Feedback Controller Design for the Four-Tank Process using Dissipative Hamiltonian Realization

Nicolas Hudon [*] **Martin Guay** [*]

[*] *Department of Chemical Engineering, Queen's University, Kingston, ON, Canada, K7L 3N6 (e-mail:* `martin.guay@chee.queensu.ca`*)*

Abstract: This paper considers the problem of stabilizing the quadruple-tank process using an approximate dissipative Hamiltonian realization. The proposed approach consists in canceling by feedback the deviation of the system from a Hamiltonian system. First, we obtain a characteristic one-form for the system by taking the interior product of a non vanishing two-form with respect to the controlled vector field. We then construct a homotopy operator on a star-shaped region centered at a desired equilibrium point. The dynamics of the system is then decomposed into an exact part and an anti-exact one. The exact part is generated by a potential, hence stability of this part is guaranteed using the generating potential as a Lyapunov function. The stabilizing feedback controller is designed by canceling the anti-exact part of the characteristic one-form. Application of the resulting controller is illustrated by numerical simulations.

Keywords: Feedback Regulation, Approximate Dissipative Hamiltonian Realization, Stability.

1. INTRODUCTION

Application of generalized Hamiltonian systems are an important approach for stability studies and controller design of nonlinear control systems (van der Schaft, 2000) and several physical problems were studied using this class of dynamical system representations. One example in chemical engineering was given recently by Otero-Muras et al. (2008) who studied the stability of a reaction network using its dissipative Hamiltonian representation. However, one limitation associated with the study of non-mechanical nonlinear systems using dissipative Hamiltonian is to derive a suitable Hamiltonian function for the problem. As discussed in (Johnsen and Allgöwer, 2007) and (Ortega et al., 1999), applications of Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC) techniques is difficult since the concept of "energy" is usually ill-defined for process control applications, for example when mass balances are considered. In (Cheng et al., 2005), it was shown that a nonlinear system of the form

$$\dot{x} = f(x) + G(x)u, \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $G(x)$ full rank, is transformable to a stable Port-Controlled Hamiltonian (PCH) system

$$\dot{x} = F(x)\nabla H(x), \quad F(x) = [J(x) - R(x)] \tag{2}$$

if there exists a feedback $\beta : \mathbb{R}^n \to \mathbb{R}^m$ such that the *matching equation*

$$f(x) + G(x)\beta(x) = F(x)\nabla H(x) \tag{3}$$

holds. In particular, for a fixed structure matrix $F(x)$ and a free Hamiltonian function $H(x)$, the problem leads to a set of PDEs parameterized by the structure matrix and the feedback controller $\beta(x)$. Relaxing the need for exact matching, a non-exact matching IDA-PBC approach was recently developed and applied successfully to chemical reactor process stabilization (Ramírez et al., 2009).

In this paper, we will address the problem of stabilizing controllers design using approximate dissipative Hamiltonian realization. In (Cheng et al., 2000), conditions for approximate Hamiltonian realizations were given in terms of a normal form. Sufficient conditions and a constructive algorithm for a generalized Hamiltonian realization for time-invariant nonlinear systems were presented in (Wang et al., 2003). In particular, the method proposed in (Wang et al., 2003) proposed a vector field decomposition along the gradient direction $\nabla H(x)$ and the tangential direction of the energy surfaces of $H(x)$, for a regular positive-definite function $H(x)$. Following the work in (Maschke et al., 2000) which related port-controlled Hamiltonian systems to the construction of Lyapunov functions, it was shown in (Wang et al., 2007) how k-th degree approximate dissipative Hamiltonian systems can be used to solve the realization problem and how associated k-th degree approximate Lyapunov functions can be used to study the stability of such systems.

In the following, we propose to use the tools of exterior calculus to construct the Hamiltonian function and design a stabilizing controller. It is shown that a stabilizing controller can be developed by canceling the anti-exact part of the dynamics (this dynamics acts tangentially to the dynamics generated by the potential). More precisely, assuming a controller structure, we obtain a characteristic one-form for the system by taking the interior product of a non vanishing two-form with respect to the vector field. A homotopy operator centered at a desired equilibrium point for the system is used to obtain an exact one form, generated by a Hamiltonian function, and an anti-exact form that generates the tangential dynamics. We design the controller in such a way that the anti-exact form vanishes. The stability argument presented in (Hudon et al., 2008) uses local equivalence between the exact part of the dynamics and a pre-defined Hamiltonian dissipative realization, viewed as a reference system to develop a local change of coordinates to derive the desired local dissipative potential for the system.

