000; Gundala, Hoo, and Piovoso, 2000).

It is obvious that model bank significantly affects control performance. Thus, it is critical to have a model bank that considers all possible operating points. Since all possible operating points are not known a priori, increasing the number of model bank members may be a solution. However, in addition to intensifying computational burden, there is a chance of deteriorating performance owing to excessive competition of unnecessary members (Li and Bar-Shalom, 1996). Another solution is model bank tuning.

Model bank tuning means that beside each fixed models there is an adaptive model which starts to adjust itself from the location of the fixed model after the fixed model was chosen by the supervisor of the control system. Consequently, a quite new control strategy is introduced and called *multiple models, switching, and tuning* (MMST) (Narendra and Balakrishnan, 1997; Narendra and Xiang, 2000).

The main objective of this paper is to study how the possibility of tuning affects the effectiveness of a multiple-model adaptive controller. To accomplish the objective, we experiment two multiple-model controllers on a pH pilot-plant. The pilot-plant was designed and constructed by members of the research group at KNTU. Figure 1 shows this plant. The secondary objective is to investigate practically a disturbance rejection supervisor, introduced firstly in (Peymani et al, 2008). Since there is no buffer stream in the pilot-plant, the process keeps its high nonlinearity such that the static gain of the process can vary more than 70 times for the entire operating range.

The paper is organized as follows. After introduction, the principles of multiple-model adaptive controllers are reviewed. Two control strategies are presented: one has tuning possibility and the other does not. In the next section, section 3, a disturbance rejection supervisor is designed and its specifications are stated. Then, section 4 is allocated for the pH pilot plant description. Application results are, also, provided in this section. Finally, conclusions end the paper.

## 2. THE BASIS OF MULTIPLE MODELS ADAPTIVE CONTROLLERS

The control strategies utilized in this paper are based on supervisory switching multiple-model adaptive controllers. In these controllers, the whole nonlinear system is divided into several linear systems which can be represented more exactly by a set of simple linear models, called *model bank*.



Fig 1. pH pilot-plant at ACSL, K. N. Toosi University of Technology

That which model and when must be selected is the duty of the *supervisor*. The supervisor determines the appropriate model according to a *switching scheme*.

Let us presume that we have the simplest form of model bank which contains  $N$  fixed-parameter models. In fact, each model is a predictor anticipating the future output of the process in accordance with the last measured input and output. The difference between the output of the real plant and of each model is sent to the supervisor to calculate the identification performance index according to eq. (1):

$$J_s(t) = \alpha e_s^2(t) + \beta \sum_{k=1}^M \lambda^k e_s^2(t-k),$$

$$0 < \lambda \leq 1, \alpha, \beta, M > 0$$
(1)

in which  $e_s = y - \hat{y}_s$ , and  $\alpha, \beta, \lambda$  and  $M$  are the free-design parameters. Pole-placement control design method is utilized in this paper. It is a two-degree of freedom dynamical output feedback controller having the form of:

$$R(q)u(t) = T(q)u_r(t) - S(q)y(t)$$
(2)

where  $R, S$ , and  $T$  are calculated after the process model was selected. The process is viewed as a first order plus time delay model (FOPDT). To find the appropriate values for the controller parameters, it is necessary to define a model reference, a priori, regarding the control objective, and solve a polynomial Diophantine equation on-line. For more details about the controller design method see (Astrom and Wittenmark, 1995; Peymani et al, 2008).

The significance of model bank on the performance of the control system is beyond dispute. The more precise a model bank represents the plant, the better the control system performs. Difficulties arise from the problem of decomposing a plant into efficient smaller linear subsystems. Moreover, a real-world plant inevitably encounters variations which are able to make new operating conditions.

