On A New Approach for Self-optimizing Control Structure Design

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Abstract
In this paper, a new method for the identification of self-optimizing control structure designs (CSDs) based on generalized singular value decomposition (GSVD) is proposed. The method is primarily dedicated to find optimal CSDs where all controlled variables (CVs) are represented by a common set of linear combinations of process variables (PVs). It is shown that the implementation of the GSVD into iterative solution approaches is beneficial in order to find CSDs where an individual PV subset is mapped to each CV. The developments will be tested on a simple process.

Keywords: Control system design; Linear control systems; Controlled variables; Optimal control; Self-optimizing control

1. INTRODUCTION

With more than 4000 completed plant projects, the Engineering Division of the Linde AG ranks among the leading international plant contractors, with focus on the key market segments olefin plants, natural gas plants, air separation plants, as well as hydrogen and synthesis gas plants. This paper relates to new process developments for liquefied natural gas plants whose steady state and dynamic behavior are currently under investigation in order to provide design guidelines and to ensure reliable and economic operation. In the context of this work, new methods for the identification of regulatory control structure designs (CSDs) have been developed. They will be presented in this paper.

Steady state process optimization by regulation was first motivated by Morari et al. [1980]. They articulated the idea that a constant set point policy will lead to optimal operation if the underlying control structure is properly designed. Skogestad and Postlethwaite [1996, p. 428-433] extended this idea and gave an approximate criterion for finding CSDs with self-optimizing abilities, the so-called minimum singular value (MSV) rule. Assuming a linear process model and a quadratic cost function, an exact local criterion for the worst-case loss of CSDs was developed by Halvorsen et al. [2003]. Based on the resulting worst-case loss criterion, a multivariate non-convex problem subject to structural constraints needs to be solved in order to obtain a self-optimizing CSD. These structural constraints refer to limitations on the size of the process variable (PV) subset and the particular selection of PVs. For instance, it must be decided whether only PV selection or PV combination is taken into account. Several methods have been developed which solve the constrained optimization problem. An optimal CSD subject to PV selection can be found by either screening all possible PV combinations or applying branch and bound (BAB) algorithms in order to avoid time consuming calculations as proposed by Cao and Saha [2005], Kariwala and Skogestad [2006/07/09-13], Cao and Kariwala [2008], Kariwala and Cao [2009]. PV combination methods have been published by Alstad and Skogestad [2007], Alstad et al. [2008], Kariwala [2007] and Kariwala et al. [2008]. They all have in common that the same PV subset is considered for all CVs.

In Section 2, the mathematical framework of self-optimizing control theory will be briefly introduced. In the following sections, two variants of a new PV combination method will be proposed. In Section 3 the focus is on CSDs where a common PV subset is considered for all CVs. Section 4 is dedicated to CSDs where individual PV subsets are mapped to each CV. For illustration, the new developments will be applied to a process example in Section 5. Concluding remarks are given in Section 6.

2. MATHEMATICAL FRAMEWORK

The scheme in Figure 1 represents a general regulatory CSD applied to an arbitrary process plant. Based thereon the exact local method by Halvorsen et al. [2003] will be introduced. The vectors $\boldsymbol{u} \in \mathbb{R}^{n_{\boldsymbol{u}}}$, $\boldsymbol{d} \in \mathbb{R}^{n_{\boldsymbol{d}}}$, $\boldsymbol{y} \in \mathbb{R}^{n_{\boldsymbol{y}}}$ and $\boldsymbol{c} \in \mathbb{R}^{n_{\boldsymbol{u}}}$, respectively, correspond to the manipulated variables (MVs), disturbance variables (DVs), measured process variables (PVs) and controlled variables (CVs). A constant set point policy is applied. That is, the MVs are adjusted by the controller(s) until, feasibility provided, the CVs equal the set point vector $\boldsymbol{c}_s$. To account for measurement errors, the PVs and CVs are affected by the implementation errors $\boldsymbol{n}^{\boldsymbol{y}} \in \mathbb{R}^{n_{\boldsymbol{y}}}$ and $\boldsymbol{n}^{\boldsymbol{c}} \in \mathbb{R}^{n_{\boldsymbol{u}}}$.

Morari et al. [1980], the inventors of self-optimizing control, state that it is desirable "(...) to find a function of PVs which when held constant, leads automatically to the optimal adjustments of the MVs, and with it, the optimal operating conditions." In other words, self-optimizing con-

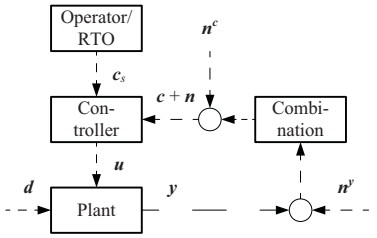

Figure 1. General representation of regulatory CSDs in chemical plants (after Alstad et al. [2008]).

trol may be achieved by an appropriate mapping of PVs towards CVs, denoted by $\boldsymbol{c} = \mathcal{H}(\boldsymbol{y})$, where $\mathcal{H} \in \mathbb{R}^{n_{\boldsymbol{u}}}$ represents the "combination" block in Figure 1. For deriving the exact local method, Halvorsen et al. [2003] considered a linear map $\boldsymbol{H} = \frac{\partial \boldsymbol{c}}{\partial \boldsymbol{y}^T}$. The cost function of a plant denoted by J are usually affected by both, MVs and DVs. In order to operate the plant optimally (at minimum cost), MVs need to be adjusted subject to variations in DVs. The solution to the problem

$$\boldsymbol{u}_{\text{opt}}(\boldsymbol{d}) = \arg\left(\min_{\boldsymbol{u}} J(\boldsymbol{u}, \boldsymbol{d})\right) \text{ s.t. } g(\boldsymbol{y}, \boldsymbol{u}, \boldsymbol{d}) = 0 \quad (1)$$

gives the best input leading to the lowest achievable cost. Problem (1) will be referred to as feed-forward re-optimization. Here $g \in \mathbb{R}^{n_{\boldsymbol{y}}}$ denotes the steady-state model equations of the plant. The following simplifying assumptions are made.

