Subspace closed loop identification using the integration of MOESP and N4SID methods

Santos Miranda, Claudio Garcia

Polytechnic School of the University of São Paulo.
e-mail: santos.borjas@poli.usp.br, clgarcia@lac.usp.br

Abstract: Linear identification of time invariant systems operating in closed loop is of special interest for a large number of engineering applications. There are different techniques and methods to carry out this type of identification. For example, modifying the N4SID method, one can derive a closed loop subspace identification method. The same can be done using the MOESP method. Based on them, the MON4SID method is introduced, which estimates the extended observability matrix and the state sequence directly from a LQ decomposition, using a combination of the techniques contained in both, MOESP and N4SID. This new method uses an algorithm to identify state space model of a plant in a closed loop system, in the same way as in MOESP method. The advantage of the proposed algorithm is that it does not require any knowledge about the controller, whereas such information is essential for other methods (e.g. N4SID). The disadvantage of this algorithm is that it needs a great amount of data to obtain better estimates. A simulated process to show the performance of this algorithm is presented.

Keywords: Subspace identification; closed loop identification; state space models.

1. INTRODUCTION

Great part of the literature referring to system identification deals with how to find polynomial models as Prediction Error Method (PEM). In case of complex systems, there is a parameterization problem in the PEM model, so the state space model appears as an alternative to PEM, such as Multivariable Output-Error State sPace (MOESP) (Verhaegen, 1994), Canonical Variate Analysis (CVA) (Larimore, 1990) and Numerical algorithms for Subspace State Space System Identification (N4SID) (Van Overschee; De Moor, 1996). Statistical properties such as consistency and efficiency of these algorithms were studied by (Bauer, 2003; Bauer; Ljung, 2002; Chiuso; Picci, 2005). One of the main assumptions of theses methods is that the process and the measurement noises are independent of the plant input. This assumption is violated when the system is working in closed loop. The closed loop identification is of special interest for a large number of engineering applications (Ljung, 1999), since closed loop experiments are necessary if the open loop plant is unstable, or the feedback is an inherent mechanism of the system (Forssell; Ljung, 1999; Van den Hof, 1997). Several closed loop identification methods have been suggested in the last years and can be broadly classified into three main groups: direct, indirect and joint input output identification methods (Forssell; Ljung, 1999). The results of any of the N4SID, MOESP and CVA methods cited above are asymptotically biased when closed loop identification is applied. To solve this problem, the MOESP method (Verhaegen, 1993) proposed a closed loop subspace identification method through the identification of an overall open loop state space. Based on it, the plant and the controller models are estimated. To do so, it is necessary to know the order of the controller. In the N4SID case (Van Overschee; De Moor, 1997) it is necessary to know a limited number of impulse response samples of the controller and, via direct identification, the plant model is estimated. There are other possible solutions to the closed loop identification problem; the reader can consult (Huang *et al.*, 2005; Katayama, 2005; Katayama *et al.*, 2002; Katayama *et al.*, 2005; Ljung; McKelvey, 1996; Qin *et al.*, 2005).

Combining the MOESP and N4SID methods, we obtain the MON4SID algorithm, which estimates the extended observability matrix in the same way it occurs in the MOESP method, the state sequence is computed through the oblique projection, as it is done in the N4SID method. From this sequence, the past and future states are obtained, and finally a consistent estimate of the system matrices is obtained, applying the least squares method. In this paper, it is proposed an algorithm to identify the state space model of a plant in a closed loop system, in the same way as it was proposed in the MOESP method, that first computes a global model from which is extracted the plant model. This method does not need any knowledge about the controller.

1.1 Open Loop Subspace identification

Consider a time discrete linear time invariant dynamic system described by the state space models in the innovation form:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ke_k \\ y_k &= Cx_k + Du_k + e_k \end{aligned} \tag{1}$$

where $u_k \in R^m$ and $y_k \in R^l$ denote the input and output signals, respectively and $x_k \in R^n$ is the vector of states. $e_k \in R^l$ is zero-mean Gaussian white noise and independent of past input and output data. *A*, *B*, *C*, *D* and *K* are matrices with appropriate dimensions.