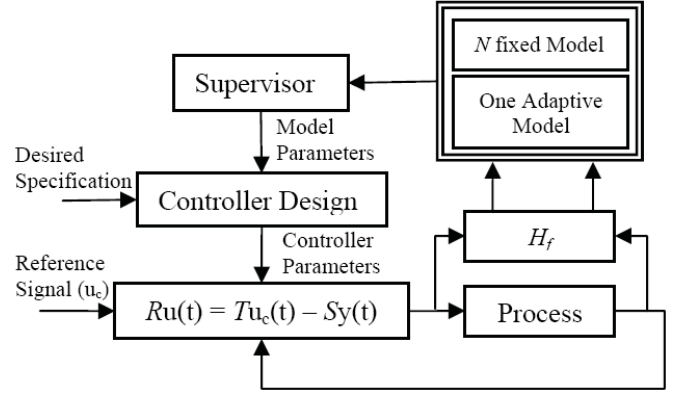


Fig 2. The block diagram of the multiple models, switching, and tuning adaptive controller

If an unpredicted one comes about, the control performance may become weak. To cover more states of the process, the number of members of the model bank should be increased. This solution is not suitable because a bank with too many members may deteriorate the performance in addition to excessive complexity burden (Li and Bar-Shalom, 1996).

Tuning of the current model is another solution to solve this problem. In this idea, after a fixed model is selected, an adaptive model starts to adjust itself to the process condition from the location of the fixed model. Thus, convergence of the adaptation algorithm may be reached soon, and the adaptive model may describe the process more precisely than the selected fixed model does. This model is called *re-initialized adaptive model*, and can be built by a recursive least-squares (RLS) identification method with exponentially forgetting factor (Astrom and Wittenmark, 1995). This control structure containing both fixed and adaptive models in the bank is also called *Multiple Models, Switching, and Tuning* (MMST) control strategy and firstly is introduced by Narendra et al. Thus, this adaptive control strategy possesses a two-stage identification unit: the switching stage to overcome sudden and large changes of the process and the tuning stage to track slow and gradual process variations. Figure 2 shows this control strategy.

**Switching Scheme:** Decision-making part in MMST is done by supervisor as follows. At each sampling time, performance indexes of  $N$  fixed models and one re-initialized adaptive model are updated. Then, in the switching stage, the best fixed model whose index is smaller than the product of other indexes by  $h_s$  is selected. The factor  $h_s$  is the hysteresis constant for the switching stage. After each change of the best fixed model, the adaptive model is reinitialized by the parameters of the fixed model unless the identification performance of the adaptive model is better than the best fixed model. The decision can be made by comparing their performance indexes.

After the switching stage, the tuning stage triggers. In this stage, the supervisor orchestrates which of the adaptive or the best fixed model is appropriate for control. This stage owns another hysteresis constant, represented by  $h_t$ . If the index of

the adaptive model is smaller than the multiplication of the index of the best fixed model by  $h_T$ , the adaptive model is used to design the controller. At this time, the process is controlled by an *adaptive pole-placement controller* (APPC). With the same logic, the supervisor determines whether it is appropriate to use the fixed model for control. As mentioned, SMMC is simpler than MMST control strategy and does not have tuning stage and adaptive model, so switching stage is simpler.

### 3. DISTURBANCE REJECTION SUPERVISOR

All adaptive controllers need the process information to adapt itself to the current condition. The process has to be excited very well in order to collect appropriate information. In process control, the set-point rarely changes. Nonetheless, disturbances occur occasionally. Hence, disturbances can be considered as the staple source of excitation.

When a disturbance happens, at first the process output deviates from the reference input. Then, the controller reacts against this action. Consequently, it is possible to divide the process output into two parts. The first part is the duration which the disturbance, as an unmeasured input, affects the process output, and the second is the one that the control signal derives the system in spite of disturbance. Excitation resulted from the first part is adverse for identification, but excitation caused by the second part which the control signal dominantly affects the system is proper for process identification.

Then, discarding irrelevant data is vital to enhance on-line system identification. This aim is achieved if the time when a disturbance occurs is detected; that is, the identification stage is interrupted in the first part of excitation caused by a disturbance. An idea to discover disturbance occurrence is proposed in (Hägglund and Astrom, 2000) and is shown in Fig. 3, in which  $y_f$  and  $u_f$  are high-pass filtered process output and input, respectively. According to this idea which is

