NMPC WITH STATE-SPACE MODELS OBTAINED THROUGH LINEARIZATION ON EQUILIBRIUM MANIFOLD

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Abstract: This paper presents a Nonlinear Model Predictive Control approach for nonlinear state-space models obtained with the modelling and identification technique recently proposed in literature as Linearization on the Equilibrium Manifold (LEM). The predictive controller that will be applied to the LEM uses the Local Linearization on the Trajectory algorithm (LLT) which simulates the nonlinear plant and calculates optimal control actions based on local linearizations around the simulated trajectory by online minimization of an objective function. The proposed combination of the LEM and LLT techniques is tested with a nonlinear SISO system.

Keywords: nonlinear systems, identification, linearization, predictive control, trajectories

1. INTRODUCTION

The importance of nonlinear approaches to control systems in industrial chemical processes has been rising significantly during the last few years and will continue to do so in the future. The high demands of today's economy in terms of process yield and the obedience of environmental standards require an increased efficiency that cannot always be achieved with linear control concepts. At the same time, the availability of nonlinear dynamic models has been recognized in the literature as one of the main obstacles, if not the most important, for the application of nonlinear control strategies. High cost and complexity of nonlinear approaches often impose restrictions on practical usability.

This situation calls for methods that take into account the well-developed linear control theory, extending it for usage with nonlinear processes. One possibility, which can be termed "grey-box" modelling, is the use of "local models", understood as approximations of the original system in a limited sub-region of the operating domain in order to construct a nonlinear model. The underlying principle is that the system behavior is "simpler" locally than globally and as a result local models can be identified more easily. Examples of this methodology are the local linear models tree (Nelles, 1997) and the identification through the decomposition into operating regimes (Johansen and Murray-Smith, 1997).

The Linearization on the Equilibrium Manifold (LEM) approach (Bolognese Fernandes and Engell, 2005) proposes a way of constructing a nonlinear model by interpolating the equilibrium manifold and the linear behavior of the system between different operating points. It has been shown that various problems of local modelling techniques can be avoided by using this method, making it an appealing way of obtaining a nonlinear model with less than the effort necessary for a first-principles modelling.

A suitable control strategy for this kind of system would also use linear models for determining control actions, as these are already available and in use for the construction of the global model. The Local Linearization on the Trajectory algorithm LLT (Duraiski, 2001) is a predictive control strategy for nonlinear models which uses local linearizations at the current point of the system state. In the following, a combination of the LLT with LEM models will be proposed and compared to the already well-tested combination of the LLT with a first-principles model.

This paper is structured as follows: section 2 presents the basics of the LEM method in the general (MIMO) form, section 3 explains the LLT control strategy. Section 4 proposes a combination of the two methods, which is evaluated with numerical experiments in section 5 using a nonlinear SISO example system. Concluding remarks and proposals for further investigations can be found in Section 6.

2. LINEARIZATION ON THE EQUILIBRIUM MANIFOLD (LEM) MODELS

Consider a continuous MIMO nonlinear dynamic system of the form

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{r}(\mathbf{x},\mathbf{u}) \\ \mathbf{y} &= \mathbf{h}(\mathbf{x})\end{aligned} \qquad (1)$$

where $\mathbf{r}: X \times U \rightarrow \Re^n$ is at least once continuously differentiable on $X \subseteq \Re^n$, $U \subseteq \Re^m$, and $\mathbf{h}: X \rightarrow \Re^p$ is at least once continuously differentiable. The output equation will be frequently omitted in the sequel for shortness. The equilibrium manifold of (1) is defined as the family of constant equilibrium points

$$\begin{aligned}\Xi = \{(\mathbf{x}_s, \mathbf{u}_s, \mathbf{y}_s) \in \Re^n \times \Re^n \times \Re^n : \\ \mathbf{r}(\mathbf{x}_s, \mathbf{u}_s) = \mathbf{0},\ \mathbf{y}_s = \mathbf{h}(\mathbf{x}_s, \mathbf{u}_s)\}\end{aligned} . \qquad (2)$$