1.2 Open Loop Subspace identification problem

The subspace identification problem is: given $u = \lfloor u_1,..,u_{nd} \rfloor$ and $y = \lfloor y_1,..,y_{nd} \rfloor$ a set of input-output measurements, determine the order *n* of the unknown system, the system matrices (*A*, *B*, *C*, *D*) up to within a similarity transformation and Kalman filter gain *K* (Van Overschee; De Moor, 1996).

1.3 Subspace matrix equation

Making successive iterations in equation (1), one can derive the following matrix equation:

$$Y_f = \Gamma_i X_f + H_i^d U_f + H_i^s E_f \tag{2}$$

where subscript f stands for the "future" and p for the "past". For the definition of the matrices H_i^d and H_i^s given in (2), see (Van Overschee; De Moor, 1996). The past and future input block-Hankel matrices are defined as:

$$\left(\frac{U_p}{U_f}\right) = \left(\begin{array}{cccc} u_0 & u_1 & \dots & u_{j-1} \\ u_1 & u_2 & \dots & u_j \\ \dots & \dots & \dots & \dots \\ u_{i-1} & u_i & \dots & u_{i+j-2} \\ \hline u_i & u_{i+1} & \dots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \dots & u_{i+j} \\ \dots & \dots & \dots & \dots \\ u_{2i-1} & u_{2i} & \dots & u_{2i+j-2} \end{array}\right) \tag{3}$$

where $U_p, U_f \in R^{mixN}$. The output and noise innovation block-Hankel matrices $Y_p, Y_f \in R^{lixN}$ and $E_p, E_f \in R^{mixN}$, respectively, are defined in a similar way to (3).

The states are defined as:

$$X_p = X_0 = \lfloor x_0,..,x_{j-1} \rfloor \tag{4}$$

$$X_f = X_i = \lfloor x_i,..,x_{i+j-1} \rfloor \tag{5}$$

The extended observability matrix Γ_i is given by:

$$\Gamma_i = \begin{pmatrix} C \\ CA \\ \dots \\ CA^{i-1} \end{pmatrix} \tag{6}$$

The orthogonal projection of the row space of A_x into the row space of B_x is:

$$A_x / B_x = A_x B_x^T (B_x B_x^T)^* B_x \tag{7}$$

where $(\bullet)^*$ denotes the Moore-Penrose pseudo-inverse of the matrix $(\bullet)$.

The projection of the row space of A_x into the orthogonal complement of the row space of B_x is:

$$A_x / B_x^\perp = A_x - A_x / B_x \tag{8}$$

The oblique projection of the row space of *G* along the row space *H* into the row space of *J* is:

$$G /_H J = (G / H^\perp) \cdot (J / H^\perp)^* \cdot J \tag{9}$$

Properties of the orthogonal and oblique projections:

$$A_x / A_x^\perp = 0 \tag{10}$$

$$A_x /_{A_x} C_x = 0 \tag{11}$$

For a proof, see (Van Overschee; De Moor, 1996).

2. PROPOSED IDENTIFICATION METHOD

2.1 MON4SID identification method

In this subsection, the MON4SID method is presented. To solve the problem in section 1.2, it is used the POMOESP method to calculate the extended observability matrix Γ_i and the N4SID method is employed to calculate the matrices *A*, *B*, *C*, *D* through the least squares method. Therefore, it is necessary to eliminate the last two terms in the right side of equation (2). That is done in two steps: first, eliminating the term $H_i^d U_f$ in (2), performing an orthogonal projection of equation (2) into the row space of $U_f^\perp$, which yields:

$$Y_f / U_f^\perp = \Gamma_i X_f / U_f^\perp + H_i^d U_f / U_f^\perp + H_i^s E_f / U_f^\perp \tag{12}$$

And by the property (10), equation (12) can be simplified to:

$$Y_f / U_f^\perp = \Gamma_i X_f / U_f^\perp + H_i^s E_f / U_f^\perp \tag{13}$$

Second, to eliminate the noises in (13), an instrumental variable $Z = (U_p^T \, Y_p^T)^T$ is defined. Multiplication of (13) by Z yields:

$$Y_f / U_f^\perp Z = \Gamma_i X_f / U_f^\perp Z + H_i^s E_f / U_f^\perp Z \tag{14}$$