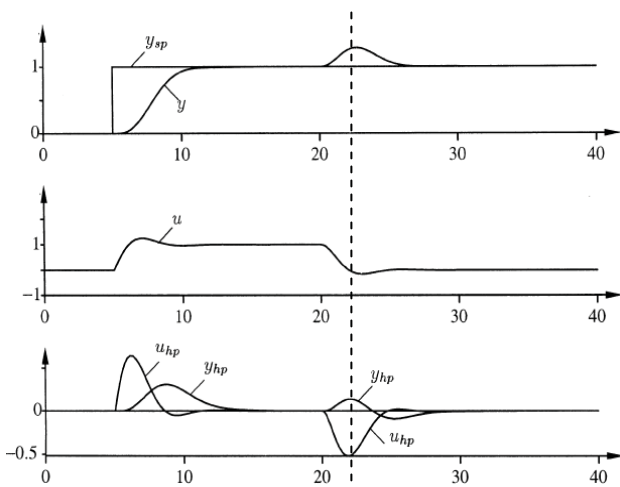


Fig 3. Response of a linear system in close-loop feedback on condition that a disturbance occurs (Hägglund and Astrom, 2000).

proposed for positive-gain systems, when filtered process input and output are larger than predefined thresholds and have opposite signs, a disturbance has occurred. The time when the control action is appropriate for identification can be estimated based on the maximum value of the filtered process output.

According to the extension of this supervisory function for multiple-model controllers, which is proposed by the authors in (Peymani et al, 2008), when a disturbance discovered, both switching and tuning stages are not allowed to work, and the controller is designed based on the best fixed model. After the first part of excitation, the tuning and switching parts are permitted again. In MMST strategy, it is possible to re-initialize the adaptive model to converge rapidly after negative excitation of disturbance.

### 4. APPLICATION RESULTS

The aim of this paper is to study the effect of tuning in the model bank of multiple-model adaptive control strategies by evaluating two multiple-model based controllers on a pH pilot-plant. After pH pilot plant description, application results are provided.

#### 4.1 pH pilot-plant Description

This plant was designed and constructed in K. N. Toosi University of Technology (Fig. 1). In this process, as shown in Fig. 4, there are four streams: acid, base, water, and effluent. The acid stream is the process stream; its concentration and flow rate are presumably constant. The base stream is the titrating stream whose concentration is constant but flow rate is calculated by the controller to regulate pH of the effluent stream. The effluent stream has a constant flow rate. Moreover, this plant is composed of a continuous stirred tank reactor (CSTR) where chemical components are mixed, a pH sensor, a level sensor, and three dosing pumps which inject water, base, and acid with precise flow rate. To keep the level of the tank constant, a classical PI controller is designed which uses water stream flow rate as the control signal. Figure 5 displays the block diagram of this control system.

It is valuable to mention that aquatic solution of acid acetic (a weak acid) and sodium hydroxide (a strong base) are used as process stream and titrating reagent, respectively. However, there is no buffer stream in this pilot-plant. This feature makes this process highly nonlinear. Figure 6 depicts the titration curve of the process with typical concentrations. As it can be seen, it has a steep and large increase in its derivative. This curve is calculated for a batch process because calculating the titration curve of a continuous process is very difficult owing to inherent nonlinearity and various external disturbances. Figure 7 reveals that the static gain can vary more than 70 times for pH in the range of 5 to 10. It is worth mentioning that the measurement noise of the process varies with pH. It is about  $5e-5$  outside the range of  $[7, 8.2]$ , whereas it is about  $1e-3$  inside the range.

## 4.2 Experimental Evaluation

In this section, two multiple models adaptive controllers, proposed earlier, are considered and their parameters are presented specifically. To have comparable situations, the parameters of both controllers are chosen the same. Indeed, the only difference between them is that MMST based controller has the possibility of model bank tuning, while supervisory switching multiple models adaptive controllers (SMMC) has a fixed-parameter model bank. Anyway, to construct a fixed model bank, the process is controlled by a classical adaptive pole-placement controller in varying pH values.

