DISTRIBUTED DECISION MAKING IN SUPPLY CHAIN NETWORKS

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Abstract: The supply chain system is modeled as a "Value Added Network" (VAN) which performs the following tasks: assembly, storage, routing, processing and transportation. Many activities interact and complexity increases as the number of business activities and links in the VAN increase. In order to develop a model which can deal with changing market conditions and evolving technology it is necessary to adapt as the supplier and demand structures change. Recent developments focus on decentralized business structures and software solutions to reduce complexity and maintain scalability. It has been claimed that decentralized decisions lead to sub-optimal solutions. We show that this is not necessarily so. We present novel abstraction of an integrated system of decision makers, software and physical devices which allows for optimal decentralized decision making. The objective function captures the idea that investment and resource use decisions in a VAN (capacity expansion expansion, how much inventory to carry, which markets to address and which technology to use) carries value. The decentralized decision making processes we cover may be quite complex and may include local feedback corrections as well as decentralized, optimal (model predictive) strategies.

Keywords: Distributed control, supply chain management, self-optimization, optimal control, inventory control, flow control.

1. INTRODUCTION

Information Technology (IT) tools can be used to improve the resource allocation, flow of materials and diffusion of knowledge within companies and entire enterprises. Enterprise Resource Planning (ERP) systems supplied by companies like SAP, i2, Oracle, J.D. Edwards and others integrate business processes, streamline production systems and provide company wide access to information related to critical work processes [13]. Such systems can also be used to track inventory levels, identify bottle necks, smooth flows and evaluate performance. Impressive gains have been reported in a great variety of industries, including the computer industry (hardware and software), discrete parts manufacture and commodity chemicals [1,4,12,14,11].

The application of ERP tools has made it apparent that it does not suffice to focus on the internal processes alone. Upsets are often created by factors beyond the control of a single company. This led to the development of Advanced Planning Systems (APS) that link the database capabilities of the ERP system to market forecasts and process models. Such tools enable a company to evaluate scenarios and respond to changes in the market place by applying feedforward and planning. However, it is clear that improved agility and better performance can be achieved by application of active feedback and tools from process systems theory, like distributed control and real time optimization [8].

In our context a supply chain is thought of as a "network of organizations that are involved, through upstream and downstream linkages, in different processes and products [3]." The objective of the supply chain is to *create value* through a sequence of operations which we refer to as activities. Such activities include assembly, storage, routing, processing and transportation. In this context Stadtler and Kilger [13] define Supply Chain Management (SCM) as "the task of integrating organizational units along a supply chain and coordinating materials, information, and financial flows in order to fulfill (ultimate) customer demands with the aim of improving competitiveness of a supply chain as a whole." The SCM perspective therefore includes the idea of two or more legally separate partners working together towards a common goal within a business sector.

A number of models of supply chain systems have been developed. Recently, control theory methods have been introduced to manage and adapt flows within the supply chain so that it remains competitive in the market place. For example, a centralized Model Predictive Control (MPC) framework for optimization of supply chains was developed in [9,8]. These models are suitable in static systems where the models and boundary conditions do not change.

In the current paper we are interested in models that are flexible, adaptive and self-optimizing. This approach leads to the study of structural properties, stability and optimality using distributed feedback in lieu of centralized planning. The study of industrial dynamics and feedback control was advanced further by Forrester [5] who elucidated an instability in supply chains referred to as demand amplification. His ideas on feedback loops and systems theory formed the basis for very fruitful developments that continue to have a significant impact to this day [10].

2. SUPPLY CHAIN NETWORKS

A supply chain system (SCS) is an integrated network of activities which transports and transforms assets so that their intrinsic values [2] are maximized. An asset may be a tangible product like a gallon of oil, a piece needed in an assembly line or an intangible item like an order, information or intellectual property. An increase in value may be the result of an asset been transported to a location closer to the customer, a transformation (e.g. chemical reaction or assembly) or because time progresses and market parameters change. The objective of this section is to describe the conservation laws that constrain the dynamic behavior of assets in the supply chain. In the next section we introduce the value function.

Consider an SCS with n distinct assets a_i. The index i identifies an asset by its name, SKU-number, chemical composition or some other index which should be unique. The asset space $A = \{a_i\}$ defines the nature of the business. The amount (inventory) of each asset is given by a non-negative real number $v_i(x,t)$, where x denotes the location and t denotes the time. The vector of inventories is represented by the vector $v^T = (v_1, ..., v_n)$.