(1) Nonlinearities of the plant are treated as locally negligible. Then, the steady-state I/O model of the plant can be represented as
$$\boldsymbol{y} = \boldsymbol{G^y}\,\boldsymbol{u} + \boldsymbol{G^y_d}\,\boldsymbol{d}, \quad (2)$$
where $\left[\,\boldsymbol{G^y}\;\;\boldsymbol{G^y_d}\,\right] = -\left(\frac{\partial g}{\partial \boldsymbol{y}^T}\right)^{-1} \frac{\partial g}{\partial \left[\,\boldsymbol{u}^T\;\;\boldsymbol{d}^T\,\right]}$.
(2) The cost function J is locally approximated by a second order Taylor series.
(3) The number of MVs might be reduced as some of them need to be spend in "a priori" controller loops in order to either stabilize the plant or fulfill optimally active constraints. It is assumed that $\boldsymbol{u}$ represents only the remaining MVs available for self-optimizing CSD. The "a priori" controller loops are considered part of the model equations g.

Figure 2 shows exemplarily the operational cost of a process plant versus one DV. The cost of feed-forward re-optimization is indicated by the solid line and gives the lower bound for the cost of feedback control with constant set points. It is thus convenient to define a loss function as

$$L(\boldsymbol{d}) = J(\boldsymbol{u}_{\boldsymbol{H}}(\boldsymbol{d}), \boldsymbol{d}) - J(\boldsymbol{u}_{\text{opt}}(\boldsymbol{d}), \boldsymbol{d}).$$

Here $\boldsymbol{u}_{\boldsymbol{H}}(\boldsymbol{d})$ represents the influence from DVs to MVs for feedback control, easily derived for the linear case. From $\boldsymbol{c} = \boldsymbol{H}\,\boldsymbol{y} \stackrel{!}{=} \boldsymbol{c}_s = \boldsymbol{0}$ and (2) it follows that

$$\boldsymbol{u}_{\boldsymbol{H}} = -(\boldsymbol{H}\,\boldsymbol{G^y})^{-1}\,\boldsymbol{H}\,\boldsymbol{G^y_d}\,\boldsymbol{d}.$$

According to Halvorsen et al. [2003] the worst-case loss is given by

$$L_{\text{worst}} = \frac{1}{2}\,\boldsymbol{z}^T\,\boldsymbol{z}, \quad (3)$$

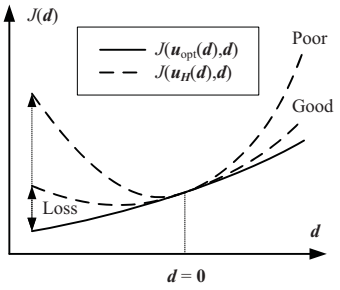

Figure 2. Objective functions for a poor and a good self-optimizing CSD compared with the case of re-optimized MVs (after Skogestad [2000]).

where the loss variables $\boldsymbol{z}$ are given by

$$\boldsymbol{z} = \boldsymbol{M}\,\boldsymbol{f}, \quad (4)$$

for feedback control with

$$\boldsymbol{M} = J_{\boldsymbol{uu}}^{1/2}\,(\boldsymbol{H}\,\boldsymbol{G^y})^{-1}\,\boldsymbol{H}\,\tilde{\boldsymbol{F}}$$
$$\tilde{\boldsymbol{F}} = \left[\,-\left(\boldsymbol{G^y}\,J_{\boldsymbol{uu}}^{-1}\,J_{\boldsymbol{ud}} - \boldsymbol{G^y_d}\right)\,\boldsymbol{W_d}\;\boldsymbol{W_{n^y}}\,\right].$$

Here, the matrices $J_{\boldsymbol{uu}}$ and $J_{\boldsymbol{ud}}$ indicate the second derivatives (Hessians) of the cost function J with respect to $\boldsymbol{u}$ and $\boldsymbol{d}$. The disturbance variation $\boldsymbol{\Delta d}$ and the implementation error $\boldsymbol{n_y}$ are commonly represented by the scaled variable $\boldsymbol{f}$, *i.e.*,

$$\begin{bmatrix}\boldsymbol{\Delta d}\\ \boldsymbol{n_y}\end{bmatrix} = \begin{bmatrix}\boldsymbol{W_d} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{W_{n^y}}\end{bmatrix}\boldsymbol{f} \text{ with } \|\boldsymbol{f}\|_2 \le 1,$$

where the matrices $\boldsymbol{W_d}$ and $\boldsymbol{W_{n^y}}$ are diagonal scaling matrices. From observation of (3) and (4) it is evident that a self-optimizing CSD with least worst-case loss may be obtained by solving the problem

$$\boldsymbol{H} = \arg\min_{\boldsymbol{H}} \overline{\sigma}(\boldsymbol{M}), \quad (5)$$

where $\overline{\sigma}$ indicates the largest singular value. In a more recent work, Kariwala et al. [2008] proved that the average loss given by

$$L_{\text{average}} = \frac{1}{6\,(n_{\boldsymbol{y}} + n_{\boldsymbol{d}})}\,\|\boldsymbol{M}\|_{\text{F}}^2 \quad (6)$$

is a better estimate of the loss as the worst-case loss (3) tends to overestimation. Here, $\|.\|_F$ indicates the Frobenius norm also known as the Euclidean norm. Besides, Kariwala et al. [2008] proved that the average loss is super-optimal in the sense that it also minimizes the worst-case loss. According to (6), they suggested solving

$$\boldsymbol{H} = \arg\min_{\boldsymbol{H}} \|\boldsymbol{M}\|_{\text{F}} \quad (7)$$

instead of (5).

The solution of problems (5) and (7) is nontrivial since the matrix $\boldsymbol{M}$ depends in a nonlinear fashion on $\boldsymbol{H}$. Moreover, the problem may be structurally constrained, as indicated in Section 1. *E.g.*, the dimension of the PV subset could be limited or special PVs may be excluded from PV subset etc. Many authors such as Alstad and Skogestad [2007], Alstad et al. [2008], Kariwala [2007] and Kariwala et al. [2008] addressed the problem of finding a global solution to either (5) or (7) with focus on PV combination. All of these

methods are limited to the structural constraint that the same PV subset is used for each CV. Based on the method developed in the next section, iterative solution strategies will be developed which focus on finding CSDs without structural limitations except for rank $(\boldsymbol{H}) = n_{\boldsymbol{u}}$.

