Application of a New Scheme for Adaptive Unfalsified Control to a CSTR with Noisy Measurements ⋆

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Abstract:
In this paper, a new scheme for adaptive unfalsified control with non-ideal measurements is presented and demonstrated for a well-known example of a nonlinear plant, the continuous stirred tank reactor (CSTR) with the van-der-Vusse reaction scheme. In our adaptive control algorithm, there are two adaptation mechanisms: 1. Switching of the active controller in a fixed set of candidate controllers by the ϵ-hysteresis switching algorithm. 2. Adaptation of the set of controllers performed by a population-based evolutionary algorithm. In this paper, the effect of measurement errors on the adaptive control scheme is investigated. The total least squares method is used to perform the deconvolution of noisy signals.

Keywords: Adaptive Control; Unfalsified Control; Nonlinear Control, Reactor Control, Measurement Error

1. INTRODUCTION

The adaptive unfalsified control scheme was initially introduced by Safonov et al. (1997). The basic idea is to switch among candidate controllers in a predefined set of controllers. This approach does not require a plant model but uses the observed plant input-output data while one controller is active to decide on the switching to the next active controller. Further developments by Wang et al. (2005) led to the concept of cost-detectability, the proposal of a cost-detectable cost function, and the ϵ-hysteresis switching algorithm. Stability of the adaptive system was proven in Wang et al. (2005) in the sense that if the set of controllers contains stabilizing controllers with satisfactory performance, the scheme will ultimately switch to one of them. In Engell et al. (2007), Manuelli et al. (2007) and Dehghani et al. (2007), it was pointed out that the scheme in Wang et al. (2005) cannot detect instability of controllers that are not in the loop and may temporarily switch to destabilizing controllers. For this reason, the cost function proposed in Wang et al. (2005) is not suitable for evaluating controllers that are not in the loop, and cannot be used to adapt the controllers in the set.

To resolve this problem, a new scheme of adaptive unfalsified control was proposed in Engell et al. (2007). The key point was the introduction of a new fictitious error signal that can be computed using the estimated sensitivity function obtained by deconvolution between the fictitious reference signal and the fictitious error signal. This new signal is used in a new cost function that can measure the true performance of non-active controllers correctly. Based upon this new cost function, an adaptation of the set of controllers using evolutionary algorithm was performed. The scheme was demonstrated to work for the well-known non-minimum phase CSTR example with undisturbed measurements in Wonghong and Engell (2008).

In this paper, we extend the approach in Engell et al. (2007) to the case of noisy measurements. In the next section, we first review the new concept of unfalsified control with the ϵ-hysteresis switching algorithm as a switching mechanism for the active controller and the adaptation of the set of candidate controllers using evolution strategies. Then we investigate the effect of noise added to the plant output signal on the observed plant input-output data, the fictitious reference signal, and the fictitious error signal. We introduce the total least squares technique to solve the noisy deconvolution problem. This leads to an estimation of the sensitivity function in the non-ideal situation. In section 5, we present simulation studies for the CSTR example. We show that the adaptation of the controller parameters can be performed successfully under non-ideal measurements.

2. A NEW SCHEME FOR ADAPTIVE UNFALSIFIED CONTROL

We consider a SISO adaptive unfalsified control system $\Sigma(P, \hat{K})$ mapping r into (u, y). The system is defined on $\Sigma(P, \hat{K}) : \mathcal{L}_{2e} \rightarrow \mathcal{L}_{2e}$. The scheme of an adaptive unfalsified control system $\Sigma(P, \hat{K})$ is shown in Fig. 1.

The disturbed unknown plant $P : \mathcal{U} \rightarrow \mathcal{Y}$ is defined by

$$\mathbf{P} = \{(u, y, w) \subset \mathcal{U} \times \mathcal{Y} \times \mathcal{W} \mid y = Pu + w\}. \quad (1)$$

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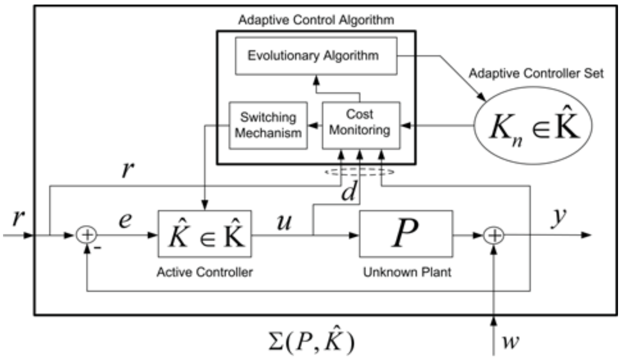