As it is assumed that the noise is uncorrelated with input and output past data (Verhaegen; Dewilde, 1992), which means that $E_f / U_f^\perp Z = 0$. Therefore, (14) is written as:

$$Y_f / U_f^\perp Z = \Gamma_i \hat{X}_f \tag{15}$$

In equation (15), $X_f / U_f^\perp Z = \hat{X}_f$ is the estimate of the Kalman filter state. Equation (15) indicates that the column space of Γ_i can be calculated by the SVD decomposition of $Y_f / U_f^\perp Z$. For further details, see (Verhaegen; Dewilde, 1992). Γ_i, given in (15), can be derived from a simple LQ factorization of a matrix constructed from the block-Hankel matrices U_f, U_p and Y_f, Y_p, in the form:

$$\begin{pmatrix} U_f \\ Z_p \\ Y_f \end{pmatrix} = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix} \tag{16}$$

and the orthogonal projection in the left side of (15) can be computed by matrix L_{32} (Verhaegen; Dewilde, 1992). The SVD of L_{32} can be given as:

$$L_{32} = (U_1\ U_2)\begin{pmatrix} S_n & 0 \\ 0 & S_2 \end{pmatrix}\begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} = USV^T \tag{17}$$

The order n of the system is equal to the number of non-zero singular values in S. The column space of U_1 approximates that of Γ_i in a consistent way (Verhaegen; Dewilde, 1992), that is:

$$\Gamma_i = U_1 \tag{18}$$

The system (1) can be written as:

$$\begin{pmatrix} \widetilde{X}_{i+1} \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} \widetilde{X}_i \\ U_{i|i} \end{pmatrix} + \begin{pmatrix} r_i \\ r_2 \end{pmatrix} \tag{19}$$

In equation (19), suppose (ideally) that $\widetilde{X}_{i+1}$ and $\widetilde{X}_i$ are given, then the system matrices (A, B, C, D) could be computed through the least squares method. Therefore, the problem now is to find the state sequences.

$\Theta_i = Y_f /_{U_f} Z_p$ is the oblique projection (Van Overschee; De Moor, 1996), which is achieved by performing an oblique projection of equation (2), along the row space U_f onto the row space of Z_p, that is:

$$Y_f /_{U_f} Z_p = \Gamma_i X_f /_{U_f} Z_p + H_i^d U_f /_{U_f} Z_p + \\ + H_i^s E_f /_{U_f} Z_p \tag{20}$$

It is easy to see that the last two terms of equation (20) are zero, by the property of the oblique projection, equation (11); and by the assumption that the noise is uncorrelated with input and output past data (Van Overschee; De Moor, 1996). Thus, equation (20) can be simplified to:

$$Y_f /_{U_f} Z_p = \Gamma_i \widetilde{X}_i \tag{21}$$

where $\widetilde{X}_i = X_f /_{U_f} Z_p$. Then equation (21) is written as:

$$\Theta_i = \Gamma_i \widetilde{X}_i \tag{22}$$

The oblique projection Θ_i given in equation (22) can be computed from (16) by:

$$\Theta_i = Y_f / U_f W_p = L_{32}(L_{22})^{-1}(L_{21}\ L_{22})\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}. \tag{23}$$

An estimate of the state sequence X is given by:

$$X = (\Gamma_i)^* L_{32}(L_{22})^{-1} Z_p . \tag{24}$$

The following matrices are defined: $\widetilde{X}_i = X(:,1:N-1)$, $\widetilde{X}_{i+1} = X(:,2:N)$. Thus, the system matrices can be estimated from equation (19). To estimate K see (Van Overschee; De Moor, 1996) or (Verhaegen; Dewilde, 1992).

2.2 Closed loop identification method

Figure 1 shows a typical standard closed loop system, where P and C denote respectively the plant and the controller, r_k is the exogenous input, u_k the input control, y_k the plant output, w_k the plant disturbance and v_k the plant noise.