3. THE GSVD METHOD

In this section a new solution method is presented for the worst-case and average loss problem, (5) and (7), subject to a common PV subset for all CVs. It will be referred to as the GSVD method. In a first step, (4) is restated as

$$\boldsymbol{z}^T \left(\boldsymbol{G}_{\boldsymbol{z}}^{\boldsymbol{y}}\right)^T \boldsymbol{H}^T = \boldsymbol{f}^T \tilde{\boldsymbol{F}}^T \boldsymbol{H}^T, \tag{8}$$

where

$$\boldsymbol{G}_{\boldsymbol{z}}^{\boldsymbol{y}} = \boldsymbol{G}^{\boldsymbol{y}} J_{\boldsymbol{uu}}^{-1/2}.$$

Suppose, that the rank condition

$$\operatorname{rank}\left(\begin{bmatrix} \left(\boldsymbol{G}_{\boldsymbol{z}}^{\boldsymbol{y}}\right)^T \\ \tilde{\boldsymbol{F}}^T \end{bmatrix}\right) = n_{\boldsymbol{y}} \tag{9}$$

is satisfied which will be generally the case if the condition $n_{\boldsymbol{u}} + n_{\boldsymbol{f}} \geq n_{\boldsymbol{y}}$ holds (the case $n_{\boldsymbol{u}} + n_{\boldsymbol{f}} < n_{\boldsymbol{y}}$ is discussed below in Remark 4). Then, according to Hogben [2007, p. 15.12f], the generalized singular value decomposition (GSVD) of the matrix pair $\left\{\left(\boldsymbol{G}_{\boldsymbol{z}}^{\boldsymbol{y}}\right)^T, \tilde{\boldsymbol{F}}^T\right\}$ exists and (8) can be written as

$$\boldsymbol{z}^T \boldsymbol{U} \boldsymbol{\Sigma} \boldsymbol{V}^T \boldsymbol{H}^T = \boldsymbol{f}^T \tilde{\boldsymbol{U}} \tilde{\boldsymbol{\Sigma}} \boldsymbol{V}^T \boldsymbol{H}^T, \tag{10}$$

where the decomposed matrices have the following properties.

The matrices $\boldsymbol{U} \in \mathbb{R}^{n_{\boldsymbol{u}} \times n_{\boldsymbol{u}}}$ and $\tilde{\boldsymbol{U}} \in \mathbb{R}^{n_{\boldsymbol{f}} \times n_{\boldsymbol{f}}}$ are unitary, *i.e.*, $\boldsymbol{U}^T \boldsymbol{U} = \boldsymbol{I}_{n_{\boldsymbol{u}}}$ and $\tilde{\boldsymbol{U}}^T \tilde{\boldsymbol{U}} = \boldsymbol{I}_{n_{\boldsymbol{f}}}$. The matrix $\boldsymbol{V} \in \mathbb{R}^{n_{\boldsymbol{y}} \times n_{\boldsymbol{y}}}$ is regular. The matrix $\boldsymbol{\Sigma} \in \mathbb{R}^{n_{\boldsymbol{u}} \times n_{\boldsymbol{y}}}$ is tailing diagonal, with $\boldsymbol{\Sigma}^T \boldsymbol{\Sigma} = \operatorname{diag}\left(\alpha_1^2, \alpha_2^2, ..., \alpha_{n_{\boldsymbol{y}}}^2\right)$ and $0 \leq \alpha_i \leq \alpha_{i+1} \leq 1$. The matrix $\tilde{\boldsymbol{\Sigma}} \in \mathbb{R}^{n_{\boldsymbol{f}} \times n_{\boldsymbol{y}}}$ is leading diagonal, with $\tilde{\boldsymbol{\Sigma}}^T \tilde{\boldsymbol{\Sigma}} = \operatorname{diag}\left(\beta_1, \beta_2, ..., \beta_{n_{\boldsymbol{y}}}\right)$ and $1 \geq \beta_i \geq \beta_{i+1} \geq 0$. Note that the number of $r = \max\left(0, n_{\boldsymbol{y}} - n_{\boldsymbol{u}}\right)$ leading α_i and β_i are 0 and 1, respectively, and that the number of $s = \max\left(0, n_{\boldsymbol{y}} - n_{\boldsymbol{f}}\right)$ tailing α_i and β_i are 1 and 0, respectively. For more information on GSVD and on how the resulting matrices can be computed, the reader is referred to standard linear algebra textbooks, *e.g.*, Golub and VanLoan [1996, pp. 465-467].

Theorem 1. If $n_{\boldsymbol{y}} \geq n_{\boldsymbol{u}}$, the minimum worst-case and average loss are given by

$$L_{\text{worst}} = \frac{1}{2} \left(\frac{\beta_{r+1}}{\alpha_{r+1}}\right)^2 \tag{11}$$

and

$$L_{\text{average}} = \frac{1}{6\ (n_{\boldsymbol{y}} + n_{\boldsymbol{d}})} \sum_{i=r+1}^{n_{\boldsymbol{y}}} \left(\frac{\beta_i}{\alpha_i}\right)^2, \tag{12}$$

respectively. They may be obtained by selecting

$$\boldsymbol{H} = \boldsymbol{M}_{\boldsymbol{n}} \begin{bmatrix} \boldsymbol{p}_{r+1} & \cdots & \boldsymbol{p}_{n_{\boldsymbol{y}}} \end{bmatrix}^T,$$

where $\boldsymbol{p}_i$ is the i^{th} column of $\boldsymbol{P} = \boldsymbol{V}^{-T}$ and $\boldsymbol{M}_{\boldsymbol{n}} \in \mathbb{R}^{n_{\boldsymbol{u}} \times n_{\boldsymbol{u}}}$ is an arbitrary regular matrix.