Similarly, the family of linearizations of (1) at the set of equilibrium points determined by (2) is given in the usual way as

$$\dot{\mathbf{x}} = \left[\frac{\partial \mathbf{r}(\mathbf{x},\mathbf{u})}{\partial \mathbf{x}}\right]_{\mathbf{x}_s,\mathbf{u}_s} (\mathbf{x} - \mathbf{x}_s) + \left[\frac{\partial \mathbf{r}(\mathbf{x},\mathbf{u})}{\partial \mathbf{u}}\right]_{\mathbf{x}_s,\mathbf{u}_s} (\mathbf{u} - \mathbf{u}_s) \qquad (3)$$

and similarly for the output equation. Under the condition that the rank of $[\partial \mathbf{r}(\mathbf{x}_s,\mathbf{u}_s)/\partial \mathbf{x}]$ is n for the set Ξ (Wang and Rugh, 1987, Bolognese Fernandes 2005), the equilibrium manifold and consequently the family of linearizations of (1) will be specified by m among the $n+m$ variables $(\mathbf{x},\mathbf{u})$. Therefore, if this matrix is full rank, the set of inputs fully parameterize both families of equilibrium points and linearizations.

Calling the steady-state map $\mathbf{\Omega}: \Re^m \rightarrow \Re^n$, such that $\mathbf{r}(\mathbf{\Omega}(\mathbf{u}),\mathbf{u}) = \mathbf{0}$ (that is, the function $\mathbf{\Omega}$ gives the steady-state $\mathbf{x}_s$ corresponding to a constant input $\mathbf{u}_s$), the input-parameterized linearization the equilibrium manifold (LEM) of (1) is defined as the system (Bolognese Fernandes and Engell, 2005)

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{u})(\mathbf{x} - \mathbf{\Omega}(\mathbf{u})) \qquad (4)$$

$\mathbf{A}(\mathbf{u})$ represents the evaluation of the Jacobian matrix $[\partial \mathbf{r}(\mathbf{x},\mathbf{u})/\partial \mathbf{x}]$ on $(\mathbf{\Omega}(\mathbf{u}),\mathbf{u})$. The focus on input parameterization is due to the fact that identification experiments to obtain $\mathbf{A}(\mathbf{u})$ and $\mathbf{\Omega}(\mathbf{u})$ from process data are carried out by exciting the plant with a designed input signal. The output equation can be linearized in an analogous way, considering the stationary output mapping: $\mathbf{\Psi}: \Re^m \rightarrow \Re^p$.

The model (4) has to be interpreted as *a* (state-affine) nonlinear system that possesses the same family of equilibrium points (2) and the same linearization family (4) as the nonlinear system (1). Following the discussion in Bolognese Fernandes (2005), the LEM system can be a good approximation of (1) in transient regimes away from the equilibrium manifold depending on the degree of nonlinearity, in that way substituting a first-principles model. Obviously, other representations that are equivalent on the equilibrium manifold can be constructed on the basis of any m distinct parameters. Moreover, these representations can be easily interchanged, provided that the inverses of the corresponding elements in $\mathbf{\Omega}(\mathbf{u})$ and $\mathbf{\Psi}(\mathbf{u})$ exist. For further information about how to obtain the equilibrium manifold and the dynamic matrix $\mathbf{A}$, please refer to Bolognese Fernandes (2005).

3. LOCAL LINERIZATIONS ON THE TRAJECTORY (LLT)

In the following section a control strategy for the model introduced above will be presented. It was developed by Duraiski (2001) and consists of a model predictive control algorithm which works in the following way: the control actions applied to the manipulated variables are obtained by optimizing an objective function of control costs using a nonlinear internal model to predict the future system outputs. The control actions, however, are determined in each iteration through the use of a set of linear models in the step response form, obtained through local linearizations around the trajectory of the system, previously obtained in the last iteration. This ensures that the optimization problem is quadratic as it is in the case of Linear Model Predictive Control, and thus easy to solve.