The paper is organized as follows. Section 2 presents the four-tank system as a motivating example. In Section 3, mathematical background is presented, recalling the elements required for the development of the radial homotopy operator that is used in the sequel. The application of this operator to discriminate the exact and anti-exact parts of the dynamics and the development of the stabilizing controller are presented in Section 4. Numerical applications to the four-tank system are given in Section 5. Conclusions and future areas of investigation are outlined in Section 6.

2. QUADRUPLE-TANK PROCESS EXAMPLE

To motivate the present paper, we use the four-tank system studied in details in (Johansson, 2000). More recently, Johnsen and Allgöwer (2007) developed an IDA-PBC controller for the system by introducing error dynamics and solving the matching equations assuming a perturbed Hamiltonian function for the closed-loop dynamics.

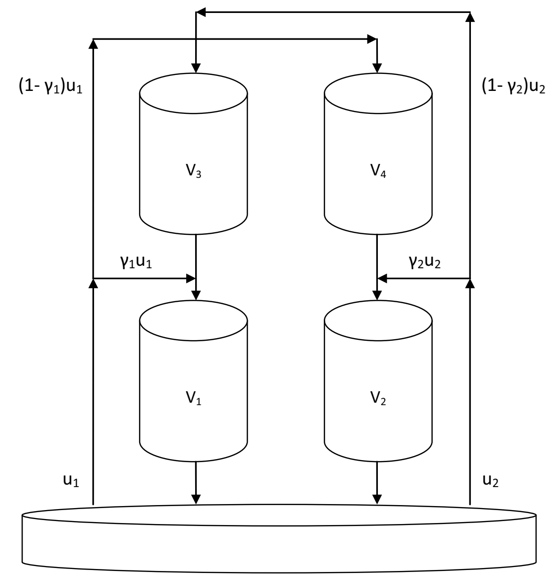

Fig. 1. Four-Tank System

The dynamic model for the four-tank system is given as a control affine nonlinear system of the form

$$\dot{x} = f(x) + G(x)u \tag{4}$$

where $x \in \mathbb{R}^4$ are the levels in the respective tanks and $u \in \mathbb{R}^2$ are the manipulated flows. Using the model proposed in (Johnsen and Allgöwer, 2007), $f(x)$ and $G(x)$ are given by

$$f(x) = \begin{pmatrix} \frac{-a_1}{A_1}\sqrt{2gx_1} + \frac{a_3}{A_1}\sqrt{2gx_3} \\ \frac{-a_2}{A_2}\sqrt{2gx_2} + \frac{a_4}{A_2}\sqrt{2gx_4} \\ \frac{-a_3}{A_3}\sqrt{2gx_3} \\ \frac{-a_4}{A_4}\sqrt{2gx_4} \end{pmatrix}, \tag{5}$$

$$G(x) = \begin{pmatrix} \frac{\gamma_1}{A_1} & 0 \\ 0 & \frac{\gamma_2}{A_2} \\ 0 & \frac{1-\gamma_2}{A_3} \\ \frac{1-\gamma_1}{A_4} & 0 \end{pmatrix}. \tag{6}$$

The parameters A_i represent the cross sections of the respective tanks $i = 1, \dots, 4$, such that the volumes are given by $V_i = A_i x_i$. The parameters a_i are the cross section of the outlet holes. The gravitational acceleration is denoted by g. The parameters $\gamma_1,\ \gamma_2 \in [0,\ 1]$ are the valve parameters that determined how much of the flows u_i are re-directed in bottom tanks $i = 1, 2$. If the levels of tanks 1 and 2 are the only measured states, it was shown in (Johansson, 2000) that the condition for stable zero dynamics is that $\gamma_1 + \gamma_2 \neq 1$.

To stabilize the system at a desired admissible steady-state, (x^*, u^*), we propose a controller of the form

$$u_1(t) = k_{11}(x) \cdot x_1(t) + k_{12}(x) \cdot x_2(t) \tag{7}$$

$$u_2(t) = k_{21}(x) \cdot x_1(t) + k_{22}(x) \cdot x_2(t). \tag{8}$$

At this point, we assume that all tanks levels are measured. In Section 5, we will discuss how this requirement can be relaxed in the case where only x_1 and x_2 are measured.