The topology of the SCS is represented by the graph $\mathcal{G} = \{\mathcal{H}, \mathcal{A}\}$. $\mathcal{H}$ represents the set of edges, along which we allow assets to flow, while $\mathcal{A}$ represents vertices where assets are stored, transformed, shipped or routed. A non-empty collection of edges and vertices is called an *activity*.

We find it sometimes useful to introduce a little more structure and distinguish among four different classes of activities. These include *transportation, manufacture, storage, terminals (shipping/receiving)* and *routing*. This additional structure allows us to define the Supply Chain Graph [1]:

$$\mathcal{G} = \{\mathcal{H}, \underbrace{\mathcal{M}, \mathcal{S}, \mathcal{T}, \mathcal{R}}_{\mathcal{A}}\}$$

Elements, $h_i \in \mathcal{H}, i = 1, ..., n_h$ denote the transportation of assets. Elements $m_i \in \mathcal{M}, i = 1, ..., n_p$ represent the manufacturing with assembly or disassembly of chemical constituents or parts into pre-cursors and products. Elements $s_i \in \mathcal{S}, i = 1, ..., n_s$ denote the storage facilities. Elements $t_i \in \mathcal{T}, i = 1, ..., n_t$ denote terminals for receiving and shipping. Elements $r_i \in \mathcal{R}, i = 1, ..., n_r$ represent points where material, energy, money and data can be routed in different directions.

Example 1. Consider the production facility shown in Figure 1. There is a terminal where materials are received from the supplier. There are storage locations for raw materials and products next to the terminal, an assembly plant, storage for finished products and a shipping terminal. All nodes are connected by edges representing flow of assets. More vertices and edges can be added to represent flow of services, orders, information, capital and energy. There are two routing points in this figure. At routing point 1 decisions are made

[1] The notation and order has been chosen in memory of our beloved hamster TicTac

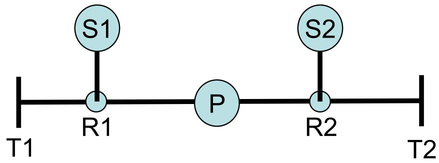

Fig. 1. Graph of an activity in a supply chain system consisting of two terminals, two storage locations, one production facility and six transportation links.

about sending raw materials to storage (Storage 1) or production. At routing point 2 decisions are made about sending finished products to the plant warehouse (Storage 2) or shipping.

We now develop the conservation laws that govern the transformation and flow of assets.

(1) *Transportation:* Asset flow is represented using the hyperbolic, partial differential equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial f}{\partial x} = 0$$

This equation model describes a pipeline where $\rho(x,t)$ is the local "density of an asset" at the point x and time t while $f(x,t)$ is the local flow rate.

(2) *Manufacture:* Manufacture is represented as a source or sink so that

$$f_i = p$$

where p is rate of production/consumption of an asset. This notation allows us to model transformation of assets via assembly or disassembly.

(3) *Storage:* The rate of change in storage is given by the differential equation

$$f_i = \frac{dv}{dt}, \qquad v(0) = v_0$$

where v_0 is the amount stored initially. Negative f_i denotes flow out of storage whereas positive f_i denotes flow into storage. This type of storage, referred to as "tanque pulmon", represents a capacitor in an electrical network.

(4) *Terminals:* Applying the conservation law to the terminal gives

$$0 = f_i + f_T \tag{1}$$

where f_T is the shipping/receiving rate. Receiving is positive and shipping is negative.

(5) *Routing points:* Asset flow through routing points, like terminals, is conserved. We therefore have

$$0 = \sum_{\text{Connections}} f_i \tag{2}$$

The summation is carried out so that the index i ranges over all edges connected to the corresponding routing point in the network.

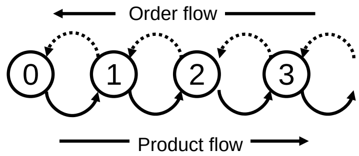

Fig. 2. Three echelon supply chain network, representing retailer, distribution center, warehouse, and production center.

