MULTIVARIABLE FUZZY IDENTIFICATION APPROACH APPLIED TO COMPLEX LIQUID RESIDUES INCINERATION PROCESS

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Abstract: This paper proposes an identification scheme for a complex liquid effluent incinerator process. This scheme is developed to obtain a MIMO (*Multiple Input Single Output*) TS (*Takagi-Sugeno*) fuzzy model where the modified Gath Geva clustering algorithm is used to determine the antecedent part as the consequent parameters are estimated by RLS (*Recursive Least Square*) algorithm.

Keywords: Multivariable system identification, Liquid effluent incinerator, Fuzzy systems, Takagi-Sugeno fuzzy model, Recursive least square.

1. INTRODUCTION

Techniques of systems identification are widely used in control systems design and successful applications have appeared at last two decades. In a typical adaptive control design, a valid model of the dynamic system, in one of some operating conditions, is identified on-line, and the controller design is carried according to this model so that some performance specifications are satisfied (Serra, and Bottura, 2006a). In systems identification literature (Ljung, 1999; Soderstrom, and Stroica, 1989), the most approaches are concerned to linear modelling and control using continuous or discrete time equations as well as state space ones. Moreover, motivated by the fact of all dynamic system present a nonlinear behaviour, several approaches have been proposed for analysis, identification and control, where fuzzy systems are key elements in these application (Khalil, 2002; Isidori, 1995; Wang, 1996; Pedrycz, and Gomide, 1998; Serra, and Bottura, 2006b).

Fuzzy systems is an effective tool for uncertain nonlinear systems identification based on measured data (Hellendoorn and Driankov, 1997). Among different fuzzy modelling techniques, the Takagi-Sugeno fuzzy model has attracted the most attention (Takagi and Sugeno, 1985). This model consists of “if-then” rules with fuzzy antecedents and mathematical functions in the consequent part. The antecedents fuzzy sets partition the input space into a number of fuzzy regions, while the consequent functions describe the system’s behavior in each region. The identification procedure of TS fuzzy models is usually done in two steps. Firstly, the antecedents parameters (membership functions parameters) are determined using knowledge of the process behavior or data-driven techniques. In the second step, the parameters of the consequent functions are estimated. As these consequent functions are linear in their parameters, the least-squares method can be applied.

A real world example of complex nonlinear dynamic system is the liquid residues incineration process (Cunha, 2003). It is part of the power unit at BASF industry, placed in Resende-Brazil. The necessity to study its dynamic behavior, which motivates a MIMO (*Multiple Input Single Output*) fuzzy identification scheme application, is due to the following reasons (Almeida and Barreto, 2004):

- Avoiding emission of gas from combustion out of ambient agency standards;
- Improving the reside burning efficiency to reduce the fuel consumption in the incinerator;
- Minimizing costs.

This paper proposes an identification scheme for a complex liquid effluent incinerator process. This scheme is developed to obtain a MIMO TS fuzzy model via modified Gath Geva clustering algorithm used to determine the antecedent parameters and RLS (*Recursive Lest Square*) method used to estimate the consequent parameters. Experimental results show the efficiency of the proposed scheme as well as the accuracy of the obtained models, so important characteristics in intelligent adaptive control design.

2. LIQUIDS EFFLUENTS INCINERATOR

The effluent liquids incinerator, whose study of its characteristics can be seen in (Cunha, 2003; Almeida and Barreto, 2005), receives residues from industrial plants. Basically, this incineration system consists in an unit which was developed by T-Thermal, Sub-X Down Fired type system, to incinerate liquid residues through oxidation in high temperature, as shown in Fig. 1. This unit is composed by: combustion chamber (1), oxidation chamber (2), cooling tank (3), initial separation tower (4), particle breaker (5), final separation tower (6) and gas washer (7).