3. EXTERIOR CALCULUS AND HOMOTOPY OPERATOR

In this section, we show how to construct a homotopy operator $\mathbb{H}$, *i.e.*, a linear operator on differential forms ω, that satisfies the identity

$$\omega = d(\mathbb{H}\omega) + \mathbb{H}d\omega. \tag{9}$$

We first recall some notions of exterior calculus on $\mathbb{R}^n$ (Edelen, 1985). We denote a smooth vector field $X \in \Gamma^\infty(\mathbb{R}^n)$ as a smooth map

$$X : \mathbb{R}^n \to T\mathbb{R}^n, \quad X|_x = \sum_{i=1}^{n} v^i(x)\partial_{x_i}|_x. \tag{10}$$

The cotangent (dual) space $T_x^*\mathbb{R}^n$ is the set of all linear functionals on $T_x\mathbb{R}^n$,

$$T_x^*\mathbb{R}^n = \{\omega|_x : T_x\mathbb{R}^n \to \mathbb{R}\} \tag{11}$$

where each $\omega|_x$ is linear, *i.e.*

$$(a\omega_1|_x + b\omega_2|_x)(X_x) = a\omega_1|_x(X|_x) + b\omega_2|_x(X|_x). \quad (12)$$

The standard basis of $T_x^*\mathbb{R}^n$ is given by $\{dx_1, \ldots, dx_n\}$, where $dx_i(\partial_{x_j}) = \delta_j^i$, δ_j^i being the Kronecker delta. An element $\omega|_x$ in the cotangent space $T_x^*\mathbb{R}^n$ can be written as

$$\omega|_x = \sum_{i=1}^{n} \omega_i dx_i, \quad \omega_i \in \mathbb{R}. \quad (13)$$

In the sequel, differential one-forms will be used. We write

$$\omega = \sum_{i=1}^{n} \omega_i(x) dx_i, \quad (14)$$

where ω_i are smooth functions on $\mathbb{R}^n$. The exterior (wedge) product $\wedge$ is defined on $\Omega^1(\mathbb{R}^n) \times \Omega^1(\mathbb{R}^n)$ by the requirements

$$dx_i \wedge dx_j = -dx_j \wedge dx_i$$
$$dx_i \wedge f(x)dx_j = f(x)dx_i \wedge dx_j$$

for all smooth functions $f(x)$ and

$$\alpha \wedge (\beta + \gamma) = \alpha \wedge \beta + \alpha \wedge \gamma, \quad (15)$$

for all $\alpha, \beta, \gamma \in T^*\mathbb{R}^n$. If $\alpha \in \Lambda^k(\mathbb{R}^n)$, then we write $\deg \alpha = k$. Notice that $\Lambda^1(\mathbb{R}^n) = T^*\mathbb{R}^n$ and $\Lambda^0(\mathbb{R}^n) = \mathcal{C}^\infty(\mathbb{R}^n)$.

The differential operator d is the unique operator on $\Lambda(\mathbb{R}^n) = \bigoplus_{k=0}^{n} \Lambda^k(\mathbb{R}^n)$ with the following properties:

$$d : \Lambda^k(\mathbb{R}^n) \to \Lambda^{k+1}(\mathbb{R}^n), \quad 0 \le k \le n-1, \quad (16)$$

1. $d(\alpha + \beta) = d\alpha + d\beta$.
2. $d(\alpha \wedge \beta) = d\alpha \wedge \beta + (-1)^{\deg \alpha} \alpha \wedge d\beta$.
3. $df = \frac{\partial f_i}{\partial x_i} dx_i, \ \forall f(x) \in \Lambda^0(\mathbb{R}^n)$.
4. $d \circ d\alpha = 0$.

A k-form α is said to be closed if $d\alpha = 0$. It is said to be exact if there exists a $(k-1)$-form β such that $d\beta = \alpha$.