3.1 Algorithm description

The LLT algorithm (Duraiski, 2001) consists of the following iterative calculation steps:

1) The first solution is based on a linearized model at the current operating conditions. Using this trajectory it is possible to simulate the nonlinear model which is used to calculate a sequence of linear models for the next iteration.
2) With the sequence of linearized models on the trajectory a new control action trajectory is calculated.

3) This sequence of control moves is applied to the simulation of the open-loop response of the internal model.
4) Based on the new trajectory, it is possible to determine a new set of linearized models in the same way as it is done in the first step. Then, this set of models is used in the next iteration step.
5) The steps 2, 3 and 4 are sequentially carried out until the algorithm converges, i.e., when the i-th control action trajectory (calculated in the current iteration) does not differ too much from the (i-1)-th, satisfying the maximum norm convergence criterion $\|u_i - u_{i-1}\|_\infty \le TolU \in \Re$. In case the algorithm does not converge after a given time and number of iterations, e. g. when the setpoint is unattainable, the best of all calculated control actions in this time step will be applied.

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3.2 Linearized Step Response Model

In this part the linear step response model used for predicting the future system output will be developed. Primarily, the following discrete time state space equation is considered.

$$(\mathbf{x}_k - \mathbf{x}^B_{k-1}) = \mathbf{A}_{k-2} \cdot (\mathbf{x}_{k-1} - \mathbf{x}^B_{k-2}) + \mathbf{B}_{k-2} \cdot (\mathbf{u}_{k-1} - \mathbf{u}^B_{k-2}) \quad (5)$$

$$(\mathbf{y}_k - \mathbf{y}^B_{k-1}) = \mathbf{C}_{k-1} \cdot (\mathbf{x}_k - \mathbf{x}^B_{k-1}) + \mathbf{D}_{k-1} \cdot (\mathbf{u}_k - \mathbf{u}^B_{k-1}) \quad (6)$$

The matrices $\mathbf{A}_{k-2}$, $\mathbf{B}_{k-2}$, $\mathbf{C}_{k-1}$ and $\mathbf{D}_{k-1}$ are obtained by *discretization* of the continuous linear state space system resulting from the Taylor linearization of equation (1). They are not to be confused with the traditional notation for the continuous state space matrices (i.e., **A**, **B**, **C** and **D)**. The variables $\mathbf{x}^B$, $\mathbf{u}^B$, $\mathbf{y}^B$ represent the variables **x**, **u**, **y** at the point of linearization. Equations (5) and (6) can now be applied iteratively for the time steps from 0 up to the simulation horizon P, yielding an output equation for each discrete time step. With this, an equation for the output **Y** from the time instant 0 to P can be constructed. Written in a compact matrix form, the following equation is obtained:

$$\mathbf{Y}_{\left[\begin{smallmatrix}P\\0\end{smallmatrix}\right]} = \mathbf{Su} \cdot \boldsymbol{\delta}\mathbf{U}_{\left[\begin{smallmatrix}P\\0\end{smallmatrix}\right]} + \mathbf{Sx} \cdot \boldsymbol{\delta}\mathbf{x}_0 + \mathbf{Y}^B_{\left[\begin{smallmatrix}P-1\\-1\end{smallmatrix}\right]} \quad (7)$$

Equation (7) will be used with some alterations within the calculation and optimization of the objective function. For details please refer to Duraiski (2001).