Proof. By selecting $\boldsymbol{H} = \boldsymbol{M}_{\boldsymbol{n}} \begin{bmatrix} \boldsymbol{p}_{r+1} & \cdots & \boldsymbol{p}_{n_{\boldsymbol{y}}} \end{bmatrix}^T$ it follows that

$$\boldsymbol{V}^T \boldsymbol{H}^T = \boldsymbol{V}^T \begin{bmatrix} \boldsymbol{p}_{r+1} & \cdots & \boldsymbol{p}_{n_{\boldsymbol{y}}} \end{bmatrix} \boldsymbol{M}_{\boldsymbol{n}}^T = \begin{bmatrix} \boldsymbol{0}_{r \times n_{\boldsymbol{u}}} \\ \boldsymbol{I}_{n_{\boldsymbol{u}}} \end{bmatrix} \boldsymbol{M}_{\boldsymbol{n}}^T. \tag{13}$$

and

$$\boldsymbol{\Sigma}\, \boldsymbol{V}^T \boldsymbol{H}^T = \operatorname{diag}\left(\alpha_{r+1}, ..., \alpha_{n_{\boldsymbol{y}}}\right) \boldsymbol{M}_{\boldsymbol{n}}^T$$

$$\tilde{\boldsymbol{\Sigma}}\, \boldsymbol{V}^T \boldsymbol{H}^T = \begin{bmatrix} \boldsymbol{0}_{r \times n_{\boldsymbol{u}}} \\ \operatorname{diag}\left(\beta_{r+1}, ..., \beta_{n_{\boldsymbol{y}}}\right) \\ \boldsymbol{0}_{\bar{s} \times n_{\boldsymbol{u}}} \end{bmatrix} \boldsymbol{M}_{\boldsymbol{n}}^T,$$

where $\bar{s} = \max\left(0, n_{\boldsymbol{f}} - n_{\boldsymbol{y}}\right)$. Inserting these results into (10) yields

$$\boldsymbol{z}^T = \boldsymbol{f}^T \underbrace{\tilde{\boldsymbol{U}} \begin{bmatrix} \boldsymbol{0}_{r \times n_{\boldsymbol{u}}} \\ \operatorname{diag}\left(\sigma_{r+1}, ..., \sigma_{n_{\boldsymbol{y}}}\right) \\ \boldsymbol{0}_{\bar{s} \times n_{\boldsymbol{u}}} \end{bmatrix} \boldsymbol{U}^T}_{=\boldsymbol{M}^T},$$

where $\sigma_i = \beta_i / \alpha_i$ indicate the i^{th} largest generalized singular value of the matrix pair $\left\{\left(\boldsymbol{G}_{\boldsymbol{z}}^{\boldsymbol{y}}\right)^T, \tilde{\boldsymbol{F}}^T\right\}$. Note that the maximum singular value and the Frobenius norm of a matrix are invariant to unitary transformations thereof. Thus, the worst-case loss (3) and the average loss (6) depend only on the selected generalized singular values and the derivation of (11) and (12) is trivial. As the minimum generalized singular values were selected, both, the worst-case loss and the average loss are minimal.

Remark 2. The GSVD method is written in terms of the complete PV set $\mathcal{Y}$. Note that it work as well for a selected PV subset $\mathcal{Y}_{\text{c}} \subseteq \mathcal{Y}$. Then, in all formulas stated above the respective rows in $\boldsymbol{G}_{\boldsymbol{z}}^{\boldsymbol{y}}$ and $\tilde{\boldsymbol{F}}$ must be extracted and $n_{\boldsymbol{y}}$ must be substituted by $n_{\mathcal{Y}_{\text{c}}}$.

Remark 3. For perfect disturbance rejection, *i.e.*, $\boldsymbol{M} = \boldsymbol{0}$, it is required that at least $n_{\boldsymbol{u}}$ tailing β_i are 0. As indicated above, the number of $s = \max\left(0, n_{\boldsymbol{y}} - n_{\boldsymbol{f}}\right)$ tailing β_i are in fact 0. Thus, perfect disturbance rejection occurs if the inequality

$$s \geq n_{\boldsymbol{u}}$$

is satisfied. The necessary condition therefore is $s > 0$, *i.e.*, $n_{\boldsymbol{f}} < n_{\boldsymbol{y}}$, which can only be satisfied if the implementation error is disregarded, *i.e.*, $\boldsymbol{W}_{\boldsymbol{n}^{\boldsymbol{y}}} = \emptyset$ and $n_{\boldsymbol{f}} = n_{\boldsymbol{d}}$. The sufficient condition for perfect disturbance rejection is then $n_{\boldsymbol{y}} \geq n_{\boldsymbol{u}} + n_{\boldsymbol{d}}$. This is in agreement with the combination methods from Alstad and Skogestad [2007], Alstad et al. [2008] and Kariwala [2007], Kariwala et al. [2008].

Remark 4. If $n_{\boldsymbol{u}} + n_{\boldsymbol{f}} < n_{\boldsymbol{y}}$, the rank condition (9) is violated and the GSVD as stated above cannot be performed. Note that this is only a formal issue and will not be treated here for the sake of brevity.

Remark 5. The GSVD method is related to the method by Kariwala et al. [2008] and the "constrained average loss minimization" method by Alstad et al. [2008]. It can be shown that all three methods minimize the average loss subject to the same structural constraint and thus provide the same results. However, it will be omitted here for the sake of brevity.

4. BEYOND COMMON PV SUBSETS

For better legibility of this section, definitions for CSDs will be introduced.

Definition 6. A CSD is said to be *column-structured*, if all CVs are linear combinations of the same PV subset $\mathcal{Y}_\mathrm{c}$ of size $n_{\mathcal{Y}_\mathrm{c}} = \dim(\mathcal{Y}_\mathrm{c})$. A *common-sized* CSD refers to a CSD in which the i^{th} CV is a linear combination of an individual PV subset $\mathcal{Y}_i$ with the constraint that all PV subsets have the same set size $n_\mathrm{s} = \dim(\mathcal{Y}_i)\,\forall i \in \{1,\dots,n_{\boldsymbol{u}}\}$. In a more general *loosely-structured* CSD, the i^{th} CV is a linear combination of an individual PV subset $\mathcal{Y}_i$ with individual set size $n_{\mathcal{Y}_i} = \dim(\mathcal{Y}_i)$.