Fig. 1. Adaptive Unfalsified Control Scheme

The set of controllers $K : \mathcal{R} \times \mathcal{Y} \rightarrow \mathcal{U}$ is defined by

$$\hat{\mathbf{K}} = \{(r, u, y) \subset \mathcal{R} \times \mathcal{U} \times \mathcal{Y} \mid u = K_n \begin{bmatrix} r \\ y \end{bmatrix}\}, n = 1, 2, ..., N. \tag{2}$$

The signals $r(t), u(t), y(t)$ are assumed to be square-integrable over every bounded interval $[0, \tau], \tau \in \mathbb{R}_+$. The adaptive control algorithm maps vector signals $d = [r(t), u(t), y(t)]^T$ into a choice of a controller $K_n \in \hat{\mathbf{K}}$, where K_n satisfies the *stable causal left invertible* (SCLI) property [Wang et al. (2005)]. The true error signal is

$$e(t) = r(t) - y(t). \tag{3}$$

The adaptive control law has the form:

$$u(t) = \hat{K}(t) * e(t) \tag{4}$$

where $\hat{K} = K_{n(t)}$ denotes the active controller. $n(t)$ is a piecewise constant function with a finite number of switchings in any finite interval and $*$ denotes the convolution integral.

Let $d = (r(t), u(t), y(t)), 0 \leq t \leq T$ denote experimental plant data collected over the time interval T, and let $\mathbf{D}$ denote the set of all possible vector signals d. d_τ denotes the truncation of d, e.g., all past plant data up to current time τ. The data set $\mathbf{D}_\tau$ is defined by

$$\mathbf{D}_\tau = \{(r, u, y) \subset \mathcal{R} \times \mathcal{U} \times \mathcal{Y} \mid d_\tau = (r_\tau, u_\tau, y_\tau)\}.$$

We consider linear time-invariant control laws of the form:

$$K_n = \{(r, u, y) \subset \mathcal{R} \times \mathcal{U} \times \mathcal{Y} \mid u = c_n * e\} \tag{5}$$

where c_n is the impulse response of the n^{th} controller. $C_n(s)$ denotes the Laplace transform of c_n.

We assume that we have observed the excitation r_τ, the plant input data u_τ and the plant output data y_τ.

In unfalsified control, these data are used to evaluate whether the controller K_n meets a specified closed-loop performance criterion

$$J_n^*(r_\tau, u_\tau, y_\tau) \leq \alpha \tag{6}$$

where α is called the unfalsification threshold. If this condition is not met, the control law switches to a different controller and the previous controller is discarded. After at most N switchings, a suitable controller is found, if there is such a controller in the set.

The key idea of unfalsified control is to compute the cost $J_n^*(d_\tau, \tau)$ based upon the available measurements. For this purpose,

$$\tilde{r}_n = c_n^{-1} * u + y \tag{7}$$

and

$$\tilde{e}_n = \tilde{r}_n - y \tag{8}$$

are defined where c_n^{-1} is the impulse response of the inverse controller transfer function $C_n^{-1}(s)$. These signals are called the *fictitious reference signal* and the *fictitious error signal*, respectively. $\tilde{r}_n$ is the reference signal that produces the measured plant input u and output y if the controller K_n is in the loop instead of the currently active controller.

In Engell et al. (2007), the *new fictitious error signal*

$$e_n^* = \tilde{s}_n * r \tag{9}$$

where $\tilde{s}_n$ is the impulse response of the sensitivity function with the n^{th} controller in the loop,

$$\tilde{S}_n(s) = \frac{1}{1 + C_n P} \tag{10}$$

was introduced. e_n^* is the error that results for the true reference signal r with the controller K_n in the loop. As $\tilde{S}_n(s) = \frac{\tilde{E}_n(s)}{\tilde{R}_n(s)}$, $\tilde{s}_n$ can be computed via the deconvolution of $\tilde{r}_n$ and $\tilde{e}_n$ [Engell et al. (2007)].

When $J_n^*(e_{n_\tau}^*, d_\tau, \tau) > \alpha$, this implies that if the controller K_n were in the loop, it would not satisfy the performance criterion (6). In this case, it is said that the controller K_n is a *falsified* controller. Otherwise the controller K_n is an *unfalsified* controller.

The new adaptive control algorithm consists of two adaptation mechanisms:

1. Switching of the active controller

The closed-loop performances of all candidate controllers are computed using sampled signals

$$J_n^*(d_{\tau_k}, \tau_k) = \max_{\tau_j \leq \tau_k} \frac{\sum_{i=0}^{j} |e_n^*(i)|^2 + \gamma \cdot \sum_{i=0}^{j} |u(i)|^2}{\sum_{i=0}^{j} |r(i)|^2} \tag{11}$$

where γ is a positive constant.