An activity is an arbitrary collection of the basic building blocks. By combining building blocks and eliminating the internal flows we see that the dynamics of activities are represented by the inventory balance

$$\frac{dv}{dt} = \sum_{\text{Terminals}} f_{T,i} + p \tag{3}$$

where

$$p = \sum_{\text{Production sites}} p_i$$

It is often convenient to use projected and transformed variables so that $\bar{v} = Tv$ where T is a linear operator. T is often non-square and projects the high dimensional asset space into a lower dimensional space. It is possible to let T be a differential operator (to allow prediction) and/or an integral operator (the Fourier-Laplace transforms for example).

Example 2. In the last decade the world changed from a marketplace with several large independent markets to a highly integrated global market that demands a large variety of products and services complying with high quality, reliability and environmental standards. Furthermore, the fast development of new products as well as customer focus and increasing competitiveness pose new challenges in the area of modeling and control of global supply chain systems. Here we will develop a model control technique carried out in cooperation with Unilever.

The problem we consider is illustrated in Figure 2. This system has three echelons corresponding to the retailer, distribution center and plant warehouse. There are two classes of flows, one corresponding to the flow of orders and the other the flow of goods in response to the demand. There is only one product is this example.

For the flow and storage of goods we have

$$\frac{dI_i}{dt} = f_{i-1,i} - f_{i,i+1}, \qquad i = 1,2,3$$

where I_1 is the inventory in the plant warehouse, I_2 is the inventory in the distribution center and I_3 is the inventory at the retailer. We develop a similar equation for the order flow so that

$$\frac{dO_i}{dt} = f^o_{i-1,i} - f^o_{i,i+1}, \qquad i = 1,2,3,4$$

Where O_0 is the backlog of orders in the plant, O_1 the backlog for the plant warehouse, O_2 is the backlog in the distribution center and O_3 is the back at the retail level.

The objective is to ensure a high level of service at the retail level. In this example we will work on the basis that we should have $f_{3,4} = f^o_{3,4}$ indicating that the demand is satisfied exactly. We furthermore want to achieve this objective without carrying too high inventory anywhere in the supply chain system.

There are 4 inventory flows and 3 inventories, 4 order flows and 4 order levels in this problem. So there are 15 variables. It follows that we have $15-7=8$ degrees of freedom. These correspond to the flows that must be managed. According to the objectives we would like to manage these flows so that inventories and back-orders follow setpoints so that

$$O_i = O_i^*, \qquad i = 1,2,3$$
$$I_i = I_i^*, \qquad i = 1,2,3$$

Ideally we would like to use $O_i^* = I_i^* = 0$ indicating the the inventory and order levels are equal to zero [2].

Starting at the retailer level we see that we should use the following inventory controller for the order buffer,

$$f_{3,4} = f^o_{3,4} + K_O(O_3 - O_3^*)$$

This means that the we deliver product at the rate of incoming orders plus a proportionality constant times the size of the back-orders. This distributed control policy quickly converges so that $f_{3,4} = f^o_{3,4}$ indicating that we deliver at the same rate as the orders come. If we want to track a specific order then the average delay in fulfilling the order is given by the number

$$d = O/f$$

The policy can only be implemented if there is sufficient material in storage to fulfill the orders at the rate given by the inventory controller. In order to ensure that the inventory is also controlled we need to use another inventory controller for the retail storage. Ideally we would like to set

$$f_{2,3} = f_{3,4} - K_I(I_3 - I_3^*)$$

This means that we use a combination of feedforward and feedback control to manage the inventory.

However, this method cannot be used exactly as indicated since the retailer does not control the rate of arrivals directly. The retailer has to send an order to the distribution center and wait until the order is fulfilled. The controller for inventory therefore becomes

$$f^o_{2,3} = f_{3,4} - K_I(I_3 - I_3^*)$$

Indicating that orders are sent to the distribution center at the same rate that material is shipped plus a term which is proportional to current inventory level.

The distribution centers and plant warehouse use a similar policy. We will assume for now that the production plant is very responsive so that we can set

$$f^o_{0,1} = f_{0,1}$$

Indicating that the plant warehouse is re-stocked as soon as an order is sent.