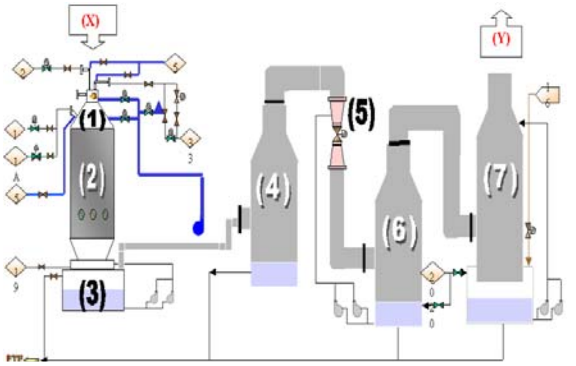

Fig. 1. Incinerator process

The capacity of the combustion chamber is of 6 million Kcal/h. The air/combustive relation is adjusted according to the stoichiometric computing. In these conditions, it is desired to obtain an efficient effluent toxics destruction at least of 99,99%. The combustion products are unloaded in the cooling tank. The gases leave the cooling tank for the duct of gases exit, passing to the initial separator, whose function is to minimize water transport, in the liquid state, presents in the gas. In the initial separator the gas follows to particle breaker. The recycled water through this washer is collected in the final separator; there are a constant draining of this water to prevent the extreme concentration of dissolved impurities. The gas leaves the final separator and follows to the gas washer. The gas washer is a tower with plastic filling where the gas flows to top, being washed and neutralized for a water solution with sodium hydroxide that is launched under sprayed form in the top of the tower, the gases leave for the chimney, located above of the gas washer gases. In this paper, we are concerned to identify the nonlinear relation between combustion chamber inputs (watery effluent, organic effluent, combustible oil, combustion air) and gas washer outputs (O_2, SO_2, CO) using a MIMO TS fuzzy model.

3. TAKAGI –SUGENO FUZZY MODEL

In the TS fuzzy model, proposed by Takagi-Sugeno in 1985, the antecedent is defined by linguistic terms of the input variables (linguistic variables), the consequent is a functional expression of these variables and the *i*-th IF-THEN rule has the following form:

$$R_i : \textit{IF } x_1 \text{ is } A_1^i \textit{ AND} \ldots \textit{AND} \ldots \text{E } x_q \text{ is } A_q^i$$
$$\textit{THEN } y_i = f_i(\mathrm{x}), \; i = 1,2,\ldots, \mathrm{c}. \quad (1)$$

where c is the number of rules.

The vector $\mathbf{x} \in \Re^q$ contains the premise variables, which has its own universe of discurse that is partitioned into fuzzy regions by the fuzzy sets describing the linguistic variable $x_j\big|^{j=1,\ldots,q}$. The premise variable x_j belongs to a fuzzy set A_q with a truth value given by a membership function $\mu_{jk} : R \to [0,1]$ for k = 1, 2,…, s_j where s_j is the number of partitions of the linguistic variable x_j for premise variable *j*. The truth value h_i for the complete rule *i* is computed using the aggregation operator , or t-norm, denoted by $\wedge : [0,1]^2 \to [0,1]$:

$$h_i(\mathbf{x}) = \mu_1^i(x_1) \wedge \mu_2^i(x_2) \wedge \ldots \mu_q^i(x_q) \quad (2)$$

Among the different t-norms available, in this work the algebrbraic product will be used:

$$h_i(\mathbf{x}) = \prod_{j=1}^{q} \mu_j^i(x_j) \quad (3)$$

The activation degree for the rule *i* is normalized as:

$$\gamma_i(\mathrm{x}) = \frac{h_i(\mathrm{x})}{\sum_{r=1}^{c} h_r(\mathrm{x})} \quad (4)$$

where *c* is the number of rules. This normalization implies that:

$$\sum_{i=1}^{c} \gamma_i(\mathrm{x}) = 1 \quad (5)$$

The response of the TS model is a weighted sum of the consequent functions, i.e, a convex combination of the local functions (models) f_i:

$$y = \sum_{i=1}^{c} \gamma_i(\mathrm{x}) \, f_i(\mathrm{x}) \quad (6)$$

1.1 Fuzzy Structure Model

In this paper is, the NARX (*Nonlinear Autoregressive with Exogenous Input*) structure, widely applied in fuzzy modeling, where the model output is a function of the past input-output data, is used:

$$\hat{y}(k+1) = f\,(y(k) \ldots \, y(k-n_y+1), u(k) \ldots$$
$$\ldots u(k-n_u+1) \quad (7)$$