The interior product $\lrcorner$ is a map

$$\lrcorner : \Gamma^\infty(\mathbb{R}^n) \times \Lambda^k(\mathbb{R}^n) \to \Lambda^{k-1}(\mathbb{R}^n) \quad (17)$$

with the following properties $\forall X \in \Gamma^\infty(\mathbb{R}^n)$ and $\forall f \in \Lambda^0(\mathbb{R}^n)$:

1. $X \lrcorner f = 0$.
2. $X \lrcorner \omega = \omega(X), \forall \omega \in \Lambda^1(\mathbb{R}^n)$.
3. $X \lrcorner (\alpha + \beta) = X \lrcorner \alpha + X \lrcorner \beta, \forall \alpha, \beta \in \Lambda^k(\mathbb{R}^n), \ k = 1, \ldots, n$.
4. $X \lrcorner (\alpha \wedge \beta) = (X \lrcorner \alpha) \wedge \beta + (-1)^{\deg(\alpha)} \alpha \wedge (X \lrcorner \beta), \forall \alpha, \beta \in \Lambda(\mathbb{R}^n)$.

The first step in the construction of a homotopy operator is to define a star-shaped domain on $\mathbb{R}^n$ (Edelen, 1985; Banaszuk and Hauser, 1996). An open subset S of $\mathbb{R}^n$ is said to be star-shaped with respect to a point $p^0 = (x_1^0, \ldots, x_n^0) \in S$ if the following conditions hold:

- S is contained in a coordinate neighborhood U of p^0.
- The coordinate functions of U assign coordinates $(x_1^0, \ldots, x_n^0)$ to p^0.
- If p is any point in S with coordinates $(x_1, \ldots, x_n)$ assigned by functions of U, then the set of points $(x^0 + \lambda(x - x^0))$ belongs to S, $\forall \lambda \in [0, 1]$.

A star-shaped region S has a natural associated vector field $\mathfrak{X}$, defined by

$$\mathfrak{X}(x) = [x_i^0 + \lambda(x_i - x_i^0)]\partial_{x_i}, \quad \forall x \in S. \quad (18)$$

In this paper, we will consider the case where the star-shaped domain is centered at the desired equilibrium point x_i^*.

For a differential form ω of degree k on a star-shaped region S centered at an equilibrium point, the homotopy operator will be defined, in coordinates, as

$$(\mathbb{H}\omega)(x) = \int_0^1 \mathfrak{X}(x) \lrcorner \, \omega(\lambda x) \lambda^{k-1} d\lambda, \quad (19)$$

where $\omega(\lambda x)$ denotes the differential form evaluated on the star-shaped domain in the local coordinates defined above.

The properties of the homotopy operator are as follows (Edelen, 1985):

H1. $\mathbb{H}$ maps $\Lambda^k(S)$ into $\Lambda^{k-1}(S)$ for $k \ge 1$ and maps $\Lambda^0(S)$ identically to zero.
H2. $d\mathbb{H} + \mathbb{H}d =$ identity for $k \ge 1$ and $(\mathbb{H}df)(x) = f(x) - f(x_0)$ for $k = 0$.
H3. $(\mathbb{H}\mathbb{H}\omega)(x_i) = 0, \quad (\mathbb{H}\omega)(x_i^0) = 0$.
H4. $\mathfrak{X} \lrcorner \mathbb{H} = 0, \quad \mathbb{H}\mathfrak{X} \lrcorner = 0$.

The first part of the right hand side of (9), $d(\mathbb{H}\omega)$, is obviously a closed form, since $d \circ d(\mathbb{H}\omega) = 0$. By property (H1), for $\omega \in \Lambda^k(S)$, we have $(\mathbb{H}\omega) \in \Lambda^{k-1}(S)$, $d(\mathbb{H}\omega)$ is also exact on S. We denote the exact part of ω by $\omega_e = d(\mathbb{H}\omega)$ and the anti-exact part by $\omega_a = \mathbb{H}d\omega$. It is possible to show that ω vanishes on $\mathbb{R}^n$ if and only if ω_e and ω_a vanish together (Edelen, 1985).

In the sequel, we will apply the homotopy operator on one-forms. Since in our applications, ω_e is an exact one-form, $(\mathbb{H}\omega)$ computed by homotopy is a dissipative potential. A non dissipative potential is associated with the anti-exact part, but on the star-shaped domain S, ω_a does not contribute to the dissipative part of the system. In other words, ω_a belongs to the kernel of $\mathbb{H}$, which can be seen by applying property (H3) from above to the definition of ω_a. In the next section, we will show how stabilization of the desired equilibrium will be ensured by canceling the dynamics associated with ω_a using feedback.