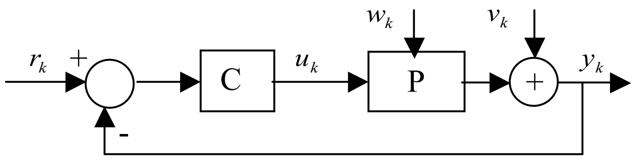

Figure1. Standard closed loop system.

P is given by equation (1) and C can be described by the following state equation:

$$\begin{aligned} s_{k+1} &= A_c s_k + B_c(r_k - y_k) \\ u_k &= C_c x_k + D_c(r_k - y_k) \end{aligned} \tag{25}$$

where A_c, B_c, C_c and D_c are matrices with appropriate dimensions.

2.3 Closed loop subspace identification problem

Given (r_k, u_k, y_k), a set of input output measurements finite data, of a well posed problem (Katayama, 2005), one considers the problem of identifying the deterministic part of the plant, that is, one determines the order n of the unknown system, the system matrices (A, B, C, D) up to within a similarity transformation.

2.3 Identification by joint input output approach

The objective of this paper is to obtain a state space model of the deterministic part of the plant P, based on finite measurement data (r_k, u_k, y_k). The present problem is practically the same as it was exposed in Verhaegen (1993), but the approach is quite different, as it is not necessary to know any information about the controller.

Using equations (1) and (25), it is possible to obtain a global state space model (Verhaegen, 1993):

$$\begin{aligned} \Psi_{k+1} &= \widetilde{A}\Psi_k + \widetilde{B}r_k + \Xi_k \\ \Phi_k &= \widetilde{C}\Psi_k + \widetilde{D}r_k + \Lambda_k \end{aligned} \tag{26}$$

where $\Psi_k = [x_k^T\ s_k^T]^T$, $\Phi_k = [u_k^T\ y_k^T]^T$. Ξ_k, Λ_k are noises and $\widetilde{A}, \widetilde{B}, \widetilde{C}, \widetilde{D}$ are matrices with appropriate dimensions.

The method MON4SID is applied to find an estimate of the matrices $\widetilde{A}, \widetilde{B}, \widetilde{C}, \widetilde{D}$, and based on them, to estimate Φ_k. Once Φ_k is known, it is easy to compute the matrices of the plant. To do so, the method POMOESP (Verhaegen, 1994) is used.

3. SIMULATION

In this section, we provide a simulation example to evaluate the performance of the MON4SID algorithm and to compare it with other existing identification algorithms PEM, N4SIDC and ARXS. N4SIDC here denotes the algorithm of Van Overschee and De Moor (1997) and ARXS the algorithm of Ljung and MacKelvey (1996). This example was used by (Huang *et al.*, 2004; Katayama, 2005; Overschee; De Moor, 1997 and Verhaegen, 1993). It is important to stress that the algorithm N4SIDC has three versions (Overschee; De Moor, 1997): two of them are unbiased and one is biased. The version implemented in this paper is the biased one, based on states. In this version, it is used two different initial conditions, one for past states and the other for future states, what causes the bias. For further details see (Overschee; De Moor, 1997).

The plant is a discrete time model of a laboratory plant setup of two circular plates rotated by an electrical servo motor with flexible shafts. For further details, see (Hakvoort, 1990). The model of the plant is given by equation (1), where:

$$A = \begin{bmatrix} 4.4 & 1 & 0 & 0 & 0 \\ -8.09 & 0 & 1 & 0 & 0 \\ 7.83 & 0 & 0 & 1 & 0 \\ -4 & 0 & 0 & 0 & 1 \\ 0.86 & 0 & 0 & 0 & 0 \end{bmatrix}, \; B = 10^{-3} \begin{bmatrix} 0.98 \\ 12.99 \\ 18.59 \\ 3.3 \\ -0.02 \end{bmatrix}, \; C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 2.3 \\ -6.64 \\ 7.515 \\ -4.0146 \\ 0.86336 \end{bmatrix} \text{ and } D = 0$$

e_k is the white noise, which generates the disturbance on the plant, with standard deviation equal to 0 (for the case of deterministic system), 0.001 (to denote a system of little noise) and 0.01 (to denote a system of high noise). The controller has a state space description as in the equation (25), where:

$$A_c = \begin{bmatrix} 2.65 & -3.11 & 1.75 & -0.39 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \; B_c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_C^T = \begin{bmatrix} -0.4135 \\ 0.8629 \\ -0.7625 \\ 0.2521 \end{bmatrix} \text{ and } D_c = 0.61$$