3.3 Objective function

The optimization problem consists of the minimization of a quadratic objective function with penalty terms for setpoint deviations and control actions, in the most general form being

$$J = \min_{\Delta U_{\left[\begin{smallmatrix}M\\0\end{smallmatrix}\right]}} \left(\sum_{i=0}^{P} \left(\gamma_i \cdot (y_i - r_i)\right)^2 + \sum_{i=0}^{M} \left(\lambda_i \cdot \Delta u_i\right)^2 \right) \quad (8)$$

In the case of the LLT method, the input difference variable $\Delta u_k = u_k - u_{k-1}$ is replaced by the deviation variable $\delta u_k = u_k - u_{k-1}^B$. Furthermore, a penalty term for soft constraints $\varphi|s|$ ($s \ge 0$ being a scalar slack variable that is only nonzero while the constraints are violated) and for the deviation of the manipulated variable from a given target z_i are introduced. With these alterations, the objective function is

$$J = \min_{\delta U_{\left[\begin{smallmatrix}M\\0\end{smallmatrix}\right]}} \left(\begin{array}{l} \sum_{i=0}^{P} \left(\gamma_i \cdot (y_i - r_i)\right)^2 + \\ \sum_{i=0}^{M} \left(\lambda_i \cdot \left((\delta u_i + u^B_{i-1}) - (\delta u_{i-1} + u^B_{i-2})\right)\right)^2 + \\ \sum_{i=0}^{M} \left(\psi_i \cdot \left((\delta u_i + u^B_{i-1}) - z_i\right)\right)^2 + \left(\varphi |s|\right)^2 \end{array} \right) \quad (9)$$

The parameters γ_i, λ_i, ψ_i and φ_i are to be determined by common MPC parameter tuning methods. For the actual implementation, equation (9) can be rewritten in a matrix form. Further details will not be discussed here and can be found in Duraiski (2001).

4. COMBINING A LEM MODEL WITH AN LLT CONTROLLER

Now a combination of models obtained through the LEM technique with a nonlinear model predictive controller using the LLT method will be proposed. In general, two possibilities exist to achieve this goal: first, using the LEM as a nonlinear model for the LLT algorithm as it is, deriving the needed Jacobians **A**, **B**, **C**, **D** through analytic or numerical differentiation of the LEM itself. A second possible approach is altering the LLT algorithm in a way that it can deal directly with the dynamic matrix **A** and the stationary manifold vector **Ω**(**u**) of the LEM. In this work only the first possibility will be investigated, as it demonstrates the feasibility of the approach with fairly low effort in terms of implementation. The control performance and computational effort of the LEM+LLT combination will be compared to an LLT controller with a nonlinear model.

It will be assumed in the sequel that a nonlinear LEM model in the autonomous state space form as stated in equation (4) has been constructed by identifying the matrix **A** and the equilibrium manifold **Ω**(**u**) using appropriate techniques. Further details about model construction can be found in Bolognese Fernandes (2005). Equation (4) will now be incorporated into the LLT algorithm, using it as a description of the nonlinear process. As can be seen in equations (5) and (6), it is necessary to derive a general linearization of the process for later discretization and the calculation of control actions by the LLT. This is achieved by a straight-forward Taylor linearization of the LEM model around an arbitrary point ($\mathbf{x}^B$, $\mathbf{u}^B$) in state space. Note that this has to be a dynamic linearization as we cannot assume the system to be at an equilibrium state at all times. However, the resulting *bias* caused by the term $\mathbf{r}(\mathbf{x}^B, \mathbf{u}^B) = \mathbf{A}(\mathbf{u}^B)(\mathbf{x}^B - \boldsymbol{\Omega}(\mathbf{u}^B))$ will cancel out in the differential equation, as shown in Duraiski (2001), Appendix B.