4. FEEDBACK CONTROLLER DESIGN

4.1 Potential Computation

We now present the central element to the proposed construction, namely using the homotopy operator to discriminate the exact and the anti-exact parts associated to a given autonomous system and then computing a feedback controller to cancel the anti-exact part of the dynamics.

Let the vector field $X|_x = \sum_{i=1}^{n} f_i(x)\partial_{x_i}$, $i = 1, \ldots, n$ be known. We assume that X is of class $\mathcal{C}^k$ with $k \ge 2$. It is also assumed that X has an equilibrium point, in the present case, an admissible steady-state for the four-tank process. First, we define a non vanishing closed two-form $\Omega = \sum_{1\leq i<j\leq n} dx_i \wedge dx_j$ on $\mathbb{R}^n$.

Taking the interior product of Ω with respect to the vector field X, we compute a one-form ω as follows

$$\omega = X \lrcorner\, \Omega \tag{20}$$
$$= \sum_{1\leq i<j\leq n} \left(f_i dx_j - f_j dx_i\right). \tag{21}$$

Given a star-shaped region centered at the origin, with associated vector field $\mathfrak{X}(x) = x_i \partial_{x_i}$, we have

$$(\mathbb{H}\omega)(x) = \int_0^1 \left(\mathfrak{X} \lrcorner \omega(\lambda x)\right) d\lambda. \tag{22}$$

Letting $\tilde{f}_i$ denote the values of the components of f after integration with respect to λ, we have

$$(\mathbb{H}\omega)(x) = \sum_{1\leq i<j\leq n} \left(\tilde{f}_i \cdot x_j - \tilde{f}_j \cdot x_i\right) := \tilde{F}(x). \tag{23}$$

Taking the exterior derivative, we have

$$\omega_e = \sum_{i=1}^{n} \frac{\partial \tilde{F}}{\partial x_i} dx_i. \tag{24}$$

The anti-exact form is then given by

$$\omega_a = \omega - \omega_e$$
$$= \sum_{1\leq i<j\leq n} \left(f_i - \frac{\partial \tilde{F}}{\partial x_j}\right) dx_j - \left(f_j + \frac{\partial \tilde{F}}{\partial x_i}\right) dx_i. \tag{25}$$

Remark 1. As a special case, if one defines Ω to be the canonical symplectic two-form and if X_H is the vector field generated by a known Hamiltonian H, ω obtained by the interior product $X_H \lrcorner\, \Omega$ is closed, and we can show that $\omega = \omega_e = -dH$.

For the quadruple-tank system, using our knowledge of the coupling between the tanks, we define the non-vanishing two-forms as

$$\Omega = dx_1 \wedge dx_3 + dx_2 \wedge dx_4. \tag{26}$$

The characteristic one-form for the system is thus given by

$$\omega = -f_3 dx_1 - f_4 dx_2 + f_1 dx_3 + f_2 dx_4. \tag{27}$$

On a star-shaped region centered at the desired equilibrium point $(x_1^*, x_2^*, x_3^*, x_4^*)$, we have

$$\mathfrak{X} = x_i^* + \lambda(x_i - x_i^*). \tag{28}$$

A net result on our notation for the sequel is that on the star-shaped domain, x denotes deviation variables from the center x^*. In (Hudon et al., 2008), the exact part

$$\omega_e = \sum_{i=1}^{4} f_{e,i}(x) dx_i \tag{29}$$

was used to compute a dissipative potential by equivalence to a normal form of dissipative Hamiltonian realization. In the present paper, we are interested in canceling the anti-exact part ω_a by feedback to ensure stability of the closed-loop dynamics.

4.2 Anti-exact Dynamics Cancelation

As mentioned before, the anti-exact part does not influence the value of the computed dissipative potential, at least on the star-shaped domain where the homotopy operator is defined. However, in order to prove stability, the anti-exact part must also vanishes at the equilibrium point of the system (and only there). In the considered example, we will show that a desired equilibrium can be rendered attractive provided that $\omega_a(x^*) = 0$.