where k denotes the time sampling, n_y and n_u are integers related to the system order, u e y are the input and output, respectively. The TS fuzzy model, in terms of IF-THEN rules, is given by:

$$R_i : \textit{IF } y(k) \text{ is } A_1^i \textit{ AND} \ldots \textit{AND } y(k-n_y+1) \text{ is } A_{n_y}^i$$
$$\textit{AND } u(k) \text{ is } B_1^i \textit{ AND} \ldots \textit{AND } u(k-n_u+1) \text{ is } B_{n_u}^i \textit{ THEN}$$

$$\hat{y}^i(k+1) = \sum_{j=1}^{n_y} a_{i,j}\, y(k-j+1) + \sum_{j=1}^{n_u} b_{i,j}\, u(k-j+1) + c_i \quad (8)$$

where $a_{i,j}$, $b_{i,j}$ e c_i are consequent parameters to be estimated by the RLS (Recursive Least Square) method (Almeida and Barreto, 2005). The inference formula of the TS model is:

$$\hat{y}(k+1) = \sum_{i=1}^{l} \gamma_i(\mathrm{x}_k)\, y^i(k+1) \quad (9)$$

$$\mathbf{x}_k = (y(k) \ldots \, y(k-n_y+1), u(k) \ldots u(k-n_u+1)) \quad (10)$$

4. RLS – RECURSIVE LEAST SQUARE

The basic idea of recursive least squares algorithm is to compute the new parameter estimate $\hat{\theta}(k+1)$ at the time k+1 by adding a correction vector to the previous parameter estimate $\hat{\theta}(k)$ at the time *k*. The estimation of the recursive weighted least squares algorithm for MISO (*Multiple Input Single Output*) systems, based on the global approach (all linear consequent parameters are estimated simultaneously), is given by:

$$\hat{\theta}(k) = \mathbf{P}(k)\, \boldsymbol{X}^{\mathrm{T}}(k)\, \boldsymbol{W}(k)\, \mathbf{y}(k) \quad (11)$$

where $\boldsymbol{X}(k)$ is the regression matrix at the time k:

$$\boldsymbol{X}(k) = \begin{bmatrix} \mathrm{x}^{\mathrm{T}}(1) \\ \mathrm{x}^{\mathrm{T}}(2) \\ \vdots \\ \mathrm{x}^{\mathrm{T}}(k) \end{bmatrix}_{N \times 1+r}$$

(12)

and

$$\mathbf{P}(k) = (\boldsymbol{X}^{\mathrm{T}}(k)\, \boldsymbol{W}(k)\, \boldsymbol{X}^{\mathrm{T}}(k))^{-1} \quad (13)$$

The matrix $\boldsymbol{W}(k)$ is the weighting matrix:

$$\boldsymbol{W}(k)= \begin{bmatrix} w(1) & 0 & \dots & 0 \\ 0 & w(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & w(k) \end{bmatrix} \quad (14)$$

Similarly, the estimation of recursive weighted least squares algorithm for MISO systems, with local approach (the consequent parameters are for each rule i), is given by:

$$\hat{\theta}_i(k) = \mathbf{P}_i(k)\, \boldsymbol{X}^{\mathrm{T}}(k)\, \boldsymbol{W}_i(k)\, \mathbf{y}(k) \quad (15)$$

where the matrix $\boldsymbol{W}_i(k)$ is the weighting matrix:

$$\boldsymbol{W}_i(k)= \begin{bmatrix} w_i(\mathrm{x}(1)) & 0 & \dots & 0 \\ 0 & w_i(\mathrm{x}(2)) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & w_i(\mathrm{x}(k)) \end{bmatrix} \quad (16)$$

and

$$\mathbf{P}_i(k) = (\boldsymbol{X}^{\mathrm{T}}(k)\, \boldsymbol{W}_i(k)\, \boldsymbol{X}^{\mathrm{T}}(k))^{-1} \quad (17)$$