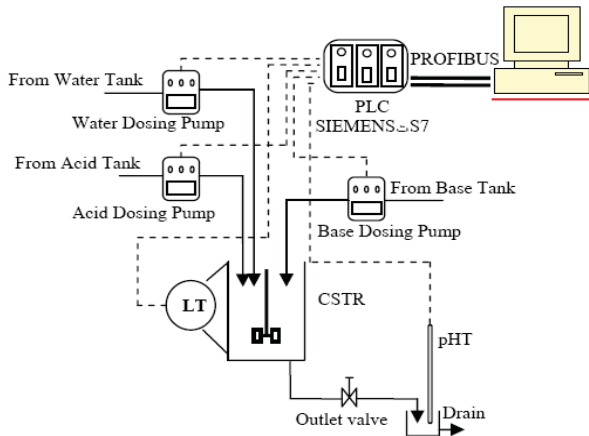


Fig 4. Schematic diagram of pH pilot-plant

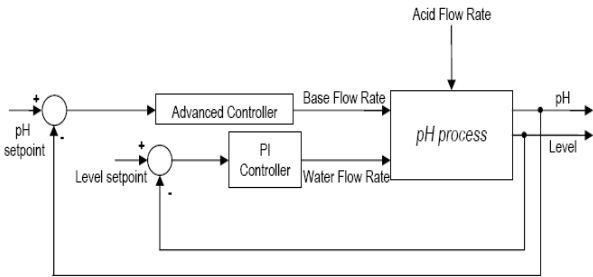


Fig 5. Block diagram of the control system

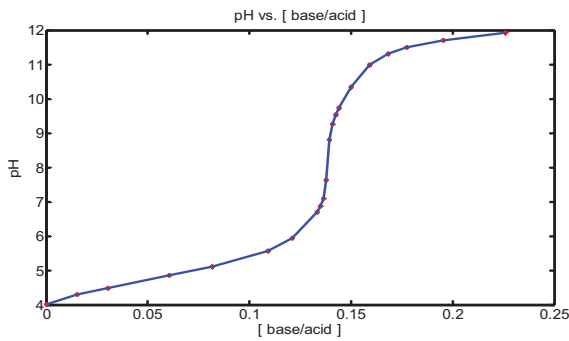


Fig 6. Titration curve of the process stream in the batch form.

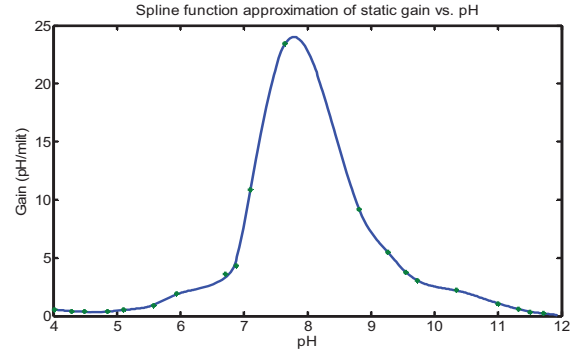


Fig 7. Variations of static gain of pH pilot-plant versus pH

After convergence of the adaptive model, the model is saved as the model of current operating condition. Table I collects models of various operating points. Before identifying, data are passed through a band-pass filter to discard bias and attenuate the adverse effect of noise. Hence, as shown in Fig. 2, filter  $H_f$  is located at the beginning of the identification loop, which is:

$$H_f(q) = \frac{(q-1)}{(q-\alpha)^2} (1-\alpha)^2 \quad (3)$$

The parameter  $\alpha$  is 0.9652. Then, the model reference for pole-placement control design method is:

$$G_m(q) = \frac{0.0012}{(q-0.9652)^2} q^{-8} \quad (4)$$

The parameters of performance index are selected as  $M = 50$ ,  $a = 320$ ,  $b = 100$ ,  $\lambda = 0.985$ . Hysteresis factor for SMMAC equals to 0.80, and switching and tuning hysteresis factors in MMST are chosen 0.85 and 0.75, respectively.

We consider a sequence for step-like set-point changes containing three kinds of variations: 1) small changes which means set-point varies from 5 to 10 one by one (Fig. 8.a); 2) medium changes which means set-point changes from 6 to 8 and to 10, and then returns to 6 in the same manner (Fig. 8.b). These variations are known as the most difficult ones in this process; 3) large changes that contain some variations larger than two units in pH (Fig 8.c). Moreover, the acid feed rate is considered as a measured disturbance. Its nominal value is 30% of maximum power of the corresponding pump. The sequence of 30%  $\rightarrow$  18%  $\rightarrow$  42%  $\rightarrow$  30% is considered as disturbance sequence.