The ϵ-hysteresis switching algorithm of Morse et al. (1992) is applied for the switching of the active controller:

(1) Initialize: Let $k = 0, \tau_0 = 0$; choose $\epsilon > 0$. Let $\hat{K}(0) = K_1, K_1 \in \hat{\mathbf{K}}(0)$, be the first active controller in the loop.
(2) $k = k + 1, \tau_k = \tau_{k+1}$
If $J^*(\hat{K}(k-1), d_{\tau_k}, \tau_k) \geq \min_{K_n \in \hat{\mathbf{K}}(k)} J_n^*(d_{\tau_k}, \tau_k) + \epsilon$,
then $\hat{K}(k) \leftarrow \arg\min_{K_n \in \hat{\mathbf{K}}(k)} J_n^*(d_{\tau_k}, \tau_k)$,
else $\hat{K}(k) \leftarrow \hat{K}(k-1)$.
(3) Go to 2.

2. Adaptation of the set of controllers $\hat{\mathbf{K}}(t^*)$

An evolutionary algorithm (EA) is used for the adaptation of the set of controllers because EA manipulate a population of candidate controllers and can handle nonconvex cost functions and are able to escape from local minima. The EA is executed only at units of time t^* after a sufficiently large change of $r(t)$ was detected. For accurate results, t^* should approximately match the settling time of the controlled system. Insufficient excitation leads to numerical problems due to an ill-conditioned matrix in the deconvolution. Thus, we restrict the activation of the EA to a suitable interval after a sufficient excitation by a change of $r(t)$. In this work, the evolutionary algorithm is a so-called *evolution strategy* where each individual is represented by a vector of controller parameters and by a vector of strategy parameters that control the mutation strength.

The *evolution strategy* as introduced by Rechenberg (1965) and later developed by Schwefel (1975) is based on a population P of μ individuals $\mathbf{a} = (\mathbf{x}, \mathbf{s})$, which represent search points $\mathbf{x} = (x_1, \ldots, x_m) \in \mathbb{R}^m$ and vectors of strategy parameters $\mathbf{s} = (s_1, \ldots, s_m) \in \mathbb{R}^m_+$ that handle the evolution of the population. The size of the population is equal to the number of candidate controllers $\mu = N$. The μ parent individuals in the parent set are randomly selected from the population. The new offspring λ are generated by recombination of two parent individuals and by subsequent perturbation of single variable $x_j, j \in \{1, ..., m\}$ with a random number drawn from a Gaussian distribution $\mathcal{N}(0, s_j)$ by

$$x'_j = x_j + s_j \cdot \mathcal{N}(0, 1). \tag{12}$$

According to the self adaptation mechanism of evolution strategies, each strategy s_j is modified log-normally

$$s'_j = s_j \cdot \exp(\delta \cdot \mathcal{N}(0, 1)) \tag{13}$$

where δ is an external parameter. Normally it is inversely proportional to the square root of the problem size ($\delta \propto \frac{1}{\sqrt{N}}$). To preserve a constant number of individuals, the survivor selection chooses the μ best $(1 \leq \mu < \lambda = 7 \cdot \mu)$ individuals out of the set of λ offspring ((μ, λ)-selection) or out of the union set of parents and offspring ($(\mu + \lambda)$-selection). The quality of each individual is evaluated by the fitness function $f(\mathbf{a}) = J^*(\mathbf{x}, d_{t^*}, t^*)$. As long as a fitness improvement $(\Delta f > min_{\Delta f})$ of the best individual within a certain number of generations can be observed, the termination criterion of the evolution strategy is not fulfilled and the μ selected individuals from the previous generation are used for the next iteration.

3. CONSIDERATION OF NOISE AT THE PLANT OUTPUT

For the scheme in Fig. 1 with a linear plant and a linear controller, in the Laplace domain,

$$\begin{aligned} Y(s) &= P(s)U(s) + W(s) \\ &= P(s)\hat{C}(s)(R(s) - Y(s)) + W(s) \\ &= \hat{T}(s)R(s) + \hat{S}(s)W(s) \\ &= Y_{true}(s) + Y_w(s) \end{aligned} \tag{14}$$

with the active complementary sensitivity function,

$$\hat{T}(s) = \frac{\hat{C}(s)P(s)}{1 + \hat{C}(s)P(s)} \tag{15}$$

and the active sensitivity function,

$$\hat{S}(s) = \frac{1}{1 + \hat{C}(s)P(s)}. \tag{16}$$

The observed disturbed plant input signal is,

$$\begin{aligned} U(s) &= \hat{C}(s)(R(s) - Y(s)) \\ &= \hat{C}(s)(R(s) - Y_{true}(s)) - \hat{C}(s)Y_w(s) \\ &= U_{true}(s) - \hat{C}(s)Y_w(s). \end{aligned} \tag{17}$$