Theorem 7. Let $\boldsymbol{H}_\mathrm{c}$ represent a column-structured CSD of size $n_{\mathcal{Y}_\mathrm{c}}$ with finite worst-case/average loss, *i.e.*, $\operatorname{rank}(\boldsymbol{H}_\mathrm{c}) = n_{\boldsymbol{u}}$. Then, for every $\boldsymbol{H}_\mathrm{c}$ there exists a common-sized CSD $\boldsymbol{H}_\mathrm{s}$ with a PV subset size of $n_\mathrm{s} = n_{\mathcal{Y}_\mathrm{c}} - (n_{\boldsymbol{u}} - 1)$ and the same worst-case/average loss as $\boldsymbol{H}_\mathrm{c}$. The proof will be omitted due to the lack of space.

Corollary 8. Let $\boldsymbol{H}_\mathrm{c}$ be a column-structured CSD with PV subset size $n_\mathrm{c} = n_{\boldsymbol{u}}$ and finite loss. Then, the worst-case/average loss of $\boldsymbol{H}_\mathrm{c}$ is independent of the coefficients in $\boldsymbol{H}_\mathrm{c}$. Rather, the worst-case/average loss of $\boldsymbol{H}_\mathrm{c}$ depends only on the selection of the PV subset $\mathcal{Y}_\mathrm{c}$. The proof will be omitted due to the lack of space.

Some advantages of common-sized and loosely-structured CSDs over column-structured CSDs are pointed out below.

(1) A smaller PV subset size is, on the one hand, favorable due to better practical acceptance but, on the other hand, usually accompanied by a larger worst-case/average loss. From Theorem 7 it can be concluded that for $n_{\boldsymbol{u}} > 1$ a reduction in PV subset size without affecting the worst-case/average loss can be achieved if, instead of a column-structured CSD, a common-sized CSD is taken into account. In particular, the PV subset size reduction with invariant worst-case/average loss can be as large as $n_{\boldsymbol{u}} - 1$ PVs.
(2) By implication of the first argument, it is evident that a smaller worst-case/average loss can be achieved if, instead of a column-structured CSD, a common-sized CSD with equal PV subset size is taken into account.
(3) For $n_{\mathcal{Y}_\mathrm{c}} = n_{\boldsymbol{u}}$ the optimality of column-structured CSDs is only a matter of PV subset selection as pointed out in Corollary 8 presented above.
(4) Column-structured CSDs $\boldsymbol{H}_\mathrm{c}$ fail if $n_{\mathcal{Y}_\mathrm{c}} < n_{\boldsymbol{u}}$ holds. This is due to the fact that $\operatorname{rank}(\boldsymbol{H}_\mathrm{c}) < n_{\boldsymbol{u}}$ which leads to a singular $\boldsymbol{H}_\mathrm{c}\,\boldsymbol{G}^{\boldsymbol{y}}$ and, by observation of (4) and (5), to infinite loss.
(5) Input/output (I/O) selection based on heuristic rules is a common practice. Physical closeness between CVs and MVs is probably the most common rule, in order to achieve good cause and effect between MVs and CVs. If decentralized controllers are used and the MVs are far apart from each other (*e.g.*, in large scale processes), it is desirable to have an individual PV subset for every CV as in common-sized and loosely-structured CSDs.
(6) PV combinations including different measurement units have poor practical acceptance. If the structural constraint was imposed on the prospective CSD that only PVs of the same type can be selected for each CV, then, in the case of a column-structured CSD with $n_{\boldsymbol{u}} > 1$, one would be forced to omit information of all PVs not part of the selected unit group. In common-sized and loosely-structured CSDs, for each CV another PV subset can be selected which allows to use information of more than only one unit group.

For loosely-structured CSDs, no explicit expression for $\boldsymbol{H}$ can be derived by the solution to problems (5) and (7). Thus, iterative solution methods need to be applied. In the following, a framework for advanced iterative methods will be presented.

In order to take loosely-structured CSDs into account, (4) is restated as

$$\boldsymbol{z}^T \sum_{i=1}^{n_{\boldsymbol{u}}} (\boldsymbol{G}_{\boldsymbol{z}}^{\boldsymbol{y}})^T\, \boldsymbol{h}_i\, \boldsymbol{e}_i^T = \boldsymbol{f}^T \sum_{i=1}^{n_{\boldsymbol{u}}} \tilde{\boldsymbol{F}}^T\, \boldsymbol{h}_i\, \boldsymbol{e}_i^T, \tag{14}$$

where $\boldsymbol{h}_i^T \in \mathbb{R}^{n_{\boldsymbol{y}}}$ is the i^{th} row vector of $\boldsymbol{H}$, *i.e.*, $\boldsymbol{H}^T = [\,\boldsymbol{h}_1\ \dots\ \boldsymbol{h}_{n_{\boldsymbol{u}}}\,]$, and $\boldsymbol{e}_i \in \mathbb{R}^{n_{\boldsymbol{u}}}$ is the i^{th} standard basis vector. The vector $\boldsymbol{h}_i$ represents the map from the PVs of the subset $\mathcal{Y}_i$ towards the i^{th} CV, hence $h_{ij} = 0 \forall j \notin \mathcal{Y}_i$. It is thus convenient to write (14) as

$$\boldsymbol{z}^T \sum_{i=1}^{n_{\boldsymbol{u}}} (\boldsymbol{G}_{\boldsymbol{z}}^{\boldsymbol{y}})_{\mathcal{Y}_i}^T\, \boldsymbol{h}_{i\mathcal{Y}_i}\, \boldsymbol{e}_i^T = \boldsymbol{f}^T \sum_{i=1}^{n_{\boldsymbol{u}}} \tilde{\boldsymbol{F}}_{\mathcal{Y}_i}^T\, \boldsymbol{h}_{i\mathcal{Y}_i}\, \boldsymbol{e}_i^T, \tag{15}$$

where the subscript $\mathcal{Y}_i$ denotes that those columns/elements of $(\boldsymbol{G}_{\boldsymbol{z}}^{\boldsymbol{y}})^T$, $\tilde{\boldsymbol{F}}^T$ and $\boldsymbol{h}_i$ are selected whose index is part of $\mathcal{Y}_i$.