PRBS was used as an exogenous input signal, that is, persistently exciting of any finite order. There were collected 3000 samples and the number of block rows $i = 20$.

The simulation results for a closed loop deterministic identification without noise is shown in figure 2, where the order of the plant is n = 5. From figure 2, one can observe that all the algorithms had a good performance, apart from the algorithm N4SIDC, which had am improvement using n =7, as it is shown in figure 4. Figure 3 shows the poles of the original open loop plant and the estimated systems, where • denotes the original poles of the plant. One can see that all the algorithms had a good performance in relation to the estimation of the poles, which are on the unit circle.

To see an advantage of the proposed algorithm, a white noise is added to the plant, first with measurement noise variance 0.001 and then with 0.01. The comparison results are shown in figures 5 and 7 respectively.

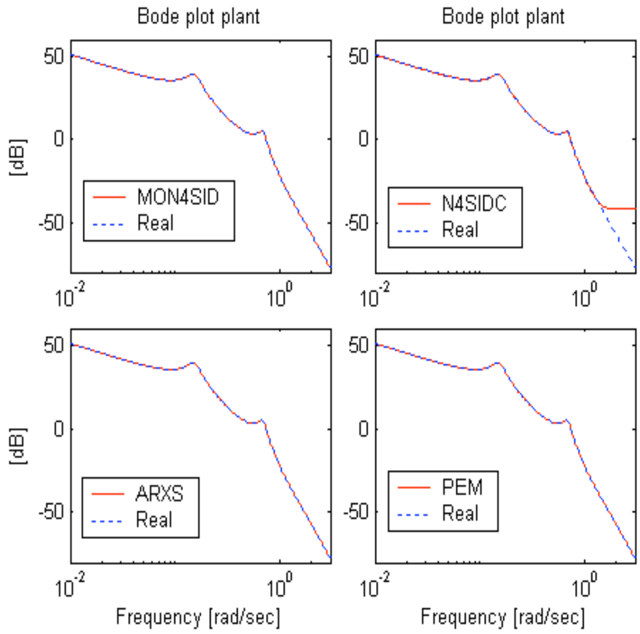

Fig.2. Bode plots of the plant P, to n=5 and no noise.

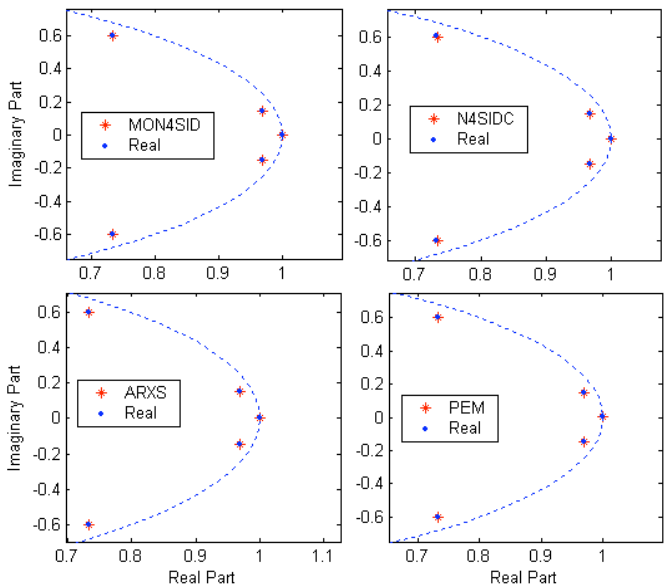

Fig. 3 Poles of the eigenvalues of the estimated A matrix.