Hence $y(t)$ and $u(t)$ consist of deterministic and stochastic components,

$$y(t) = y_{true}(t) + y_w(t) \tag{18}$$

$$u(t) = u_{true}(t) - \hat{c}(t) * y_w(t) \tag{19}$$

where

$$y_{true}(t) = \hat{t}(t) * r(t) \tag{20}$$

$$u_{true}(t) = \hat{c}(t) * (r(t) - y_{true}(t)) \tag{21}$$

$$y_w(t) = \hat{s}(t) * w(t). \tag{22}$$

Therefore, the error propagation in the measured plant input-output data depends on the closed-loop performance of the active controller.

4. STOCHASTIC DECONVOLUTION

4.1 Stochastic Fictitious Signals

Using (7), the stochastic fictitious reference signal $\tilde{R}_{i,w}$ of C_i using the noisy observed plant input-output data (U, Y) while controller $\hat{C}$ is active results as

$$\begin{aligned} \tilde{R}_{i,w} &= C_i^{-1}U + Y \\ &= C_i^{-1}\hat{C}(R - Y) + Y \\ &= C_i^{-1}\hat{C}(R - Y_{true} - Y_w) + Y_{true} + Y_w \\ &= \tilde{R}_{i,true} + \Delta\tilde{R}_i \end{aligned}$$

where $\Delta\tilde{R}_i = (1 - C_i^{-1}\hat{C})\hat{S}W$. Note that $\Delta\tilde{R}_{\hat{C}} = 0$.

Using (8), the stochastic fictitious error signal $\tilde{E}_{i,w}$ of C_i can be computed from (U, Y),

$$\begin{aligned} \tilde{E}_{i,w} &= C_i^{-1}U \\ &= C_i^{-1}\hat{C}(R - Y) \\ &= C_i^{-1}\hat{C}(R - Y_{true} - Y_w) \\ &= \tilde{E}_{i,true} + \Delta\tilde{E}_i \end{aligned}$$

where $\Delta\tilde{E}_i = -C_i^{-1}\hat{C}\hat{S}W$. Note that $\Delta\tilde{E}_{\hat{C}} = -Y_w \neq 0$.

Using (9), the new fictitious error signal $e^*_{i,w}(t)$ of C_i is

$$E^*_{i,w} = \tilde{S}_{i,w}R. \tag{23}$$

$\tilde{S}_{i,w}$ can be obtained using (10),

$$\begin{aligned} \tilde{E}_{i,w} &= \tilde{S}_{i,w}\tilde{R}_{i,w} \\ \tilde{e}_{i,w}(t) &= \tilde{s}_{i,w}(t) * \tilde{r}_{i,w}(t). \end{aligned} \tag{24}$$

From (24), $\tilde{s}_{i,w}(t)$ can be computed from $u(t)$ and $y(t)$ via $\tilde{e}_{i,w}(t)$ and $\tilde{r}_{i,w}(t)$. The noisy deconvolution is performed using sampled signals:

$$\begin{aligned} \tilde{\mathbf{R}}_{i,w} \cdot \tilde{\mathbf{s}}_{i,w} &= \tilde{\mathbf{e}}_{i,w} \\ (\tilde{\mathbf{R}}_{i,true} + \Delta\tilde{\mathbf{R}}_i) \cdot \tilde{\mathbf{s}}_{i,w} &= \tilde{\mathbf{e}}_{i,true} + \Delta\tilde{\mathbf{e}}_i \end{aligned} \tag{25}$$

where

$$\tilde{\mathbf{R}}_{i,true} = \begin{bmatrix} \tilde{r}_{i,true}(0) & 0 & \cdots & 0 \\ \vdots & \tilde{r}_{i,true}(0) & 0 & 0 \\ \tilde{r}_{i,true}(l-1) & \ddots & \tilde{r}_{i,true}(0) & 0 \\ \tilde{r}_{i,true}(l) & \tilde{r}_{i,true}(l-1) & \cdots & \tilde{r}_{i,true}(0) \end{bmatrix}$$

and $\tilde{\mathbf{e}}_{i,true} = [\tilde{e}_{i,true}(0) \cdots \tilde{e}_{i,true}(l-1)\ \tilde{e}_{i,true}(l)]^T$. The unknown matrix $\Delta\tilde{\mathbf{R}}_i$ is defined similar to $\tilde{\mathbf{R}}_{i,true}$ and the unknown vector $\Delta\tilde{\mathbf{e}}_i$ is defined similar to $\tilde{\mathbf{e}}_{i,true}$. Examples of the computation of $\tilde{\mathbf{R}}_{i,true}, \tilde{\mathbf{e}}_{i,true}, \tilde{\mathbf{s}}_{i,true}$ can be seen in Engell et al. (2007) and Wonghong and Engell (2008).