Applying these idea to the entire supply chain gives

$$f_{i,i+1} = f^0_{i+1,i} + K_O(O_i - O_i^*), \qquad i = 0,1,2$$
$$f_{i-1,i} = f^0_{i-1,i} - K_I(I_i - I_i^*), \qquad i = 0,1,2$$

There are seven of these controllers so there is now one degree of freedom, corresponding to the demand rate at the retail level, which acts as a disturbance. This effect can be seen by developing the closed loop expression for the supply chain. The inventories are seen to satisfy the expression

$$\frac{dI_i}{dt} = -K_I(I_i - I_i^*) + \Delta_i(t)$$

where

$$\Delta_i(t) = f^0_{i-1,i} - f_{i-1,i}$$

represents the discrepancy between the order rate and supply rate to node i. If this is equal to zero then the supply chain system is stable. If this number is not equal to zero then the supply chain dynamics may exhibit instabilities, and disturbances may even be amplified causing bullwhip.

The problem we consider is how to manage the flow through, routing points, terminals, storage and production sites so that assets flow through the system and are distributed in the best manner. In order to solve this problem we must assign values to the assets as functions of time and location in the SCS.

3. VALUE ADDED NETWORKS

The instantaneous profit is the difference between the revenues from sales and the activity costs:

$$P = R - C \qquad (4)$$

This measure is also called the *rate of accounting earnings*. Integrated and discounted over time into

[2] Walmart is a company that has been able to move in this direction by elimination of distribution centers.

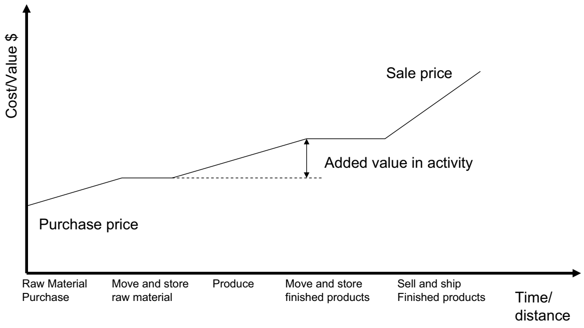

Fig. 3. The generation of value through the production process.

the future the expected accounting earnings gives an indication of the performance of the SCS.

Approaching the supply chain management problem from the point of view of maximizing the discounted income in a distributed network of activities in this way results in a type of analysis, called activity based analysis [6].

We now need to make some basic assumptions about the supply chain system.

Assumption 1. *Consider a supply chain system.*

(1) The inventory of assets represents the state of the system.
(2) There a exists a positively homogeneous degree one function $A(v)$ which defines the value of the assets.
(3) Any activity cost is positive.

The first assumption provides the concept of state. The basic idea here is that the state of a company can be defined by determining the magnitude of its assets. The second assumption implies that the value of the company (for example the discounted cash value) can be expressed in terms of its current state and that it is a homogeneous function. The third assumption states that all activities cost something. The cornerstone for our developments is then given by the Legendre-Fenchel dual

$$A^*(c) = \max_v (A(v) - c^T v) \tag{5}$$

The vector c represents the value of adding one unit of the corresponding asset to the inventory at location x. We see that c represents the Lagrange multiplier corresponding to the inventory vector.

Euler's theorem for homogeneous functions gives

$$A = v^T c = \sum_{\text{Assets}} v_i^T c_i \tag{6}$$

We connect this equation with the inventory balances (3) which define the dynamics of the SCS process. First we note the following orthogonality relationship

$$v^T dc = 0 \tag{7}$$

which is referred to as the Gibbs-Duhem equation. By differentiating $A(v)$ we have

$$\frac{dA}{dt} = c^T \frac{dv}{dt}$$

We now use equation (3) with equation (7) to give

$$\frac{dA}{dt} = c_r^T r - c_p^T s + p_A \tag{8}$$

where c_r and c_p represent the per unit value of the resource and the product and the variable

$$p_A = \underbrace{(c - c_r)^T r - (c - c_p)^T s}_{\text{Transportation}} + \underbrace{c^T p}_{\text{Production}} \tag{9}$$

represents the activity cost. The activity cost is positive (in accordance with Assumption A3. This leads to the following important result.

Lemma 1. *The value $A(v)$ is concave.*

Proof. Follows from the homogeneous degree one property and positivity of p_A. □

The result is important since it shows that value based analysis in supply chain systems can be approached using convex optimization.