The estimator equation for the time $k+1$ is:

$$\hat{\theta}_i(k+1) = \mathbf{P}_i(k+1)\, \boldsymbol{X}^{\mathrm{T}}(k+1)\, \boldsymbol{W}_i(k+1)\, \mathbf{y}(k+1) \quad (18)$$

which can be rewritten as:

$$\hat{\theta}_i(k+1) = \mathbf{P}_i(k+1)$$

$$\begin{pmatrix} X(k) \\ \mathrm{x}^T(k+1) \end{pmatrix}^T \begin{bmatrix} W_i(k) & 0 \\ 0 & w_i(\mathrm{x}(k+1)) \end{bmatrix} \begin{pmatrix} \mathrm{y}(k) \\ \mathrm{y}(k+1) \end{pmatrix} \quad (19)$$

$$= \mathbf{P}_i(k+1)[\boldsymbol{X}^{\mathrm{T}}(k)\, \boldsymbol{W}_i(k)\, \mathbf{y}(k) + \mathrm{x}(k+1)\, w_i(\mathrm{x}(k+1))\, \mathbf{y}(k+1)]$$

Substituting $\boldsymbol{X}^{\mathrm{T}}(k)\, \boldsymbol{W}_i(k)\, \mathbf{y}(k) = \mathbf{P}_i^{-1}(k)\, \hat{\theta}_i(k)$ in (19),

adding and subtracting $\hat{\theta}_i(k)$ on the right side, results:

$$\hat{\theta}_i(k+1) = \hat{\theta}_i(k) + [\mathbf{P}_i(k+1)\, \mathbf{P}_i^{-1}(k) - I]\, \hat{\theta}_i(k) + \mathbf{P}_i(k+1)\, \mathrm{x}(k+1)\, w_i(\mathrm{x}(k+1))\, \mathbf{y}(k+1) \quad (20)$$

where according to (17):

$$\mathbf{P}_i(k+1) = (\mathbf{P}_i(k)^{-1} + \mathrm{x}(k+1)\, w_i(\mathrm{x}(k+1))\, \mathrm{x}(k+1)^{\mathrm{T}})^{-1} \quad (21)$$

Taking the inverse on both sides in (21), we obtain:

$$\mathbf{P}_i(k)^{-1} = \mathbf{P}_i(k+1)^{-1} - \mathrm{x}(k+1)\, w_i(\mathrm{x}(k+1))\, \mathrm{x}(k+1)^{\mathrm{T}} \quad (22)$$

Substituting (22) in (20), the recursive estimator equation is obtained by:

$$\hat{\theta}_i(k+1) = \hat{\theta}_i(k) + \mathbf{P}_i(k+1)\, \mathrm{x}(k+1)\, w_i(\mathrm{x}(k+1))\, (\mathbf{y}(k+1) - \mathrm{x}(k+1)^{\mathrm{T}}\, \hat{\theta}_i(k)) \quad (23)$$

The RLS algorithm requires the inversion of the matrix **P**. Utilizing the matrix-inversion theorem, this procedure provides a lower computational cost and the equation (22) can be rewritten as:

$$\mathbf{P}_i(k+1) = \mathbf{P}_i(k) - \mathbf{P}_i(k)\, \mathrm{x}(k+1)\, (1/w_i(\mathrm{x}(k+1)) + \mathrm{x}(k+1)^{\mathrm{T}}\, \mathbf{P}_i(k)\, \mathrm{x}(k+1))^{-1}\, \mathrm{x}(k+1)^{\mathrm{T}}\, \mathbf{P}_i(k) \quad (24)$$

After some simplifications, results:

$$\mathbf{P}_i(k+1) = \mathbf{P}_i(k) - \frac{w_i(\mathrm{x}(k+1))\, \mathrm{P}_i(k)\, \mathrm{x}(k+1)\, \mathrm{x}(k+1)^T\, \mathrm{P}_i(k)}{1 + w_i(\mathrm{x}(k+1))\, \mathrm{x}(k+1)^T\, \mathrm{P}_i(k)\, \mathrm{x}(k+1)} \quad (25)$$

The recursive weighted least squares algorithm used for consequent parameters estimation is given by (20) and (25), where $w_i(\mathrm{x}(k+1)$ it is the activation degree for each rule.
In order to get the activation degree for each rule, to determine the antecedent parameters of the fuzzy model, is required.