The deconvolution problem (25) contains error terms both in the matrix and in the right hand side, and therefore it is not adequate to approach it as an ordinary least squares problem. Instead we employ the *total least squares* (TLS) method.

The total least squares method was originally proposed by Golub et al. (1980). The motivation comes from the asymmetry of the *least squares* (LS) method where no error term in the matrix $\tilde{\mathbf{R}}$ is taken into account. The idea of TLS is to find the minimal (in the Frobenius norm sense) error terms $\Delta\tilde{\mathbf{R}}$ and $\Delta\tilde{\mathbf{e}}$ in the matrix $\tilde{\mathbf{R}}$ and in the vector $\tilde{\mathbf{e}}$ that make the linear equations system (25) solvable, i.e.,

$$\{\tilde{\mathbf{s}}_{TLS}, \Delta\tilde{\mathbf{R}}_{TLS}, \Delta\tilde{\mathbf{e}}_{TLS}\} = \arg \min_{\tilde{\mathbf{s}}, \Delta\tilde{\mathbf{R}}, \Delta\tilde{\mathbf{e}}} \|\Delta\tilde{\mathbf{R}} \;\; \Delta\tilde{\mathbf{e}}\|_F$$

$$\text{subject to } (\tilde{\mathbf{R}} + \Delta\tilde{\mathbf{R}}) \cdot \tilde{\mathbf{s}} = \tilde{\mathbf{e}} + \Delta\tilde{\mathbf{e}}.$$

4.2 Solution of the Total Least Squares Problem

The conditions for the existence and the uniqueness of a TLS solution can be found in Markovsky et al. (2007):

$$\mathbf{Z} = [\tilde{\mathbf{R}} \;\; \tilde{\mathbf{e}}] = \mathbf{U}\Sigma\mathbf{V}^T$$

where $\Sigma = diag(\sigma_1, ..., \sigma_{l+1})$ is a singular value decomposition of $\mathbf{Z}$, $\sigma_1 \geq \cdots \geq \sigma_{l+1}$ are the singular values of $\mathbf{Z}$. Partitioned matrices are defined as

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_{11} & \vdots & \mathbf{v}_{12} \\ \cdots & & \cdots \\ \mathbf{v}_{21} & \vdots & v_{22} \end{bmatrix}, \Sigma = \begin{bmatrix} \Sigma_{11} & \vdots & \mathbf{0}_{12} \\ \cdots & & \cdots \\ \mathbf{0}_{21} & \vdots & \sigma_{l+1} \end{bmatrix},$$

where $\mathbf{V}_{11}, \Sigma_{11} = diag(\sigma_1, ..., \sigma_l) \in \mathbb{R}^{l\times l}, \mathbf{v}_{12}, \mathbf{0}_{12} \in \mathbb{R}^{l\times 1}, \mathbf{v}_{21}, \mathbf{0}_{21} \in \mathbb{R}^{1\times l}, v_{22}, \sigma_{l+1} \in \mathbb{R}$. A TLS solution exists if and only if v_{22} is not zero. In addition, it is unique if and only if $\sigma_l \neq \sigma_{l+1}$. In the case when a TLS solution exists and is unique, the solution is given by

$$\tilde{\mathbf{s}}_{TLS} = -\frac{\mathbf{v}_{12}}{v_{22}}. \tag{26}$$

Therefore, the new fictitious error signal $e^*_{i,w}(t)$ for controller C_i can be computed by

$$\mathbf{e}^*_{i,w} = \mathbf{R} \cdot \tilde{\mathbf{s}}_{i,w} = \mathbf{R} \cdot \tilde{\mathbf{s}}_{i_{TLS}}. \tag{27}$$

4.3 Ill-conditioned Matrix $\tilde{R}_{i,w}$

In the error-free case, the computation fails if $\tilde{\mathbf{R}}_{i,true}$ is ill-conditioned, in particular if $\tilde{r}_{i,true}(0) \to 0$. Using the relationship of $\tilde{r}_{i,true}(t)$ and $r(t)$

$$\tilde{R}_{i,true} = \frac{\hat{C}}{C_i}\frac{1 + C_i P}{1 + \hat{C}P} R \tag{28}$$

and applying the initial value theorem,

$$\tilde{r}_{i,true}(0) = \lim_{s\to\infty} s \frac{\hat{C}}{C_i}\frac{1 + C_i P}{1 + \hat{C}P} R. \tag{29}$$