From figures 5 and 7 one can see that the algorithm MON4SID performs better in the presence of noise. The order for identification of the plant is $n = 7$. Figures 6 and 8 show the pole estimates and the true poles of the plant. From figure 5 one can see that the algorithm ARXS does not have a good performance.

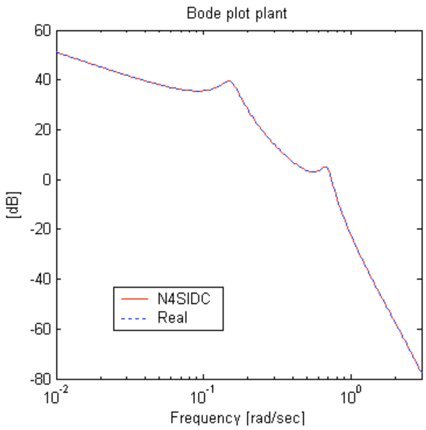

Fig.4. Bode plots of the plant P, to n=7 and no noise.

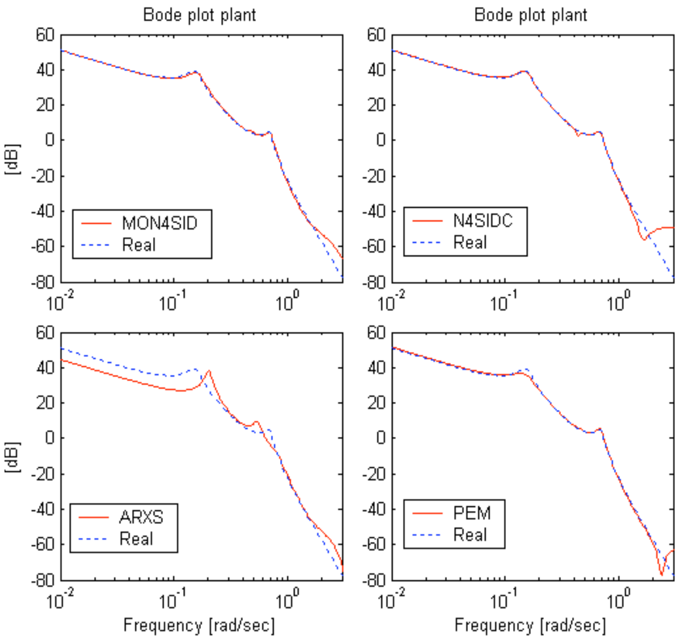

Fig.5. Bode plots of the plant P, to n=7 and little noise.

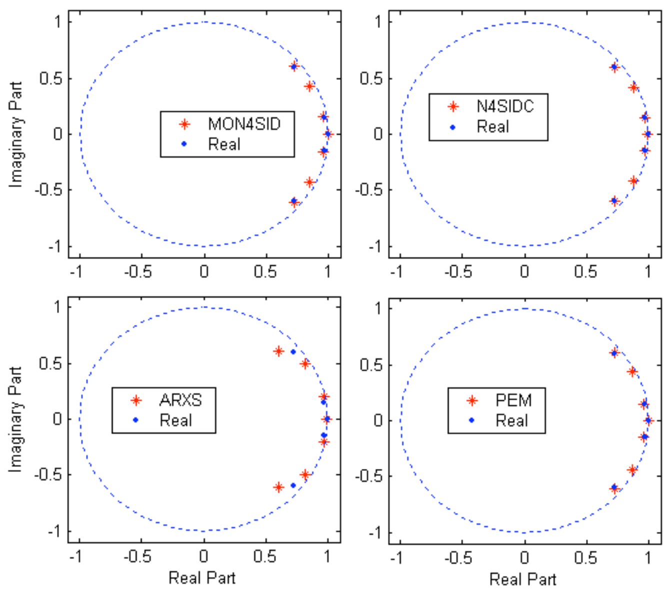

Fig.6 Poles of the eigenvalues of the estimated A matrix.

As can be noticed in figure 6 for the ARXS model, there is a difference between the estimated and real poles, what causes the difference between the real and estimated plots in figure 5.

