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RUN-TO-RUN CONTROL OF MEMBRANE FILTRATION PROCESSES [1]

Jan Busch, **Wolfgang Marquardt** [2]

Lehrstuhl für Prozesstechnik, RWTH Aachen University, Turmstr. 46, 52064 Aachen, Germany

Abstract: Membrane filtration processes are often operated cyclically, where one cycle comprises a filtration and a backwashing phase. Due to the complex mechanisms involved, these filtration processes are mostly operated with fixed values of the manipulated variables. In this paper, a model-based process control approach is introduced, which is based upon run-to-run control theory. To evaluate the controller, a suitable model of submerged membrane filtration in wastewater applications is developed, which describes the main process phenomena while being computationally inexpensive. The model-based controller is then tested in a simulation environment employing a validated reference model. Excellent results with respect to prediction quality and optimality are obtained.

Keywords: run-to-run control, self-adaptive control, online control, online optimization, filtration, membrane filtration

1. INTRODUCTION

Filtration, and more recently, membrane filtration, are well-known and established technologies for the separation of particles, macromolecules or even dissolved molecules from fluids. Depending on the size of the separable substances, different technologies are known as filtration, microfiltration (MF), ultrafiltration (UF), nanofiltration (NF), and reverse osmosis (RO). This paper addresses filtration technologies where the separation principle is based on the difference in size of macromolecules/particles and of the diameter of the pores of the filtration medium. This includes regular filtration applications as well as MF and UF. For simplicity, all filters belonging to this broad class will subsequently be termed *membranes*. For these applications, the driving force facilitating filtration is usually a pressure difference across the membrane, that drives those particles through the membrane pores which are small enough to pass. Together with the solvent fluid, they leave the system as *permeate*, while particles larger than the pores are held back as *retentate*.

In most applications, the repelled particles concentrate on the feed side of the membrane and build a filter cake, which increases the filtration resistance (organic fouling). Furthermore, pores can be blocked by intruding particles (pore blocking). Finally, microorganisms can grow on the membrane and pore surfaces, leading to biofilms, which decrease the performance and can also damage the membrane (biofouling). When repelled by the membrane, soluble substances concentrate on the feed side of the membrane, and after reaching maximum solubility, they crystallize and add to cake layer formation (scaling, anorganic fouling).

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[2] Corresponding author: marquardt@lpt.rwth-aachen.de

All of these phenomena, which are known to contribute to *membrane fouling*, can be counteracted by membrane and module design as well as by appropriate process control strategies. In this paper, the focus will be on the process control aspect.

There are three main concepts for limiting membrane fouling. Firstly, high cross- ow velocities along the membrane surface (perpendicular to the pores) decrease the deposition of substances. Secondly, the ow direction through the membrane is periodically reversed, such that the membrane pores are ushed with uid (usually permeate). The third measure is to chemically or mechanically clean the membranes, which is usually performed at a much lower frequency.

1.1 Filtration process control: State-of-the-art

State-of-the-art process control for ltration processes usually employs xed values for the manipulated variables, which are only adjusted to meet the required net ux

$$J_{\mathrm{net}} = \frac{J_f \quad t_f \quad J_b \quad t_b}{t_f + \quad t_b}. \qquad (1)$$

The manipulated variables are the permeate and the backwashing uxes J_f and J_b and the ltration and backwashing durations t_f and t_b, respectively. A further manipulated variable is the cross- ow velocity u_c.

The reason for the rather simple control strategies lies in the high complexity of the ltration process. It is characterized by the periodic change between ltration and backwashing, the drift of the membrane permeability due to irreversible membrane fouling, and the typically non-steady-state operation. Furthermore, only very limited measurement information is available in industrial installations.

1.2 Membrane ltration modeling

The rigorous modeling of ltration processes is a highly complex task due to the many physical and possibly chemical and biological phenomena, which take place on very di erent time scales. There are numerous works in literature, which deal with the detailed modeling of various aspects of ltration processes. At the same time, there are several approaches to describe ltration processes only from a phenomenological point of view using simple, empirical correlations.

From a model-based process control point of view, both the mechanistic and the empirical models have advantages and drawbacks. If the uncertainty can be su ciently reduced by measurements, mechanistic models can yield a much higher prediction quality. Simpler, possibly empirical models have a lower computational demand and can often be identi ed from less data.

1.3 Outline of the paper

Our aim is to operate the ltration process at its economical optimum at every point in time while regarding safety constraints. In the framework of nonlinear model predictive control (NMPC), this objective is achieved by repeatedly solving a nonlinear, dynamic, and constrained optimization problem on a moving horizon. Its objective function resembles the operational cost, and its constraints re ect operational limitations. In the general case of measurement and process uncertainty, the optimization model needs to be regularly updated with current state information. Furthermore, the model has to be adapted to current process behavior, which is usually achieved by tting the parameters to past measurements on a suitably chosen estimation horizon. The success of the approach strongly depends on the ful llment of the following objectives:

- Satisfactory prediction and optimization,
- online applicability,
- robustness against disturbances, and
- adaptation to process drift and changes.