If a unit step function is applied to $r(t)$ and P is strictly proper and $C_i(s) = k_{p_i}(1 + \frac{1}{T_{n_i}s})$,

$$\tilde{r}_{i,true}(0) = \frac{\hat{k}_p}{k_{p_i}}, \tag{30}$$

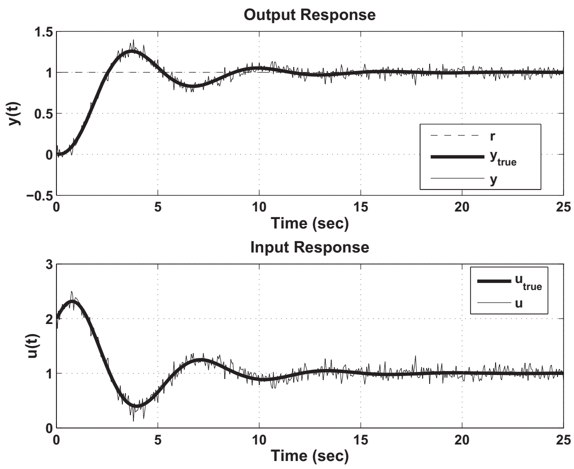

Fig. 2. The measured plant input-output data

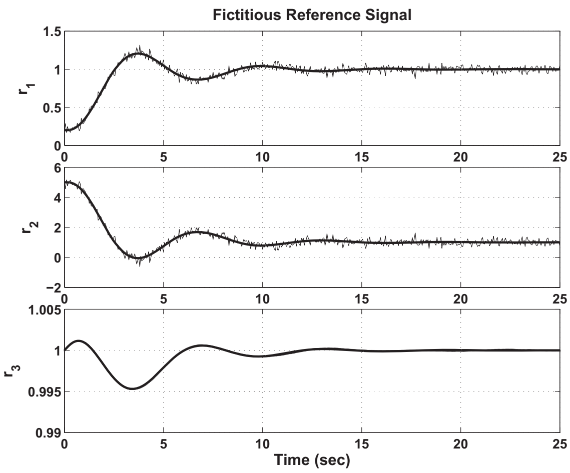

Fig. 3. Fictitious reference signals

so $|k_{p_i}| \gg |\hat{k}_p|$ leads to ill-conditioned matrices $\tilde{\mathbf{R}}_{i,true}$. For $\tilde{\mathbf{R}}_{i,w}$

$$\tilde{r}_{i,w}(0) = \tilde{r}_{i,true}(0) + \Delta\tilde{r}_i(0) = \frac{\hat{k}_p}{k_{p_i}} + \Delta\tilde{r}_i(0). \tag{31}$$

Hence the presence of measurement noise alleviates the ill-conditioning problem for the deconvolution technique.

4.4 Example of Estimated Fictitious Error Signals

We assume that $P = \frac{1}{(s+1)^3}$ and $\hat{C} = 2(1 + \frac{1}{3s})$ and a unit step was applied to $r(t)$. $d_\tau = (r_\tau, u_\tau, y_\tau)$ was observed up to $\tau = 25s$. We assume measurement errors in the plant output data as shown in Fig. 2. Three candidate controllers are tested: 1. $C_1 = 10(1+\frac{1}{3s})$ 2. $C_2 = 0.4(1+\frac{1}{3s})$ 3. $C_3 = 2(1 + \frac{1}{3s})$. $\tilde{r}_{i,true}(t)$ and $\tilde{r}_{i,w}(t)$ result as shown in Fig. 3. Note that the computation of $\tilde{r}_{3,w}(t)$ is error-free. $e^*_{i,true}(t)$ and $e^*_{i,w}(t)$ are shown in Fig. 4. The closed-loop instability of the loop with C_1 is detected and the performances of C_2 and C_3 are estimated well.

5. ADAPTIVE CONTROL OF A CSTR WITH NONMINIMUM PHASE BEHAVIOR

As an example of the application of the new adaptive control scheme to a nonlinear process we investigate the