The cost is the sum of the cost of resources and activities so that

$$C = p_A + c_r^T r$$

Combining this expression with equations (4) and (8) gives

$$\frac{dP}{dt} = R - \left(\frac{dA}{dt} + c_p^T s\right) \tag{10}$$

By using the definition for the income from sales we get

$$\frac{dP}{dt} = \sum_{\text{Sales}} (c_{s,i} - c_{p,i}) s_i - \frac{dA}{dt} \tag{11}$$

In this expression we let $c_{s,i}$ denote the ith component of the vector c_s and $c_{p,i}$ denote the ith component of the vector p_s. We note that $c_{s,i}$ denotes the sales price whereas as $c_{p,i}$ denotes the price "at cost" for item i. We therefore have

$$c_{s,i} - c_{p,i} = \begin{cases} > 0, & \text{sell @ profit} \\ = 0, & \text{sell @ cost} \\ < 0, & \text{sell @ loss} \end{cases}$$

In the case $c_s - c_p = 0$ there is no mark-up. This is often the case for internal customers and the cost c_p is then referred to as a "transfer price".

Expressions (9) and (11) highlight the main issues in supply-chain management [3] :

(1) The profit increases at a faster rate when the markup $c_s - c_p > 0$ is large and sales volume high. Larger markup can be achieved by raising the per unit price of the item sold. But

[3] There can be a considerable phase shift between the movement of goods and the associated financial transaction. Ignoring this phase shift is referred to as accrual.

higher prices also tend to give reduced sales. This expression emphasize the importance of marketing and sales.

(2) The profit can be increased by reducing current inventory and fixed assets since we have $dA/dt < 0$. This expression emphasizes the importance of being "lean" [14].

(3) The transportation and production costs as defined in (9) should be minimized. This expression emphasizes the importance of planning, scheduling and process control and new process technology.

The added value of a path consisting of several activities is given by the formula

$$w = \sum_{\text{Segments}} w_i \tag{12}$$

where w_i represents the added value of each subactivity. This number may be positive, zero or negative and it does not depend on the path taken since the function $A(v)$ is unique. It follows that for a cyclical activity we have

$$0 = \sum_{\text{Loop}} w_i \tag{13}$$

This expression conveys the idea that there is no value in a cyclical activity. However there is cost associated with every activity, and cyclical activities therefore add cost but no value.

Just like for the conservation laws (3), it is convenient to project or transform the activity costs $\bar{w} = Lw$, and $\bar{c} = Lc$. Equation (13) still holds if these transformations are linear.

We now have the following extremely important result for transportation, storage and production in an SCS.

Theorem 1. *Consider an supply chain network with with linear network operators T and L. We have*

$$\sum_{storage} \frac{d\bar{v}}{dt}^T \bar{c} = \sum_{transportation} \bar{f}^T \bar{w} + \sum_{production} \bar{p}^T \bar{c} + \sum_{terminals} \bar{c}^T \bar{f}$$

Proof. See [7] □

This result expresses the interesting fact that the spaces of inventories and are cost variables are orthogonal.

4. OPTIMALITY OF DECENTRALIZED DECISION MAKING

The problem we want to solve is how to stabilize the dynamics and balance the load in the supply chain while maximizing the intrinsic value. The discussion given above shows that we can formulate this problem so that

$$\min_{f_i, p_i} \sum_{i=1}^{M} A(v_i)$$

subject to equations (3) and (5). In centralized decision making all information is collected and the problem is solved using all available information. In decentralized decision making the problem is solved by distributing computational effort amongst the node points. In either case we want to implement the strategies using feedback laws of the type

$$f = \hat{f}(w), p = \hat{p}(c)$$

where $\hat{f}, \hat{p}$ determine the transportation and production rates as functions of the cost.

In order to develop production schedules that balance system load, we need to evaluate the activity costs and their sensitivity with respect to changes in the activity rate. In the simplest case this may be a linear function with a downward trend to reflect discounts for larger volumes. Let Δ be the difference operator so that for any variable z we have $\Delta z = z_2 - z_1$.

Definition 1. *An activity is said to be* positive *if for any f_1 and f_2*

$$\Delta f \Delta w \geq 0$$

and for any p_1 and p_2

$$\Delta p \Delta c \geq 0$$

Positive rate ensures that the cost of a given activity does not increase with increasing traffic. Examples include the barrier function, which describes capacity constraints, gradient directions that result from optimization of convex cost functions and more generally any cost which is monotonic in the sense that higher added value gives incentive to larger shipments. We may for example have

$$f = 0, \text{ if } w < w_{\min} \text{ and } f = f_{\max} \text{ otherwise}$$

and

$$p = 0, \text{ if } c < c_{\min} \text{ and } p = p_{\max} \text{ otherwise}$$

In this case there is no activity if the value added is below a certain threshold and we operate at maximum capacity otherwise.