5. MODIFIED GATH-GEVA ALGORITHM

The previous section has shown how the consequent parameters of the TS fuzzy model can be estimated by the recursive least squares algorithm when the antecedent parameters are given. In this section, in order to form an easily interpretable fuzzy model, the modified Gath-Geva clustering algorithm, which is based on the Expectation Maximization (EM) identification of Gaussian mixture models (Abonyi, *et al.*,2002; Abonyi and Szeifert, 2001) is presented. In this paper, this technique is extended for MIMO fuzzy models identification, that will be applied in the incineration system, where each cluster contains an input distribution, a local model and an output distribution:

$$p(\phi, y) = \sum_{i=1}^{c} p(\phi, y, n_i) = \sum_{i=1}^{c} p(y \mid \phi, n_i)\, p(x \mid n_i)\, p(n_i) \quad (26)$$

with $p(n_i)$ the *a priori* probability of the cluster, $p(x \mid n_i)$ the input distribution and $p(y \mid \phi, n_i)$ the output distribution. The clustering is based on the minimization of the sum of weighted squared distances between the data points $\mathbf{x}_k$ and the cluster centers $\mathbf{v}_i$

$$J_m(\mathbf{X}, \mathbf{U}, \mathbf{V}) = \sum_{k=1}^{n} \sum_{i=1}^{c} w_{ki}{}^{m} d_{ki}{}^{2}, 1 < m < \infty \quad (27)$$

where $\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_c]$ contains the cluster centers and m is a weighting expoent that determines the fuzziness of the resulting clusters and it is often chosen as $m = 2$. The fuzzy partition matrix has to satisfy the following conditions:

$$U \in [0,1]^{nc} \mid w_{k,i} \in [0,1], \forall k, i;$$

$$\sum_{i=1}^{c} w_{ki} = 1, \forall k; 0 < \sum_{k=1}^{n} w_{ki} < n, \forall i \quad (28)$$

The minimization of (27) represents a non-linear optimization problem subject to constrains defined by (28) and can be solved by using a variety of available methods. The modified Gath-Geva algorithm is formulated as follows:

Initialization: Given a set of data matrix **X**, specify c, choose the weighting exponent $m>1$ and the tolerance $\varepsilon > 0$. Initialize the partition matrix such that (28) holds.
Repeat for $l = 1,2,...$

Step 1: Comput the parameters of the clusters:

- Center of membership functions

$$v_i^l = \frac{\sum_{k=1}^{n} [w_{ki}^{l-1}]^m x_k}{\sum_{k=1}^{n} [w_{ki}^{l-1}]^m}, i = 1,... c. \quad (29)$$

- Standard deviations of the Gaussian membership functions

$$\sigma_{j,i}^2 = \frac{\sum_{k=1}^{n} [w_{ki}^{l-1}]^m (x_{k,j} - v_{k,j})^2}{\sum_{k=1}^{n} [w_{ki}^{l-1}]^m}, i = 1,...c. \quad (30)$$

- Parameters of local models given by (20)
- A priori probabilities of the clusters

$$p(n_i) = \frac{1}{n} \sum_{k=1}^{n} [w_{ki}^{l-1}]^m \quad (31)$$

- Covariance matrix of the modeling error

$$F_i^y = \frac{\sum_{k=1}^{n} [w_{ki}^{l-1}]^m (y_k - \hat{y}_k)(y_k - \hat{y}_k)^T}{\sum_{k=1}^{n} [w_{ki}^{l-1}]^m} \quad (32)$$

Step 2: Compute the distance measure $d_{k,i}^2$:

The distance measure consists in two terms. The first one is the distance between the cluster centers and **x**, while the second one quantifies the performance of the local linear models.

$$\frac{1}{d_{ki}^2} = p(n_i) \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi\sigma_{j,i}^2}} \exp\left(\frac{1}{2} \frac{(x_{j,k} - v_{i,j})^2}{\sigma_{i,j}^2} \right) \cdot$$

$$\frac{\exp(-(y_k - \hat{y}_k)^T (F_i^y)^{-1} (y_k - \hat{y}_k))}{(2\pi)^{\frac{no}{2}} \sqrt{|F_i^y|}} \quad (33)$$

Step 3: Update the partition matrix

$$w_{ki}^l = \frac{1}{\sum_{j=1}^{c} (d_{ki} / d_{kj})^{2/(m-1)}}, \quad (34)$$

until $\| U^l - U^{l-1} \| < \varepsilon$.