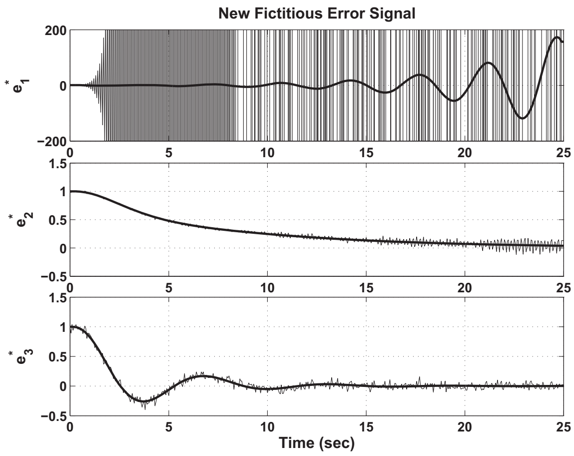

Fig. 4. New fictitious error signals

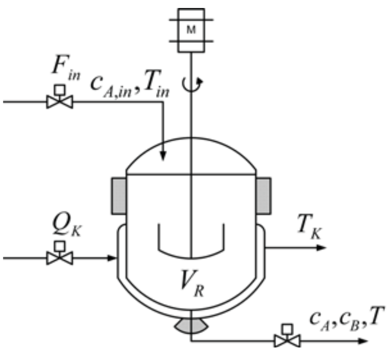

Fig. 5. Continuous Stirred Tank Reactor

well-known case study of the control of a CSTR with the van-der-Vusse reaction scheme. The parameters of the model are the same as in Engell et al. (1993), Klatt et al. (1998).

A sketch of the reactor is shown in Fig. 5. The reaction scheme is

$$\begin{aligned} A &\xrightarrow{k_1} B \xrightarrow{k_2} C, \\ 2A &\xrightarrow{k_3} D. \end{aligned}$$

The reactor is operated at a constant holdup, i.e., the volume of the contents is constant. The manipulated input $u(t)$ is the flow through the reactor, represented by the inverse of the residence time (F_{in}/V_R). u is in the range $0 \leq u(t) \leq 30h^{-1}$. We assume that the temperature control is tight so that the dependency of the kinetic parameters on the reactor temperature can be neglected. Under these assumptions, a SISO nonlinear model results from mass balances for the components A and B:

$$\begin{aligned} \dot{x}_1 &= -k_1 x_1 - k_3 x_1^2 + (x_{1,in} - x_1)u \\ \dot{x}_2 &= k_1 x_1 - k_2 x_2 - x_2 u \\ y &= x_2 \end{aligned} \tag{32}$$

where x_1 is the concentration of component A, x_2 is the concentration of component B and $x_{1,in}$ is the feed concentration of A, assumed to be constant. The parameter values are $k_1 = 15.0345h^{-1}, k_2 = 15.0345h^{-1}, k_3 = 2.324l \cdot mol^{-1} \cdot h^{-1}, x_{1,in} = 5.1mol \cdot l^{-1}$.

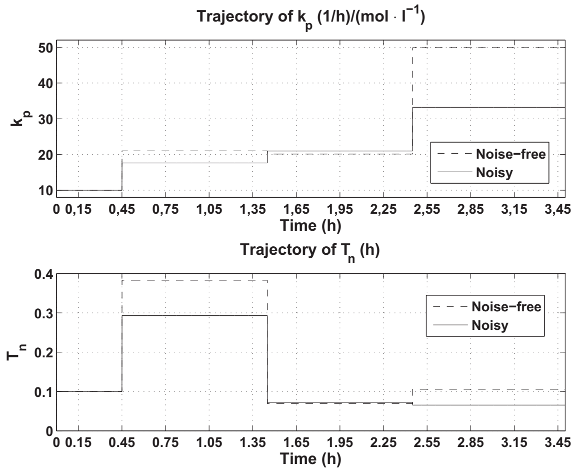

Fig. 6. Evolution of the active controller

We assume that the unknown plant P consists of the continuous stirred tank reactor as described by the above model plus a delay of $0.02h$ for the analytic instrument. The controller structure is a PI-controller defined by

$$C(s) = k_p(1 + \frac{1}{T_n s}).$$

The initial set of the controller parameters is given by the proportional gains $\mathbf{k}_{p_a} = \{10, 50, 100\}$ and the integral times $\mathbf{T}_{n_b} = \{0.1, 0.5, 1\}$. The initial set of candidate controllers consists of 9 candidate controllers with PI-controller parameter vectors $\Theta = \{\theta_i = [k_{p_a}, T_{n_b}]^T, 1 \leq a, b \leq 3\}$. The first active controller assigned to the feedback loop is $\theta_1 = [10, 0.1]^T \in \Theta$. All initial conditions at $\tau = 0$ are zero and the simulation horizon is $t_f = 3.5h$. The constant ϵ in the ϵ-hysteresis switching algorithm is 0.1 and $\gamma = 10^{-9}$ in the cost function J_i^*.