The activity costs are used to solve load balancing and resource allocation problems since they show how the cost varies with respect to production volume. Without such costs load balancing is not a well posed problem.

We now show that the decentralized policy solves the optimal control problem. We proceed in two steps. We first show that the decentralized control system is stable and converges to a unique solution provided the boundary conditions are fixed. We then show that the obtained stationary point is optimal.

Theorem 2. *Consider an enterprise network with fixed boundary costs and positive feedback controls. The inventories are then stable and converge to stationary values.*

Proof. Details given in full length paper available from the authors. □

Theorem 3. *Consider an enterprise network with fixed boundary costs and positive feedback controls. The total activity cost is then minimized.*

Proof. Details given in full length paper available from the authors. □

These two theorems show that there exists a unique, stationary solution to the enterprise network. This solution, furthermore is stable and optimized under decentralized control policies. The concavity result given in the previous section shows that optimum is global due to the concavity of A.

5. SUMMARY AND CONCLUSIONS

Distributed decision making in supply chain systems arises naturally in several ways: The systems we want to model are distributed since process segments, business units and enterprizes are integrated into a complex, diverse and highly dynamic global market. Information, physical infrastructure and human resources are distributed across the globe and the computer networks we use for information exchange are also distributed. It is often thought that decentralized decision making is sub-optimal. In this paper we show that this not necessarily the case. Optimal and stable decision making processes can be constructed when we modeled the SCN as a VAN with assembly, storage, routing, processing and transportation. The decentralized decision making processes may be quite complex and may include local feedback corrections as well as decentralized, optimal (MPC) strategies. The use of distributed decision making allows the topology of the network to change and adapt as new needs arise. Old subsystems can be exchanged with newer ones, new products and processes can be brought on line and new businesses can be added or old ones closed without changing the overall management strategy.

REFERENCES

[1] Backx, T., O. Bosgra and W. Marquardt (1998), "Towards intentional dynamics in suplly chain conscious process operations", *Proc. 3rd Int. Conf. on Foundations of Computer Aided Process Operations, AIChE* , p. 5

[2] Buffett, W. (2002), "Berkshire Hathaway, Shareholder Letters", `www.berkshirehathaway.com/letters.html`

[3] Christopher, M. (1998), *Logistics and Supply Chain Management - strategies for reducing cost and improving service, 2nd Ed.*, London et al. UK.

[4] Ettl, M., G.E. Feigin, G. Y. Lin and D.D. Yao (1996), " A supply network model with base stock control and service requirements ", *IBM Research Report* , RC 20473 (06/04/96) Computer Science/Mathematics

[5] Forrester, J. W. . (1979), *Industrial Dynamics*, MIT Press, Cambridge MA

[6] Helfert, E. A. (2001), *Financial Analysis Tools and Techniques*, McGraw Hill, New York

[7] Jillson, K. R. and Ydstie, B. E.(2005), Complex Process Networks: Passivity and Optimality, *IFAC Triennal World Congress*, Prague, July 2005.

[8] Perea Lopez, E. (2002), *Centralized and decentralized approaches for Supply Chain Management based on Dynamic Optimization and Control*, PhD Thesis, Carnegie Mellon

[9] Perea Lopez, E., I, E. Grossmann and B.E. Ydstie (2000), "Towards the integration of dynamics and control for supply chain management ", *Computers and Chemical Engineering*, Vol 24, pp 1143-1150.

[10] Sterman, J. D. (2000), *Business Dynamics: Systems Thinking and Modeling for Complex World*, McGraw Hill, NY.

[11] Sery, S., V. Presti and D. E. Shobrys (2001), "Optimization models for restructuring BASF North America's distribution system ", *Interfaces*, Vol 31, pp 55-65.

[12] Selcuk Erenguc, S, N.C. Simpson and A .J. Vakharia(1999), "Integrated production/distribution planning in supply chains: an invited review", *European Journal of Operations Research* Vol 115, pp. 219- 236

[13] Stadtler, H. and C. Kilger (2000), *Supply Chain Management and Advanced Planning* , Springer Verlag, Berlin.

[14] Taylor, D. and D. Brunt (2000), *Manufacturing operations and supply chain management: The lean approach*, Thomson Learning, London.