A criterion is defined in order to compare the results numerically, so we choose a two-part measure. Each part is calculated individually for each change in set-point or each disturbance. It is:

$$C = \sum MSWE + 10 \sum O.C. \quad (5)$$

$$MSWE = \frac{100}{n-1} \sum (y(t) - y_r(t))^2 e^{0.005t}$$

$$O.C. = 1 - \left| \frac{\sum (y_f)}{\sum |y_f|} \right|$$



in which  $O.C.$  is a measure of oscillation. Mean square weighted error ( $MSWE$ ) shows how much the process output is similar to  $y_r$ , the model reference output or desired output. It weights the errors exponentially in time.

At first, our aim is to regulate pH of effluent process at different values. Thus, we use the same set-point sequence with various changes to assess how much each controller is able to drive the process similar to the desired output. Fig. 8 shows the results, and table II gathers numerical analysis.

The second test is to evaluate the ability of the controllers to reject external disturbances. Figures 9 to 11 show the results of this test. The effect of disturbance rejection supervisor is demonstrated in Fig. 10 and Fig. 11. Table III compares the disturbance rejection abilities numerically.

#### 4.3 Discussion

This section is allocated for application results. According to Fig. 8, the presence of tuning in the model bank of a multiple-model adaptive control can improve transient response. In this figure, MMST control algorithm has a smoother response, especially for pH around 7.5. Changing from 6 to 8 (and vice versa) is the most difficult change in this plant because the control system has to drive the process from a low-gain operating point to the highest one. The same result can be derived from disturbance rejection part. Hence, it is evident that MMST strategy has a better performance than the other strategy without tuning possibility does.

Figures 9, 10, and 11, and table III demonstrate that the disturbance rejection supervisor can help the supervisor to make a better decision, so this additional supervisory function is satisfactory. Generally, according to Fig. 11, the disturbance rejection supervisor is the reason to a faster rejection.

**Table 1 Local models of the process, time delay neglected to be shown equals to 9 sampling times (45 seconds).**

Model no.	1	2	3	4
Local Model	$\frac{0.0070}{z-0.985}$	$\frac{0.0166}{z-0.990}$	$\frac{0.0044}{z-0.970}$	$\frac{0.001}{z-0.970}$

## 5. CONCLUSION

The proposed MMST control strategy is modified slightly from the original version and the switching stage is transparently separated from the tuning phase. Moreover, to constrain the speed of switching, hysteresis constants are used. The presence of tuning in multiple-model adaptive controllers has positive effect in performance. The application results, which are provided for both tracking and disturbance rejection problems by implementation two multiple-model controllers on a pH pilot-plant, prove that the response of MMST is smoother and more similar to desired output. Also, they uncover that the possibility of tuning in identification loop prevents the model bank to be specified to a certain process. In fact, tuning tries to generalize the model

bank. Furthermore, the application results reveal that the presence of the disturbance rejection supervisor can augment the effectiveness of multiple-model adaptive controllers to face load disturbances without imposing significant computational burden. The disturbance rejection supervisor is designed for positive-gain systems.

It can be claimed on condition that the model bank has a severe deficiency to describe the process, SMMC may result in instability, while MMST can stabilize the process because of the presence of tuning.

**Table 2 Comparison between SMMC and MMST in tracking**

	Small	Medium	Large	Overall
SMMC	32.37	52.61	63.29	148.27
MMST	27.81	25.79	63.50	117.10