6. IDENTIFICATION OF THE INCINETARION PROCESS

The identification of the liquid effluent incineration process will be made by the Takagi-Sugeno fuzzy model using the RLS algorithm for consequent parameters estimation and the modified Gath-Geva algorithm to obtain the antecedent parameters, partitioning the multivariable input space in valid fuzzy regions for the consequent sub-models, both presented in sections 4 and 5, respectively.

6.1 Process Characteristics

In order to get a better structure of TS fuzzy model for the process, we verify some pertinent characteristics of the incineration system (Cunha, 2003), such as:

- ***MIMO System***: 4 inputs and 3 outputs:

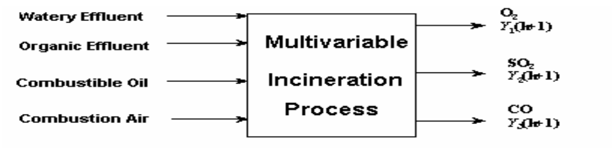

Fig. 2. Incineration system

- ***Correlation in the input variables***: The four input variables are all correlated, the burning reason according to stoichiometric parameters is of 4kg of watery effluent for 1 kg of fuel (combustible oil + organic effluent) and 11,32 m^3/h of combustion air. All the inputs variables influence in the outputs variables.

- ***Correlation in the output variables***: There are certain particularitities, such as:

- The first output variable, O_2 concentration, haven't correlation with the others two output variables, its value is given directly for the gas analyzer;

- The second output variable, SO_2 concentration, is obtained by computing in Feema-RJ (State Foundation of Environment Engineering) resolution for analysis SO_2 for dry base in 11% O_2:

$$SO_2 \text{ corrected} = \frac{SO_2 \text{ analysed.}(O_2 \text{ atmosphere} - 11\,\%)}{(O_2 \text{ atmosphere} - O_2 \text{ analyzed})} \quad (35)$$

where we can observe that second output is correlated with first output; therefore, the computing of SO_2 concentration depends to the O_2 value;

- The thirth output variable, CO concentration, is obtained by the calculation in Feema-RJ resolution for analysis CO for dry base in 11% O_2:

$$CO \text{ corrected} = \frac{CO \text{ analysed.}(O_2 \text{ atmosphere} - 11\,\%)}{(O_2 \text{ atmosphere} - O_2 \text{ analyzed})} \quad (36)$$

where we can observe that thirth output is correlated with first output; therefore the computing of the CO concentration depends to the O_2 value;

6.2 Structure of TS Fuzzy Model

Due to the characteristics in item 6.1, we could define the TS fuzzy model in the form of MIMO structure as 3 connected MISO fuzzy models, where we verify the correlation among the data of the system. We search this form to optimize the identification process. The multivariable fuzzy model is shown in Fig.3.

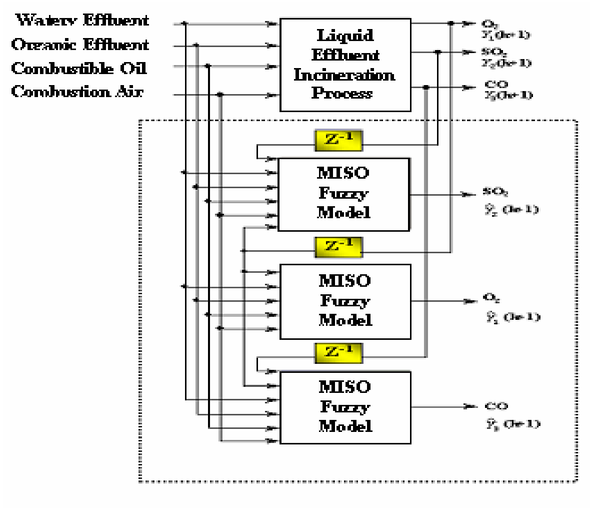

Fig. 3. MIMO fuzzy model

Once structure is known the fuzzy sets (the regions of operation of the local model) are defined in the domain of the outputs of the incinerator. For each output the following configuration is:

Output (O_2): R_i: IF $y_1(k)$ IS A_1^i THEN $\hat{y}_1{}^i(k+1)=$

$$\alpha_1^i\, y_1(k) + \beta_1^i\, u_1(k) + \beta_2^i\, u_2(k) + \beta_3^i\, u_3(k) + \beta_4^i\, u_4(k) + \gamma^i \tag{37}$$

Output (SO_2): R_i: IF $y_2(k)$ IS A_2^i THEN $\hat{y}_2{}^i(k+1)=$

$$\alpha_1^i\, y_2(k) + \beta_1^i\, u_1(k) + \beta_2^i\, u_2(k) + \beta_3^i\, u_3(k) + \beta_4^i\, u_4(k) + \beta_5^i\, u_5(k) + \gamma^i \tag{38}$$

Output (CO): R_i: IF $y_3(k)$ IS A_3^i THEN $\hat{y}_3{}^i(k+1)=$

$$\alpha_1^i\, y_3(k) + \beta_1^i\, u_1(k) + \beta_2^i\, u_2(k) + \beta_3^i\, u_3(k) + \beta_4^i\, u_4(k) + \beta_5^i\, u_5(k) + \gamma^i \tag{39}$$

where $i = 1,\dots,c$, is the number of rules and A_1^i, A_2^i, e A_3^i are sets fuzzy of antecedent variables for each TS model. The consequent parameters for each rule α_j^i, β_j^i e γ^i are estimated by RLS algorithm.

7. EXPERIMENTATION AND RESULTS

The system identification using TS MIMO fuzzy model was realized. For the modeling stage of the parameters, 600 samples (100 hours of incineration process operation) are collected from experiment. For the validation stage, others 600 samples were used. Two criteria had been used for the validation of the fuzzy models:

-VAF : (*Variance Accounted For*)

$$\textbf{VAF}\ (\%) = 100 \times \left[1 - \frac{\operatorname{var}(Y - \hat{Y})}{\operatorname{var}(Y)} \right] \tag{40}$$

where Y is the nominal output of the incineration process, $\hat{Y}$ is the estimate output of the model and *var* is the variance of the signal.

-MSE(*Mean Square Error*)

$$\textbf{MSE} = \frac{1}{N} \sum_{K=1}^{N} (Y_k - \hat{Y}_k)^2 \tag{41}$$

where Y_k is the nominal output of the incineration process, $\hat{Y}_k$ is the estimate output of the model and N is the number of points.

Five different models were identified: (1) MIMO ARX model, (2) MIMO TS fuzzy model with FCM (*Fuzzy C-Means*) clustering algorithm, (3) MIMO TS fuzzy model with GK (Gustafson and Kessel, 1979) clustering algorithm, (4) MIMO TS fuzzy model with GG (Gath and Geva, 1989) clustering algorithm, (5) MIMO TS fuzzy model with modified GG clustering algorithm. A comparative analysis is established between these models. The Table 1,2, and 3, presents the efficiency of the models that had been used in the liquid incineration process identification system for each output variable:

Table 1 Efficiency of the models – Output (O_2)

Model	VAF(%)	MSE	Rules N.
MIMO ARX	90,52	0,751	-
MIMO TS (FCM)	94,95	0,411	4
MIMO TS (GK)	96,21	0,312	4
MIMO TS (GG)	95,24	0,375	4
MIMO TS (Mod. GG)	98,89	0,236	4

Table 2 Efficiency of the models – Output (SO_2)

Model	VAF(%)	MSE	Rules N.
MIMO ARX	91,34	3,286	-
MIMO TS (FCM)	95,45	2,617	4
MIMO TS (GK)	97,81	2,409	4
MIMO TS (GG)	95,33	2,620	4
MIMO TS (Mod. GG)	98,12	2,377	4

Table 3 Efficiency of the models – Output (CO)