By performing the GSVD of the corresponding matrix pairs $\left\{(\boldsymbol{G}_{\boldsymbol{z}}^{\boldsymbol{y}})_{\mathcal{Y}_i}^T, \tilde{\boldsymbol{F}}_{\mathcal{Y}_i}^T\right\}$, (15) can be written as

$$\boldsymbol{z}^T \underbrace{\sum_{i=1}^{n_{\boldsymbol{u}}} \boldsymbol{U}_i\, \boldsymbol{I}_{n_{\boldsymbol{u}} \times n_{\mathcal{Y}_i}}\, \tilde{\boldsymbol{h}}_i\, \boldsymbol{e}_i^T}_{=\boldsymbol{X}} = \boldsymbol{f}^T \sum_{i=1}^{n_{\boldsymbol{u}}} \tilde{\boldsymbol{U}}_i\, \boldsymbol{\mathcal{S}}_i\, \tilde{\boldsymbol{h}}_i\, \boldsymbol{e}_i^T, \tag{16}$$

where $\boldsymbol{\mathcal{S}}_i = \left[\, \boldsymbol{0}_{n_{\mathcal{Y}_i} \times r_i}\ \operatorname{diag}\left(\sigma_{i1}, ..., \sigma_{in_{\mathcal{Y}_i}}\right)\ \boldsymbol{0}_{n_{\mathcal{Y}_i} \times \bar{s}_i}\, \right]^T$, $r_i = \max(0, n_{\mathcal{Y}_i} - n_{\boldsymbol{u}})$, $\bar{s}_i = \max(0, n_{\boldsymbol{f}} - n_{\mathcal{Y}_i})$ and $\sigma_{ij} = \begin{cases} \alpha_{ij}/\beta_{ij} & \text{if } \beta_{ij} \neq 0 \\ 1 & \text{otherwise} \end{cases}$; $\boldsymbol{U}_i \in \mathbb{R}^{n_{\boldsymbol{u}} \times n_{\boldsymbol{u}}}$ and $\tilde{\boldsymbol{U}}_i \in \mathbb{R}^{n_{\boldsymbol{f}} \times n_{\boldsymbol{f}}}$ are unitary; $\boldsymbol{I}_{n_{\boldsymbol{u}} \times n_{\mathcal{Y}_i}}$ is the $n_{\boldsymbol{u}} \times n_{\mathcal{Y}_i}$ tailing diagonal identity matrix; and $\tilde{\boldsymbol{h}}_i = \operatorname{diag}\left(\beta_{i(r_i+1)}, ..., \beta_{in_{\mathcal{Y}_i}}\right) \boldsymbol{V}_i^T\, \boldsymbol{h}_{i\mathcal{Y}_i}$, with $\boldsymbol{V}_i \in \mathbb{R}^{n_{\mathcal{Y}_i} \times n_{\mathcal{Y}_i}}$ if $n_{\boldsymbol{u}} + n_{\boldsymbol{f}} \geq n_{\mathcal{Y}_i}$. The decomposed formulation (16) has several advantages over (10) as pointed out below.

(1) From observation of (16) it can be seen that the first $r_i = \max(0, n_{\mathcal{Y}_i} - n_{\boldsymbol{u}})$ elements in $\tilde{\boldsymbol{h}}_i$ do not contribute to $\boldsymbol{X}$. It is generally close to optimal to set them to zero as the corresponding columns in $\tilde{\boldsymbol{U}}_i\, \boldsymbol{\mathcal{S}}_i$ vanish. Thus, if any $r_i > 0$ then one can reduced the variable space from $\sum_{i=1}^{n_{\boldsymbol{u}}} n_{\mathcal{Y}_i}$ to $\sum_{i=1}^{n_{\boldsymbol{u}}} n_{\mathcal{Y}_i} - r_i$. The maximum dimension of the reduced space is $n_{\boldsymbol{u}} \times n_{\boldsymbol{u}}$. (16). This approach will be referred to as the reduced space (RC) method. The starting values for the RC method will be $\tilde{\boldsymbol{h}}_i = \boldsymbol{e}_{n_{\mathcal{Y}_i}}$ which corresponds to the selection of the smallest generalized singular values.
(2) From (16), it can be shown that a suboptimal solution to (7) can be obtained by setting the first r_i elements in $\tilde{\boldsymbol{h}}_i$ are zero and solving the substitute problem

$$\boldsymbol{X} = \arg\min_{\boldsymbol{X}} \sum_{i=1}^{n_{\boldsymbol{u}}} \boldsymbol{X}_i^T\, \tilde{\boldsymbol{\mathcal{S}}}_i^T\, \tilde{\boldsymbol{\mathcal{S}}}_i\, \boldsymbol{X}_i \text{ s.t. } \boldsymbol{X}^T\, \boldsymbol{X} = \boldsymbol{I}, \tag{17}$$

where $\tilde{\boldsymbol{\mathcal{S}}}_i = \boldsymbol{\mathcal{S}}_i\, \boldsymbol{I}_{n_{\boldsymbol{u}} \times n_{\mathcal{Y}_i}}^T\, \boldsymbol{U}_i^T$ and $\boldsymbol{X}_i$ is the i^{th} column of $\boldsymbol{X}$. Problem (17) is still nonconvex but has the

advantage that an efficient steepest descend method can be developed. Due to the lack of space, the method cannot be outlined here but will be an issue of a subsequent publication. It will be referred to as the unitary matrix constraint (UMC) method. The starting value for the iterative solution of $\boldsymbol{X}$ will be the identity matrix.

(3) From (16) a lower bound for the minimum worst-case/average loss can be derived. This is particularly helpful in reducing computational expense as described below.