The reference signal is

$$r(t) = \begin{cases} 0mol \cdot l^{-1} & : \quad 0 \leq t < 0.15h; \\ 0.7mol \cdot l^{-1} & : \quad 0.15h \leq t < 1.15h; \\ 0.9mol \cdot l^{-1} & : \quad 1.15h \leq t < 2.15h; \\ 1.09mol \cdot l^{-1} & : \quad 2.15h \leq t < 3.5h. \end{cases}$$

EA activation times are at $t^* = 0.3h$ after each change of $r(t)$ at $t = 0.15h, 1.15h, 2.15h$.

CSTR with adaptation of the set of controllers with noisy measurements

The EA is executed three times at $0.45h$, $1.45h$, and $2.45h$. The EA used is a standard evolution strategy (ES) with adaptation of the search parameters according to Schwefel (1995) and Quagliarella et al. (1998). In this application, the size of the population is equal to the number of candidate controllers $\mu = N$. The $(\mu + \lambda)$ selection is chosen with $\mu = 9$ and $\lambda = 63$. This means that the best controllers are kept from the set of the old controllers and 63 offspring. The search space of solutions $\mathbf{k}_p \times \mathbf{T}_n$ is restricted to $[-100, 100] \times [0.01, 1]$ and the initial strategy parameters are set to 10% of the ranges of the variables. We assume Gaussian i.i.d. measurement errors and the TLS solution is used to compute the estimated sensitivity functions.

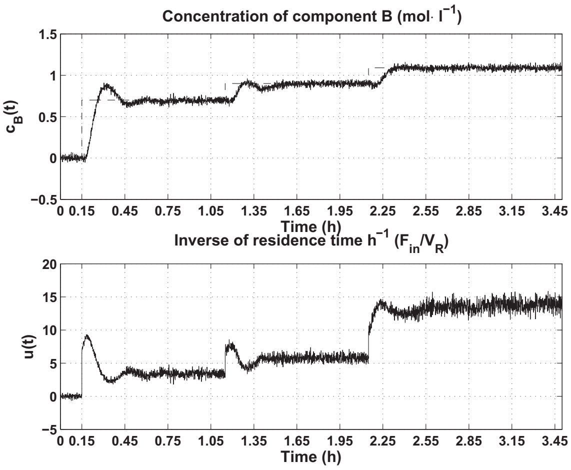

Fig. 7. Noisy control performance

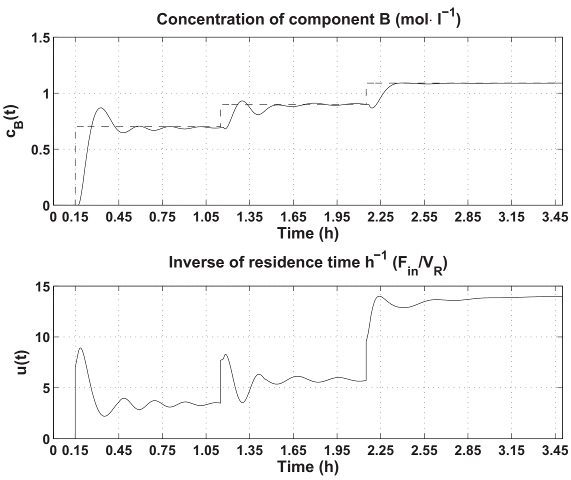

Fig. 8. Noise-free control performance

The first execution of the evolutionary algorithm was performed using measured data $d_{(0.15h,0.45h)}$ obtained with the first active controller θ_1 that was in the loop during $t \in (0.00h, 0.45h)$. At the first execution of the ES at $t = 0.45h$, the evolution strategy returns a new set of controllers for the first operating point after 38 generations. As shown in Fig. 6, the new active controller is $\theta^*_{p_{1_w}} = [17.6214, 0.2929]^T$.

The evolutionary algorithm was executed for the second time using the data $d_{(1.15h,1.45h)}$ with the active controller $\theta^*_{p_1}$. After 19 generations, the new active controller is $\theta^*_{p_{2_w}} = [20.9662, 0.0725]^T$ (see Fig. 6).

The evolutionary algorithm was executed for the third time using the data $d_{(2.15h,2.45h)}$ with the active controller $\theta^*_{p_2}$. After 12 generations the ES returned a new set of controllers and the new active controller is $\theta^*_{p_{3_w}} = [33.1989, 0.0652]^T$ (see Fig. 6). The control performance and the manipulated variable for the case with measurement noise are shown in Fig. 7 and can be compared with the noise-free case in Fig. 8. We can see that the active controller is well adapted to the change of the dynamics of the unknown plant under measurement error.

6. CONCLUSIONS

In this paper, the new scheme for adaptive unfalsified control was investigated for the case with noisy measurement. The deconvolution with noisy plant data can be solved by the total least squares method. The example of a CSTR with nonminimum phase nonlinear dynamics showed that a good performance can still be achieved for noisy measurements.

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