Linearizing equation (4) around an arbitrary point ($\mathbf{x}^B$,$\mathbf{u}^B$) in state space yields:

$$\Delta\dot{\mathbf{x}} = \frac{\partial(\mathbf{r}(\mathbf{x},\mathbf{u})}{\partial\mathbf{x}}\Big|_{\mathbf{x}^B\mathbf{u}^B}\cdot\Delta\mathbf{x} + \frac{\partial(\mathbf{r}(\mathbf{x},\mathbf{u}))}{\partial\mathbf{u}}\Big|_{\mathbf{x}^B\mathbf{u}^B}\cdot\Delta\mathbf{u}$$
$$= \mathbf{A}(\mathbf{u}^B)\Delta\mathbf{x} + \frac{\partial\mathbf{A}(\mathbf{u})}{\partial\mathbf{u}}\Big|_{\mathbf{u}^B}\cdot\mathbf{x}^B\cdot\Delta\mathbf{u} \tag{10}$$
$$-\frac{\partial\mathbf{A}(\mathbf{u})}{\partial\mathbf{u}}\Big|_{\mathbf{u}^B}\cdot\mathbf{\Omega}(\mathbf{u}^B)\cdot\Delta\mathbf{u} - \mathbf{A}(\mathbf{u}^B)\cdot\frac{\partial\mathbf{\Omega}(\mathbf{u})}{\partial\mathbf{u}}\Big|_{\mathbf{u}^B}\Delta\mathbf{u}$$

With this the following matrixes are obtained:

$$\mathbf{A}^B = \mathbf{A}(\mathbf{u}^B)$$
$$\mathbf{B}^B = \frac{\partial\mathbf{A}(\mathbf{u})}{\partial\mathbf{u}}\Big|_{\mathbf{u}^B}\cdot\left(\mathbf{x}^B - \mathbf{\Omega}(\mathbf{u}^B)\right) - \mathbf{A}(\mathbf{u}^B)\cdot\frac{\partial\mathbf{\Omega}(\mathbf{u})}{\partial\mathbf{u}}\Big|_{\mathbf{u}^B} \tag{11}$$

The output equation from (1) can also be linearized in a straight-forward way, yielding matrices $\mathbf{C}^B$ and $\mathbf{D}^B$ for the linear state space model.

$$\Delta\mathbf{y} = \frac{\partial(\mathbf{h}(\mathbf{x}))}{\partial\mathbf{x}}\Big|_{\mathbf{x}^B}\cdot\Delta\mathbf{x} + \frac{\partial(\mathbf{h}(\mathbf{x}))}{\partial\mathbf{u}}\Big|_{\mathbf{x}^B\mathbf{u}^B}\cdot\Delta\mathbf{u} \tag{12}$$

$$\mathbf{C}^B = \frac{\partial(\mathbf{h}(\mathbf{x}))}{\partial\mathbf{x}}\Big|_{\mathbf{x}^B}, \quad \mathbf{D}^B = \mathbf{0}. \tag{13}$$

Equations (11) and (13) will be discretized for each instant of time, yielding matrixes $\mathbf{A}_k$, $\mathbf{B}_k$ and $\mathbf{C}_k$. As we assume the output equation to be only dependent on $\mathbf{x}$, $\mathbf{D}_k$ will always be 0. With this, equations (5) and (6), which serve the purpose of determining the control actions of the predictive controller, can be easily constructed. The trajectory simulation is performed with the original nonlinear model from equation (4).

5. CASE STUDY

To prove the applicability of the proposed combination of the LEM and LLT methods, a nonlinear SISO system will be considered as an example.

5.1 Methodology and control objectives

The system under consideration will be tested and compared in four different forms:

a) Nonlinear model. A nonlinear differential equation derived from first-principle modelling techniques. This model also represents the real plant that is to be controlled. This holds for all the five cases a)-d).

b) Analytic LEM model. A nonlinear LEM model is constructed by analytic calculation of the dynamic matrix **A** and the equilibrium manifold $\mathbf{\Omega}(u)$.

c) Interpolated LEM model. A nonlinear LEM model is constructed by spline interpolation of the dynamic matrix **A** and the equilibrium manifold $\mathbf{\Omega}(u)$ between different operating points. These operating points and their linear dynamics are determined analytically.

d) Linearized model. One of the linear models from b) at one point of operation only, without LEM. This yields actually a purely linear MPC problem.