The large number of alternative control structures can be reduced by excluding candidate CVs which cannot lead to an optimal solution. This strategy is known as the BAB principle. BAB algorithms have been formerly applied to CSD problems by several authors such as Cao and Saha [2005], Kariwala and Skogestad [2006/07/09-13], Cao and Kariwala [2008] and Kariwala and Cao [2009]. Lower bounds on the minimal worst-case/average loss are helpful for discriminating candidate CVs. From the conclusion that the lower bounds corresponds to the ideal case that $\tilde{\boldsymbol{h}}_i = \boldsymbol{e}_{n_{\mathcal{Y}_i}}, \boldsymbol{U}_{i\mathcal{Y}_i} \perp \boldsymbol{U}_{j\mathcal{Y}_j}$ and $\tilde{\boldsymbol{U}}_{i\mathcal{Y}_i} \perp \tilde{\boldsymbol{U}}_{j\mathcal{Y}_j} \forall i \neq j$ it follows from observation of (16) that

$$\boldsymbol{M} = \sum_{i=1}^{n_{\boldsymbol{u}}} \sigma_{in_{\mathcal{Y}_i}} \boldsymbol{e}_i \boldsymbol{e}_i^T .$$

This yields the inequalities

$$L_{\text{worst}} \geq \frac{1}{2} \max_i \left(\sigma^2_{in_{\mathcal{Y}_i}} \right) \tag{18}$$

$$L_{\text{average}} \geq \frac{1}{6\ (n_{\mathcal{Y}_\cap} + n_{\boldsymbol{d}})} \sum_{i=1}^{n_{\boldsymbol{u}}} \sigma^2_{in_{\mathcal{Y}_i}} . \tag{19}$$

Note that $n_{\mathcal{Y}_\cap}$ indicates the size of the merged PV subset $\mathcal{Y}_1 \cap \cdots \cap \mathcal{Y}_{n_{\boldsymbol{u}}}$. It is important to state that (18) and (19) also hold for incomplete set of candidate CVs, *i.e.*, the lower bound of one candidate CV is also the lower bound of all possible control structures which include this CV. If an upper bound for the worts-case/average loss of all alternatives L_{ub} is known, the evaluation of structures (and substructures) can be omitted which show a lower bound $L_{\text{lb}} > L_{\text{ub}}$. Unfortunately, the bounds given above are not very tight, so that computational savings are relatively small.

5. EVAPORATOR CASE STUDY

In this section, the proposed CSD methods will be applied to the evaporation process presented in Figure 3. This forced-circulation evaporation was originally treated by Newell and Lee [1989] and has been investigated subsequently by Heath et al. [2000] and Kariwala et al. [2008], among others. The purpose of the process is the concentration of dilute liquor from the feed to the product stream by evaporation and separation of the solvent. The analytic model equations including the cost function and operational constraints can be found in Kariwala et al. [2008]. The process model has three state variables, the level L_2, the composition X_2 and the pressure P_2 with eight degrees of freedom. Table 1 lists the important stream properties, their value at the nominal operating point and

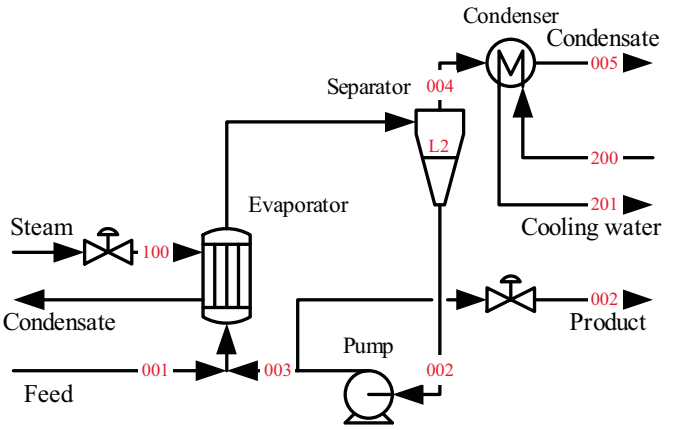

Figure 3. Evaporation process scheme

Var.	Description	Nominal value	Classification
F_1	Feed flow rate	9.469 kg/min	MV, PV (±2 %)
F_2	Product flow rate	1.334 kg/min	MV†, PV (±2 %)
F_3	Circulating flow rate	24.721 kg/min	MV†, PV (±2 %)
F_4	Vapor flow rate	8.135 kg/min	
F_5	Condensate flow rate	8.135 kg/min	PV (±2 %)
X_1	Feed composition	5.00 %	DV (±5 %)
X_2	Product composition	35.50 %	
T_1	Feed temperature	40.0 °C	DV (±20 %)
T_2	Product temperature	88.4 °C	PV (±1 °C)
T_3	Vapor temperature	81.066 °C	PV (±1 °C)
p_2	Operating pressure	51.412 kPa	PV (±2.5 %)
F_{100}	Steam flow rate	9.434 kg/min	PV (±2 %)
T_{100}	Steam temperature	151.52 °C	
p_{100}	Steam pressure	400.0 kPa	MV†
Q_{100}	Heat duty	345.292 kW	
F_{200}	Water flow rate	217.738 kg/min	MV, PV (±2 %)
T_{200}	Water inlet temp.	25.0 °C	DV (±20 %)
T_{201}	Water outlet temp.	45.55 °C	PV (±1 °C)
Q_{200}	Condenser duty	313.21 kW	
J	Operational cost	-582.233 $/h	

Table 1. Key process variables in the evaporation process

their classification into MVs, DVs and PVs. Three out of five MVs indicated by † are used to keep the three PVs L_2, X_2 and P_{100} at their set points. Note that the level in the separator L_2 has no steady-state effect but needs to be controlled for stabilization. The other two controlled PVs need to be kept at their constraints in order to achieve optimality over the given disturbance region. Generality is not lost by this particular selection of the unconstrained MVs. In Table 1, the (embraced) expected variations of the DVs and measurement errors of the PVs are given in % from their nominal value except for temperature measurement errors which are indicated on an absolute scale.