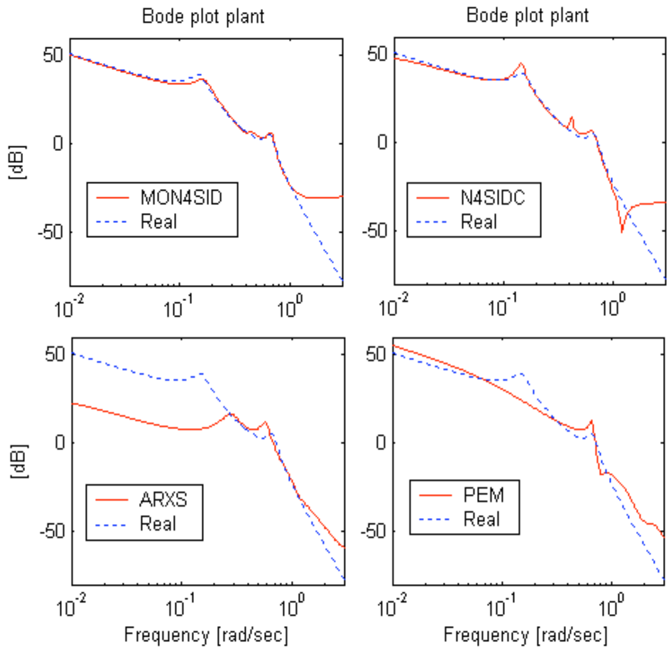

Fig.7. Bode plots of the plant P, to n=7 and high noise.

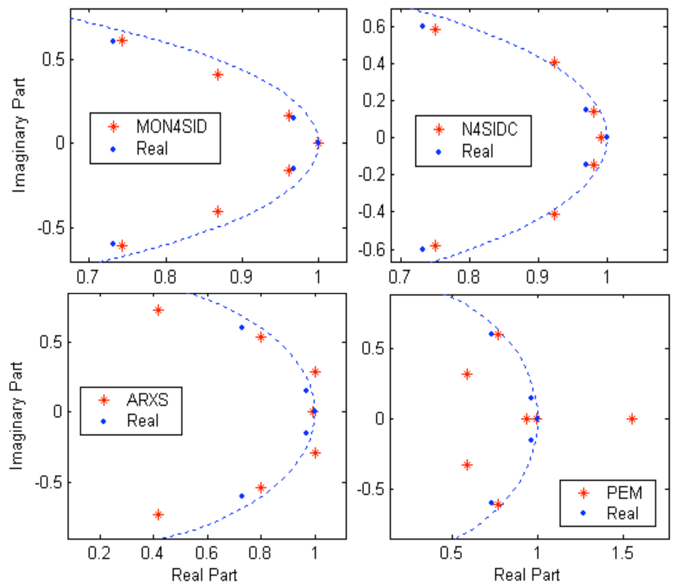

Fig.8 Poles of the eigenvalues of the estimated A matrix.

From figure 8 it can be seen that the MON4SID method provides a better estimation of the most crucial pole and all the poles are inside the unit circle. This does not happen for the other methods. Based on figure 8 one can say that direct identification models do not have a good performance for closed loop identification in the presence of high noises.

6. CONCLUSIONS

In this work, the MON4SID algorithm is presented, which uses LQ factorization in the same way as the MOESP method, which is used to compute the oblique and orthogonal projections; these projections are used to compute the state sequence and the extended observability matrix, respectively. The past and future state sequences are computed from the state sequences, which have only one initial state. It does not happen in the N4SID method, because for each oblique projection (Θ_i and Θ_{i+1}) different state sequences ($\widetilde{X}_1$ and $\widetilde{X}_{i+1}$) are computed, generating a problem of bias in the estimates.

This algorithm was compared with three identification algorithms (PEM, N4SIDC, ARXS), when applied to a simulated example, which was used in (Van Overschee; De Moor, 1997) to identify a plant model in discrete time state space. Their results were compared by means of Bode plot and the comparison of the estimated poles with the true poles. The algorithm MON4SID presented good performance in all the cases. This algorithm has an advantage over the N4SIDC algorithm in the sense that it does not need any knowledge about the controller.

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