For each of the mentioned models, numerical experiments with various LLT controllers in different operation domains are conducted. The controller parameters are determined using the RPN methodology developed by Trierweiler and Farina (2003), the control objective being a decrease of the closed-loop rise time to 1/6 of the open-loop rise time. For testing the closed-loop system, a series of random set point changes is applied to the controlled variable of the system. Furthermore, the total cost J_{total} accumulated during the simulation time is compared, as well as the total necessary iterations.

5.2 The example system: isothermal CTSR reactor with Van de Vusse reaction scheme

The Van de Vusse reaction scheme is a well-known benchmark problem for nonlinear control algorithms and has been studied extensively by various researchers. A detailed model for this system was presented in Engell and Klatt (1993). For shortness, only the differential equations will be shown here.

$$\dot{x}_1 = -k_1x_1 - k_3x_1^2 + (x_{1,in} - x_1)u$$
$$\dot{x}_2 = k_1x_1 - k_2x_2 - x_2u \tag{14}$$
$$y = x_2$$

In these equations x_1 is the concentration of component A, x_2 is the concentration of component B and $x_{1,in}$ is the feed concentration of A, assumed to remain constant. The parameter values are k_1=15.0345 h^{-1}, k_2 = 15.0345 h^{-1}, k_3 = 2.324 l·mol^{-1}·h^{-1} $x_{1,in}$ = 5.1 mol·l^{-1} (Engell and Klatt, 1993). In this example only the operating range of 3 < u_s < 35 h^{-1} is investigated.

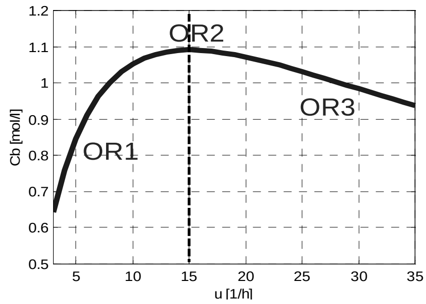

Fig. 1: Steady state C_B (x_2) concentration vs. dilution rate u for $x_{1,in}$ = 5.1 mol·l^{-1}

A particularity of the Van de Vusse reaction is the division of the operating domain in two parts with different dynamic behaviours. Fig. 1 shows the steady state C_B solutions as a function of the dilution rate u with the three typical operating regions (i.e., OR1, OR2, and OR3). For OR1 (i.e., u_s < 15 h^{-1}) a non-minimum phase behaviour can be observed, while for OR3 (i.e., u_s > 15 h^{-1}) the behaviour is minimum-phase. Close to the peak the zero gets close to the origin, intensifying the non-minimum phase behaviour of the left side and making the controller design more difficult. At the peak, the zero is null as well as the static gain, which is positive for OR1 and negative for OR3.

5.3 Numerical results

Details on obtaining the different LEM systems are not discussed here and can be found in Bolognese Fernandes (2005). First, OR1 and OR2 are investigated. In this region the proposed control goal could only be achieved by accepting excessive control action and inverse responses. Thus, the closed-loop rise time will only be reduced to about 3/4 of the open-loop value. The following LLT parameters are used:

Table 1: non-minimum-phase LLT parameters

Prediction / Control horizon (P / M)	136 / 34
Output / Input variable weight (Γ / Λ)	0.7 / 0.06
Sampling time (Ts)	0.15 min

The system is subjected to a series of setpoint changes in the region $0.7M < c_B < 1.11M$.