The controlled vector field for the four-tank system is given as in Johnsen and Allgöwer (2007) by Equations (4-6). We fixed the controller to be

$$u_1(t) = k_{11}(x) \cdot x_1(t) + k_{12}(x) \cdot x_2(t) \tag{30}$$
$$u_2(t) = k_{21}(x) \cdot x_1(t) + k_{22}(x) \cdot x_2(t). \tag{31}$$

From Section 4.1, we are left we an anti-exact part of the form:

$$\omega_a = \sum_{i=1}^{4} f_{a,i}(x) dx_i. \tag{32}$$

It is desired to make this form closed by using the elements $k_{i,j}(x)$ of the proposed controller. A one-form is closed if

$$\frac{\partial f_{a,i}}{\partial x_j} = \frac{\partial f_{a,j}}{\partial x_i}. \tag{33}$$

For the four-tank system, it leads us to 5 equations with 4 unknown:

$$k_{22}\frac{\gamma_2 - 1}{A_3} - k_{11}\frac{\gamma_1 - 1}{A_4} = 0 \tag{34}$$
$$\frac{1}{2}\frac{a_3 A_1 \sqrt{2g}}{A_3 \sqrt{x_3}} + \frac{1}{2}\frac{a_1 \sqrt{2g}}{\sqrt{x_1}} - \gamma_1 k_{11} = 0 \tag{35}$$
$$-\frac{k_{21}\gamma_2}{A_2} = 0 \tag{36}$$
$$-\frac{k_{12}\gamma_1}{A_1} = 0 \tag{37}$$
$$\frac{1}{2}\frac{a_4 A_2 \sqrt{2g}}{A_4 \sqrt{x_4}} + \frac{1}{2}\frac{a_2 \sqrt{2g}}{\sqrt{x_2}} - \gamma_2 k_{22} = 0. \tag{38}$$

From Equations (36-37), we have that $k_{12} = k_{21} = 0$. From Equations (35) and (38), we have that

$$k_{11}(x) = -\kappa_1 \frac{A_3 \gamma_1 \sqrt{2 x_1 x_3}}{a_3 A_1 \sqrt{g x_1} + a_1 A_3 \sqrt{g x_3}} \tag{39}$$
$$k_{22}(x) = -\kappa_2 \frac{A_4 \gamma_2 \sqrt{2 x_2 x_4}}{a_4 A_2 \sqrt{g x_2} + a_2 A_4 \sqrt{g x_4}} \tag{40}$$

where the gains κ_1 and κ_2 are used to guarantee the first equality (34).

The stability argument for the closed loop system uses the Barbashin-Krasovskiĭ, hence the requirement that ω_a vanishes only at the desired equilibrium point. In fact, the condition that $\omega = \omega_a + \omega_e$ be closed along with the requirement that ω vanishes at the desired equilibrium is essentially a convexity condition of a generating potential. In that sense, decomposition of the dynamics using a characteristic one-form is related to the stability requirements for IDA-PBC as presented in (Ortega et al., 1999) and (Ortega et al., 2002). In the next section, we will illustrate the application of the proposed stabilizing controller.

5. NUMERICAL SIMULATION RESULTS

We now present some numerical applications of the feedback controllers derived in the previous section. Simulation parameters are taken from (Johnsen and Allgöwer, 2007) and are presented in Table 1. We will look at 3 different cases parameterized by the values of γ_1 and γ_2.

Table 1. System Parameters (Johnsen and Allgöwer (2007))

	A_i (cm^2)	a_i (cm^2)
$i = 1, 2$	50.3	0.233
$i = 3, 4$	28.3	0.127

First, we look at the case where $\omega_1 = \omega_2 = 0.6$. For these values, an admissible steady-state x^* is computed to be approximately $x^* = [9,\ 9,\ 4.8,\ 4.8]^T$, and we initialize the system at $x = [4,\ 7,\ 6.8,\ 6.8]^T$. Figures 2 and 3 show that the controller (even with small gains) drives the trajectory to the desired equilibrium and the controller to the consistent steady-state value u^*. Hence by canceling the anti-exact part, the center of the star-shaped domain is attractive.