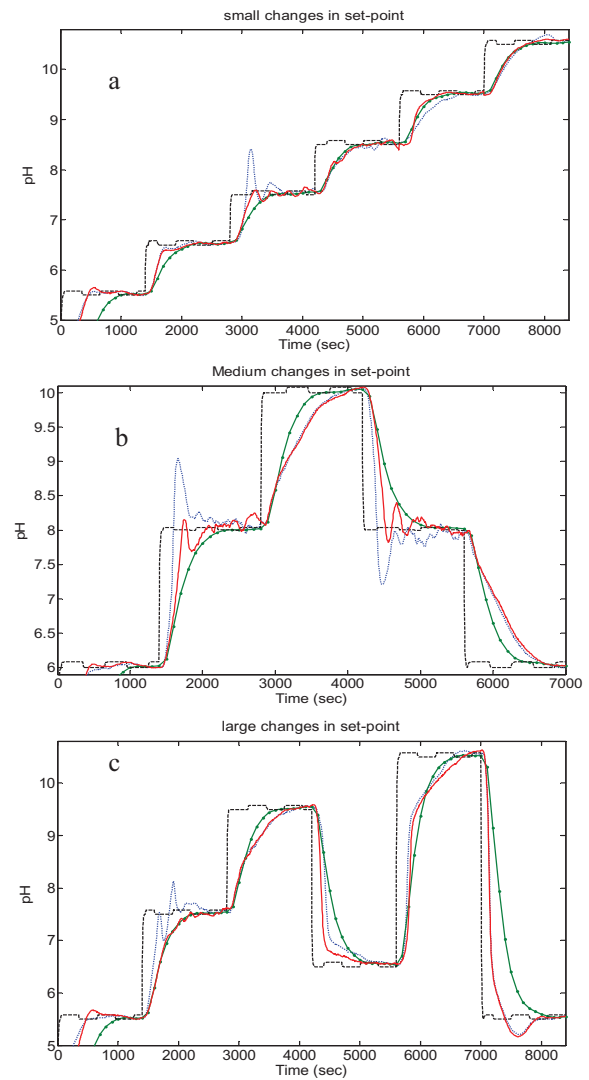


Fig 8. Evaluation of the effect of model bank tuning in tracking problem; a) small, b) medium, c) large changes in set-point; solid line (MMST), dash line (SMMC), and dash-dot line (desired output)

**Table 3 Comparison of disturbance rejection ability between SMMC and MMST.**

Disturbance Rejection	Disturbance Rejection Supervisor	
	OFF	ON
SMMC	167.38	90.31
MMST	75.07	66.31

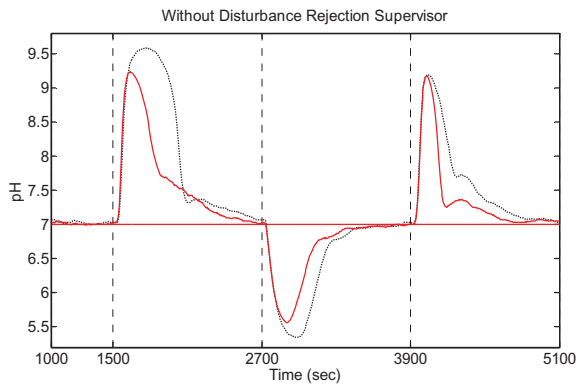


Fig 9. Disturbance rejection of both controllers without disturbance rejection supervisor; solid line (MMST) and dotted line (SMMC)

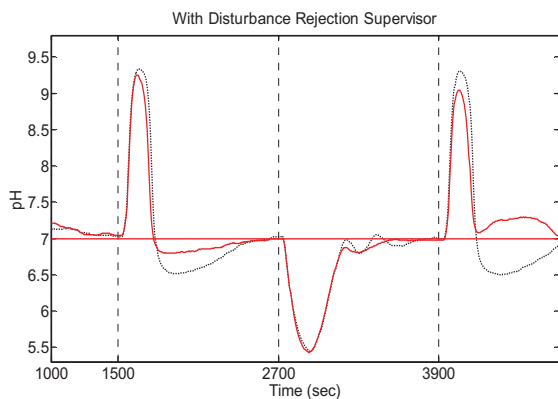


Fig 10. The effect of disturbance rejection supervisor; solid line (MMST) and dash line (SMMC)

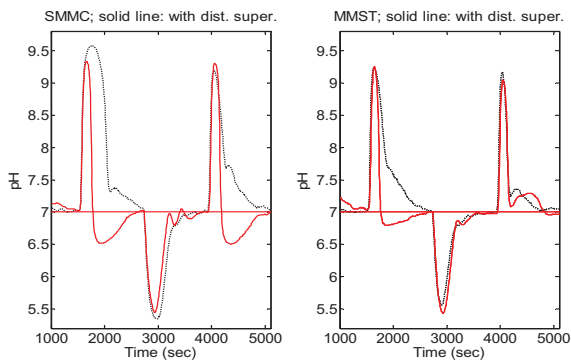


Fig 11. The effect of disturbance rejection supervisor, comparison in each controllers, dash line (without the supervisor) and solid line (with the supervisor); left: SMMC and right: MMST

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