Model	VAF(%)	MSE	Rules N.
MIMO ARX	90,87	1,394	-
MIMO TS (FCM)	93,59	0,977	4
MIMO TS (GK)	95,18	0,764	4
MIMO TS (GG)	94,11	0,883	4
MIMO TS (Mod. GG)	98,88	0,530	4

In Table 1,2 and 3, we can observe that MIMO TS fuzzy model with modified GG clustering algorithm, had a better performance than others ones. A comparative analysis between the real outputs and the estimate output for this model, is showed in Figures 4, 5 and 6:

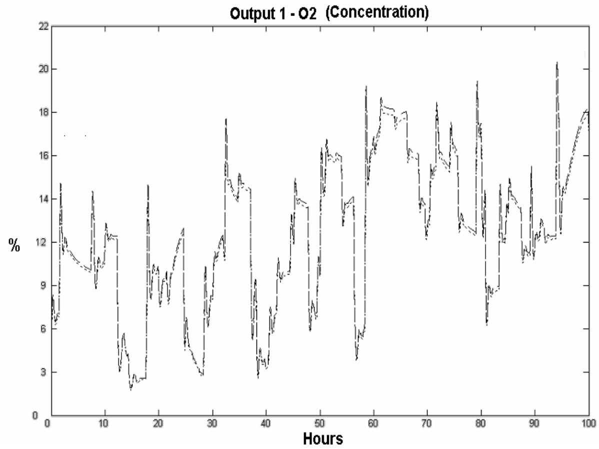

Fig. 6. Measured(-) and predicted(- -)process outputs

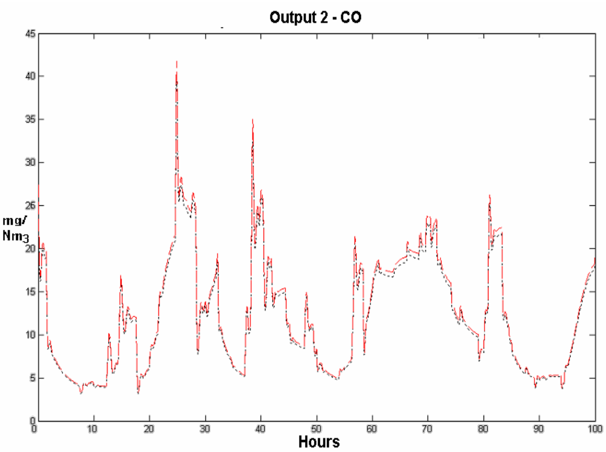

Fig. 7. Measured(-) and predicted(- -)process outputs

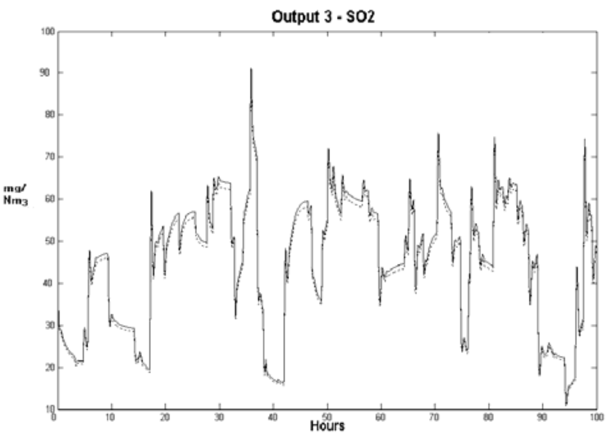

Fig. 8. Measured(-) and predicted(- -)process outputs

8. CONCLUSIONS

In this paper, the identification of complex nonlinear multivariable sistem is discussed. A fuzzy model structure has been proposed, where the liquids effluents incineration process at the BASF industry, is represented by a MIMO TS fuzzy model. The modified Gath-Geva clustering algorithm was used to determine the antecedent part of the MIMO fuzzy model and the consequent parameters were estimated by RLS algorithm. The obtained MIMO fuzzy model was able to represent the dynamic behaviour of the MIMO nonlinear dynamic system due to, mainly, the chosen structure based on the correlation analysis of input-output data. For future works, the development of a adaptive controller for the combustion system using obtained model of the incineration process makes necessary.

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