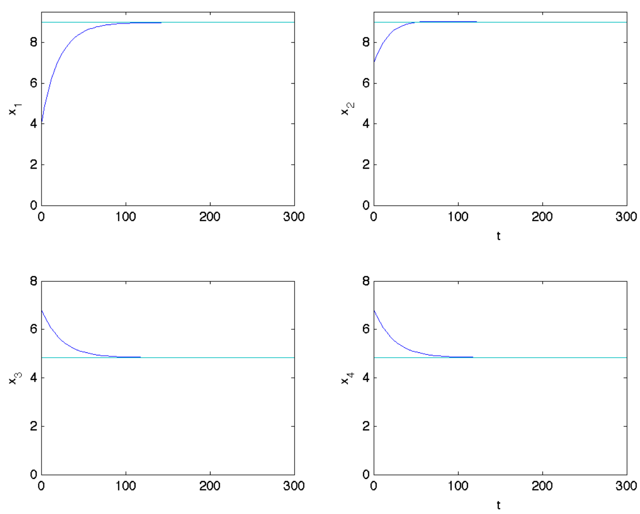

Fig. 2. Full state stabilization of case $\gamma_1 = 0.6,\ \gamma_2 = 0.6$

We now consider the case where $\gamma_1 = \gamma_2 = 0.5$. An admissible steady-state x^* is computed to be approximately $x^* = [10.9,\ 10.9,\ 9.17,\ 9.17]^T$. Initializing the simulation at $x = [5.9,\ 9.9,\ 11.2,\ 11.2]^T$, the proposed controller drives the system to the desired equilibrium (Figures Figures 4 and 5). This case is interesting since, as mentioned in Section 2, if we had considered only output feedback, the zero dynamics for the system are unstable at those values.

To consider output feedback for the case $\gamma_1 = \gamma_2 = 0.5$, we replace $x_3(t)$ and $x_4(t)$ in the controller expressions (39-40) by their desired steady-state values x_3^* and x_4^*. In this particular case, since the zero dynamics is unstable,

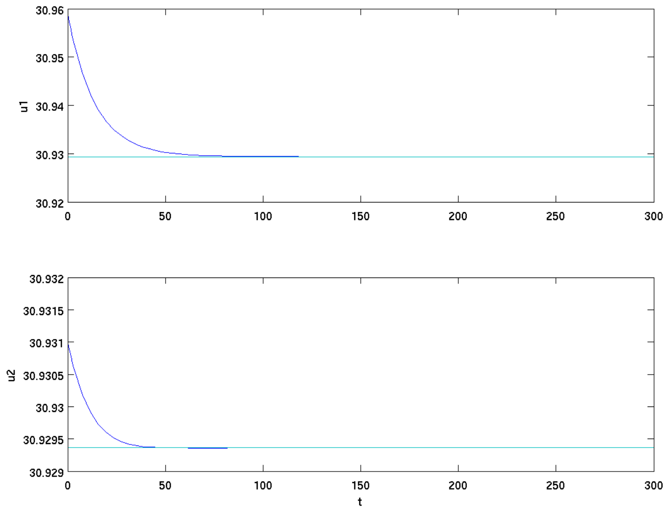

Fig. 3. Control variables values for case $\gamma_1 = 0.6,\ \gamma_2 = 0.6$

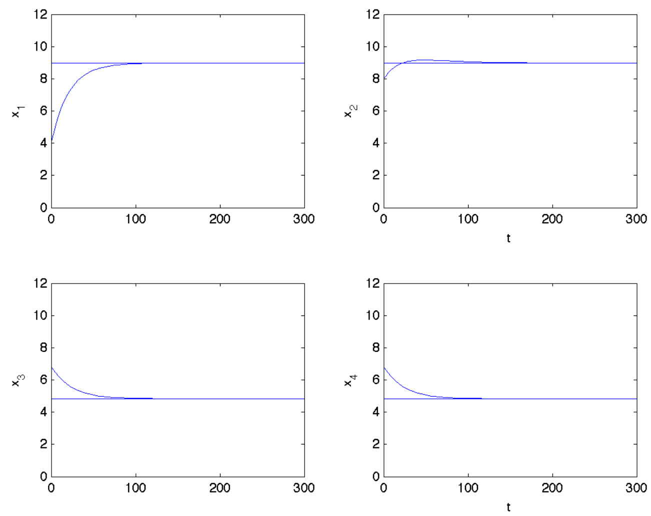

Fig. 4. Full state feedback stabilization of case $\gamma_1 = 0.5,\ \gamma_2 = 0.5$

we use the design parameters κ_1 and κ_2 to make the dynamics of the system associated with the anti-exact part dominated by the gradient term. We seek to reach the same equilibrium point as above $x^* = [10.9,\ 10.9,\ 9.17,\ 9.17]^T$ from two different initial states: $[5.9,\ 9.9,\ 7.2,\ 7.2]^T$ and $[15.9,\ 11.9,\ 11.2,\ 11.2]^T$. Results are presented in Figure 6. As argued in (Ramírez et al., 2009) for a related design approach, the stabilization results present here still hold locally since the proposed controller design procedure does not involve inversion of the dynamics.