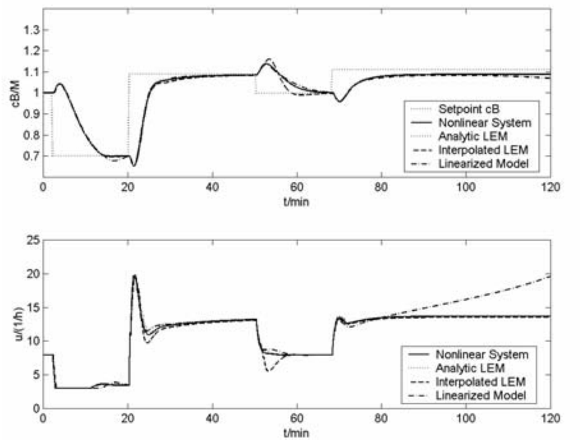

Fig. 2: time response in OR1 and OR2

In the first 20 minutes of the simulation the closed-loop velocity is limited by the lower constraint of $u = 3h^{-1}$. Up to this point, the behaviour of all the systems is very similar. The next setpoint is c_B=1.09M which is the theoretical maximum of the concentration c_B (OR2, see Fig. 1) and causes a rapid almost step-like control action. Note that the response of any input-parameterized LEM system to a step in the manipulated variable is comparable to the behaviour of the linear model at the equilibrium point corresponding *to the new input* (Bolognese Fernandes 2005). All four systems approach the equilibrium state with a similar velocity. From the 75th minute, the system is subjected to a setpoint of $c_B = 1.11h^{-1}$ that is not attainable (see Fig. 1). With the two LEM model versions the controller stabilizes the system at the maximum value of $c_B = 1.09h^{-1}$. This is a clear advantage in comparison to the linear MPC shown in Fig. 2, or any other linear (e.g. PI) controller, which is not able to keep the system at the point of maximum yield and trespasses gradually into OR3.

Now the region OR3 for $1M < c_B < 1.09M$ is investigated. Minimum-phase behaviour causes a better overall performance. First, the controller shown above is tested with a series of setpoint changes of random magnitude to prove the usability of one set of control parameters for the whole operation domain. Fig. 3 shows these results for OR3, the linearized model being designed for OR1, while Fig. 4 shows the same setpoint changes applied to OR1. Table 2 below summarizes the quantitative performance of the different models.

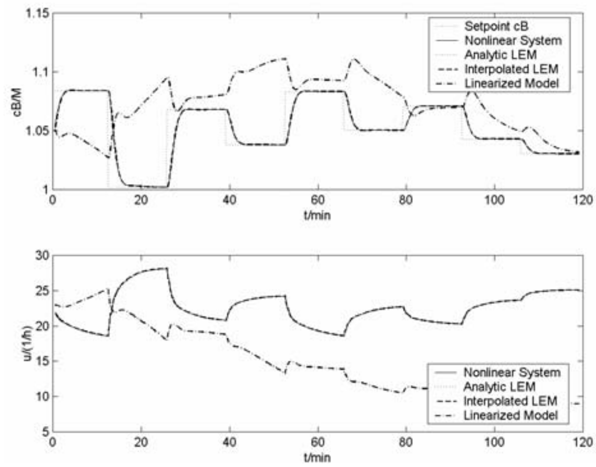

Fig. 3: minimum-phase responses, $1M < c_B < 1.09M$.

Fig. 3 shows clearly that the linear controller for OR1 cannot operate in OR3. All the other models follow the perturbations in the same way.

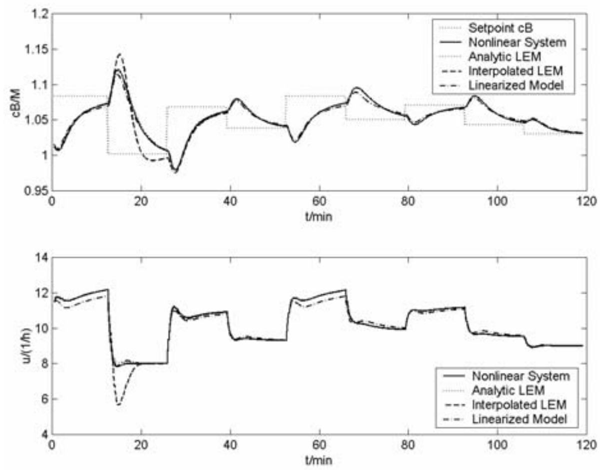

Fig. 4: non-minimum-phase responses, $1M < c_B < 1.09M$

Table 2: minimum-phase and non-minimum-phase performance, $1M < c_B < 1.09M$

	NL	ANA	INT	LIN
Iterations, min.-phase	1055	1055	1053	1030
J_{total}, m.p.	131.82	131.73	131.99	$3.33 \cdot 10^4$
Iterations, n.min.phase	831	831	858	1047
J_{total}, n.m.p.	183.37	183.69	177.48	$7.01 \cdot 10^6$