The model equations were implemented in a modeling environment (ME) of the in-house tool OPTISIM® [1], an equation-oriented process simulator. The model has been optimized with respect to the DVs' nominal values given in Table 1 and operational constraints. This led to the operating conditions of the MVs and DVs presented in Table 1. As the ME provides first derivatives by automatic differentiation, the linear I/O gains $G_{\boldsymbol{u}}^{\boldsymbol{y}}$ and $G_{\boldsymbol{u}}^{\boldsymbol{y}}$ at the operating point are directly available. Second derivatives $J_{\boldsymbol{uu}}$ and $J_{\boldsymbol{ud}}$ were estimated by finite difference approx-

[1] OPTISIM® is a registered trademark of the Linde AG. (Burr [1991/4/7-11])

$n_{\mathcal{Y}_c}$	Best PV set	L_{average} (in \$/h)	L_{worst} (in \$/h)
2	F_3, F_{200}	3.8079	56.7126
3	F_2, F_{100}, F_{200}	0.6533	11.6643
4	F_2, T_{201}, F_3, F_{200}	0.4545	9.4516
...			
10	All PVs	0.1941	7.5015

Table 2. Worst-case/average loss of best column-structured CSDs

imation and the use of the NAG routine E04XAF. The numerical results of the I/O gains and the Hessians are in agreement with those of Kariwala et al. [2008]. CSDs for the evaporation process have been identified using the methods presented in Sections 3 and 4. The calculations were conducted in Matlab®R2008b using a Windows XP SP2 desktop with an Intel®Core™Duo CPU E8400 (3.0 Ghz, 3.5 GB RAM).

At first, column-structured CSDs were identified by average loss minimization using the GSVD method. The best control structure was determined by screening over all possibilities satisfying the PV subset size condition imposed. Some results are given in Table 2. They reproduce the results by Kariwala et al. [2008] with an deviation of less than 0.6 %. Both, the minimum worst-case and average loss of the best structure decrease with the PV subset size and approach a lower bound (at $n_{\mathcal{Y}_c} = 10$) asymptotically. According to Corollary 8, the case $n_{\mathcal{Y}_c} = n_{\boldsymbol{u}} = 2$ indicates as well the best PV selection structure.

Next, common-sized CSDs PVs were sought. According to Theorem 7, for each $n_{\mathcal{Y}_c}$-sized column-structured CSD there exists a common-sized CSD of size $n_s = (n_{\mathcal{Y}_c} - n_{\boldsymbol{u}} + 1)$ which can be obtained by a simple linear transformation of the former. Thus, the results in Table 2 indicate also possible common-sized CSDs of PV subset size n_s from one to nine. For instance, Table 3 shows the transformation of the best column-structured CSD with set size three, indicated by $\boldsymbol{H}_{c3}$, into $\boldsymbol{H}_{s2}$ where only combinations of two PVs per CV occur. Despite its small PV subset size, $\boldsymbol{H}_{s2}$ achieves a considerably small average loss. Note that CSDs obtained by this approach are generally not the best among all CSDs satisfying the particular structural constraint of a common-sized set with set size two. In order to find a CSD with lower average loss, the RC method was applied. The best solution found among the $\left(C_{10}^2\right)\left(C_{10}^2-1\right)/2 = 990$ alternatives is indicated as $\hat{\boldsymbol{H}}_{s2}$ in Table 3. Due to the BAB algorithm only 103 problems with an average of 0.07 s expense per problem had to be solved. The total computation time was 8.3 s. Using the UMC method, the computational efficiency could be reduced to 6 ms expense per problem leading to a total computation time of 3.3 s at 144 iterative problem solutions. The solution $\tilde{\boldsymbol{H}}_{s2}$ showed a larger average loss than $\hat{\boldsymbol{H}}_{s2}$ structurally identical though.

Suppose that due to cost issues, only one flow meter can be afforded. Since temperature and pressure indicators are rather cheap, their numbers are not limited by cost considerations. In this situation, the task is to find the two best CVs out of $C_6^1 + 2$ candidates, *i.e.*, one out of six flows, one pressure and one temperature set. The best CSD indicated as $\boldsymbol{H}_{s1F}$ in Table 3 was found by applying

CSD	L_{average} (in \$/h)
$\boldsymbol{H}_{c3} = \begin{bmatrix} -0.99 & 0.15 & 0.00 \\ -0.99 & -0.12 & 0.01 \end{bmatrix} \begin{bmatrix} F_2 & F_{100} & F_{200} \end{bmatrix}^T$	0.6533
$\boldsymbol{H}_{s2} = \begin{bmatrix} -6.27 & 1.0 \\ -143.08 & 1.0 \end{bmatrix} \begin{bmatrix} F_2 & F_{100} & F_{200} \end{bmatrix}^T$	0.6533
$\hat{\boldsymbol{H}}_{s2} = \begin{bmatrix} -6.27 & 1.0 & \\ & & 1.0 & -23.30 \end{bmatrix} \begin{bmatrix} F_2 & F_{100} & F_{200} & F_1 \end{bmatrix}^T$	0.5673
$\tilde{\boldsymbol{H}}_{s2} = \begin{bmatrix} -6.22 & 1.0 & \\ & & 1.0 & -13.34 \end{bmatrix} \begin{bmatrix} F_2 & F_{100} & F_{200} & F_1 \end{bmatrix}^T$	0.6682
$\boldsymbol{H}_{s1F} = \begin{bmatrix} 1.0 & & & \\ & 0.36 & 0.33 & 0.87 \end{bmatrix} \begin{bmatrix} F_3 & T_2 & T_3 & T_{201} \end{bmatrix}^T$	2.9704
$\boldsymbol{H}_{c3T} = \begin{bmatrix} 0.59 & 0.53 & -0.61 \\ 0.02 & 0.01 & 1.0 \end{bmatrix} \begin{bmatrix} T_2 & T_3 & T_{201} \end{bmatrix}^T$	3.6573

Table 3. CSD results

the RC method. It shows slightly better average loss than $\boldsymbol{H}_{c3T}$ which is the best column-structured CSD where all temperatures are used.

6. CONCLUSION

In this paper new insights into the identification problem of self-optimizing CSDs were given. The GSVD method was proposed which allows finding CVs, altogether linear combinations of a common PV subset. It minimizes the average loss super-optimal to the worst-case loss by taking expected disturbances and measurement errors into account. The GSVD method can be beneficially implemented into iterative solution approaches in order to find loosely-structured CSDs where for each CV an individual PV subsets is taken into account. The new methods were successfully applied to an evaporation process. It could be shown that loosely structured CSDs are favorable in terms of flexibility, practical acceptance and economic considerations.

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