6. CONCLUSION

In this paper, a procedure to construct stabilizing controllers using local dissipative Hamiltonian realization for nonlinear dynamical systems was presented. The proposed approach can be seen as an extension of the approximate feedback linearization approach proposed by Banaszuk

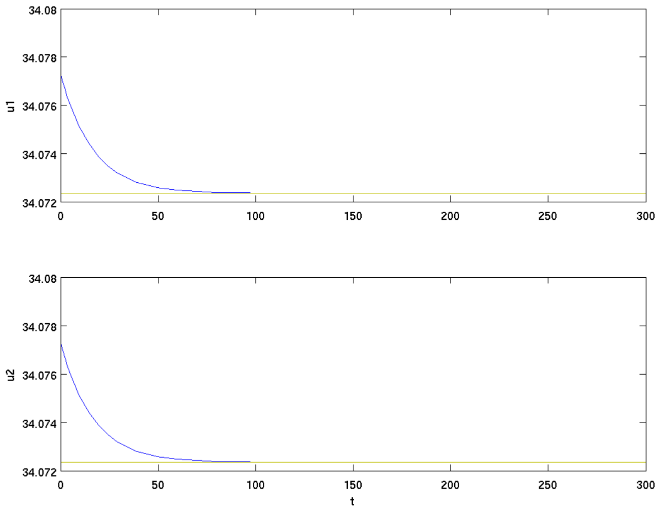

Fig. 5. Control variables values for case $\gamma_1 = 0.5,\ \gamma_2 = 0.5$

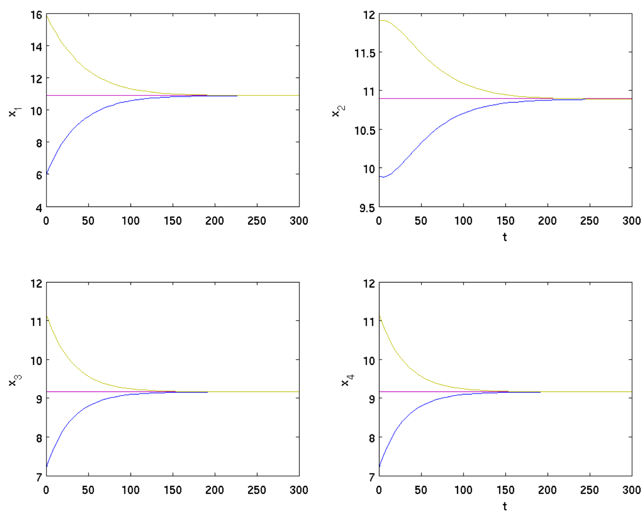

Fig. 6. Output feedback stabilization of case $\gamma_1 = 0.5,\ \gamma_2 = 0.5$

and Hauser (1996). Taking the interior product of a non vanishing two-form with respect to the vector field defining the system, we first obtained a (possibly) non-closed characteristic one-form for the system. Constructing a locally defined homotopy operator on a star-shaped domain centered at the desired equilibrium point, we presented how to decompose locally the obtained form into an exact and an anti-exact one-forms. From (Hudon et al., 2008), we know that the exact part is associated to a dissipative (stable) potential. The obtained anti-exact form is associated to a non dissipative potential which generated tangential dynamics that do not contribute to the value of the dissipative potential on the star-shaped domain. However, using a pre-defined feedback controller to make this error one-form exact, it was shown, using the four-tank system example, that the procedure enables us to construct a stabilizing control. Future research will focus on the limitations of the technique, especially cases where the controller information does not appear in the expression of the anti-exact form, for example the nonisothermal CSTR system presented in (Ramírez et al., 2009).

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