Finally, a controller designed especially for the minimum-phase region will be tested in order to see how the performance can be improved when the right tuning parameters are applied. Table 3 summarizes the new controller parameters.

Table 3: minimum-phase LLT parameters

Prediction / Control horizon (P / M)	3 / 1
Output / Input variable weight (Γ / Λ)	0.7 / 0.02
Sampling time (Ts)	0.15 min

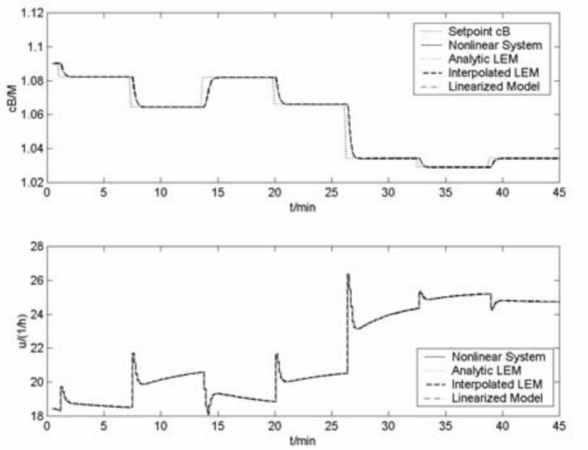

Fig. 5: minimum-phase responses, faster controller

It is evident that this controller performs much better than the one designed for non-minimum-phase behaviour, which makes it the preferable option for operation near the maximum yield of c_B=1.09M. Even the linear MPC controller shows almost exactly the same behaviour as the controller with the true nonlinear model. However, it has to be assured that the system does not trespass into the non-minimum-phase region, since in this case it would cause excessive inverse responses. This can be achieved by defining a 'target' for the manipulated variable in the minimum-phase domain within the LLT algorithm.

Table 4: minimum-phase performance, $1M < c_B < 1.09M$

	NL	ANA	INT	LIN
Iterations	810	810	810	814
J_{total}	4.2630	4.2630	4.2324	6.3172

As expected, the LEM models perform well in all the shown cases with only slight deviations from the original nonlinear model.

6. CONCLUSIONS AND OUTLOOK

In this paper the proposal for a combined use of the nonlinear modelling and identification technique LEM and the nonlinear predictive control algorithm LLT was made and tested with a SISO example. Analytical considerations and numerical results suggest a good applicability of this combination to lower order SISO systems, performing fairly well where purely linear methods fail completely. Losses in control performance are mainly due to the inevitable deviation of the identified and interpolated equilibrium manifold and the associated dynamics from the true nonlinear system's dynamics. The necessary iterations for convergence of the algorithm, as well as the accumulated values of the objective function are in all cases in the same scale, they can vary slightly because of the described deviations. It is clear that large efforts have to be made to obtain good approximations for the equilibrium manifold and the associated dynamics. Taking this into account, it can be concluded that further investigation of the demonstrated combination appears promising. Future work will be done on the testing of the technique with MIMO models and models of higher order, the goal being the proof of applicability to real industrial processes. The second possibility of implementation mentioned in section 4, the alteration of the LLT algorithm to be directly suitable for the LEM structure, will also be explored.

ACKNOWLEDGMENTS

The authors thank PETROBRAS and FINEP for their financial support. The first author thanks the HEINRICH-BOELL-FOUNDATION, the third author thanks the GERMAN ACADEMIC EXCHANGE SERVICE (DAAD).

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