The key idea pursued throughout this paper is the following: A simple model is required to ful ll the online requirements of low computational cost and su cient identi ability. The lack of prediction precision is overcome by a frequent adaptation to plant measurements. This allows decent predictions at least in the vicinity of the current operating point. The ltration process is divided into ltration and backwashing phases. One ltration phase followed by one backwashing phase makes up one ltration cycle. The sequence of cycles can be exploited to update the process model after each cycle based on the available measurement data from the last cycle. In order to make the approach widely applicable in the process industry, only the transmembrane pressure (TMP) across the entire membrane module is assumed to be measured. The manipulated variables are then optimized for each upcoming cycle based on a model identi ed on the previous cycle. This concept is known as *run-to-run control*. It is introduced and adapted to ltration processes in Section 2. The resulting controller is evaluated in Section 3.

2. RUN-TO-RUN CONTROL FOR FILTRATION PROCESSES

Run-to-run process control is the strategy of applying one control action between two batches (cycles) in a process, while continuous control actions during the cycle are taken by base controllers. The task of the run-to-run controller is to issue set-points for the base controllers. Fig. 1 illustrates the embedding of the run-to-run controller into the control system. The run-to-run controller is activated only once between two cycles. The parameter update of the model is performed employing the measurement information of the previous cycle. The updated model is used to determine optimal set-points for the next cycle.

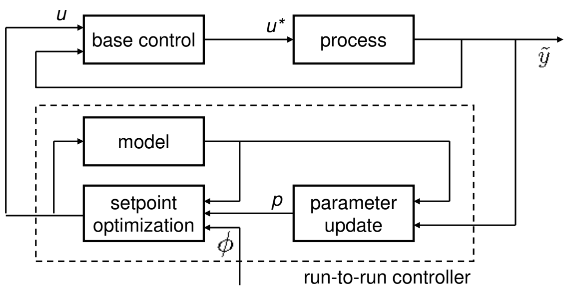

Fig. 1. Run-to-run control

An extensive review of theory and applications of run-to-run theory is provided by del Castillo and Hurwitz (1997). For the filtration systems treated in this paper, a very general problem formulation is required, which also has to account for the fact that each cycle is divided into a filtration (index f) and a backwashing phase (index b). First, the optimal control problem is formulated. To correctly represent the repeated solution of the problem on a moving horizon, the cycle index k should be introduced for every variable. The parameters $\mathbf{p}$ should be stated as $\mathbf{p}_{k|k-1}$, indicating that the parameters used in cycle k were estimated on the measurements of cycle $k-1$. However, to simplify the notation, this correct indexing is omitted:

$$\min_{\mathbf{u}_j, t_{j,e}} \phi \tag{P1}$$

$$\text{s.t. } \mathbf{f}_j\left(\dot{\mathbf{x}}_j, \mathbf{x}_j, \mathbf{y}_j, \mathbf{u}_j, \mathbf{p}_j, \mathbf{d}_j, t\right) = \mathbf{0}, \tag{2}$$

$$\mathbf{g}_j\left(\mathbf{x}_j, \mathbf{y}_j, \mathbf{u}_j, \mathbf{p}_j, \mathbf{d}_j, t\right) \leq \mathbf{0}, \tag{3}$$

$$\mathbf{h}_{j,\mathrm{eq}}\left(\mathbf{x}_j, \mathbf{y}_j, \mathbf{u}_j, \mathbf{p}_j, \mathbf{d}_j, t_{j,e}\right) = \mathbf{0}, \tag{4}$$

$$\mathbf{h}_j\left(\mathbf{x}_j, \mathbf{y}_j, \mathbf{u}_j, \mathbf{p}_j, \mathbf{d}_j, t_{j,e}\right) \leq \mathbf{0}, \tag{5}$$

$$\Gamma\left(\mathbf{x}_f\left(t_{f,e}\right), \mathbf{x}_b\left(t_{b,0}\right)\right) = \mathbf{0},\ \mathbf{x}\left(t_0\right) = \mathbf{x}_0, \tag{6}$$

$$t_0 = t_{f,0} \leq t_{f,e} = t_{b,0} \leq t_{b,e} = t_e, \tag{7}$$

$$t \in [t_0, t_e],\ j = \begin{cases} f & \text{for } t \in [t_{f,0}, t_{f,e}], \\ b & \text{for } t \in [t_{b,0}, t_{b,e}]. \end{cases} \tag{8}$$

$\mathbf{x}$ are differential and $\mathbf{y}$ are algebraic variables, $\mathbf{d}$ are disturbances, and t is the time. ϕ is the objective function representing the operational cost. $\mathbf{f}_j$ is the set of differential-algebraic equations of index 1 describing the respective process, and $\mathbf{g}_j$, $\mathbf{h}_j$, and $\mathbf{h}_{j,\mathrm{eq}}$ represent equality (endpoint) and inequality (path and endpoint) constraints. Eq. (6) states initial conditions and linking conditions between the differential states at the end of the filtration phase and the beginning of the backwashing phase. Eqs. (7)-(8) define the optimization horizon and the phase durations.

In a similar fashion the parameter estimation problem is formulated. With the additional assumption of white process and measurement noise, the parameter estimation problem reduces to a least squares optimization problem, whose formulation is omitted here for brevity.

The run-to-run control framework up to this point is generic for a wide range of filtration applications. In the following, its application to submerged MF/UF membrane filtration in wastewater applications is demonstrated.

2.1 Process model

The model proposed in the following is based on simple descriptions of the main phenomena of MF/UF membrane filtration processes. The transmembrane pressure Δp is commonly described using Darcy's law,

$$\Delta p = J\eta R, \tag{9}$$

where J is the flux, η is the fluid's viscosity, and R is the membrane resistance (e.g. Broeckmann *et al.*, 2005). While J is a manipulated variable, η depends on the feed suspension properties. As the TMP is assumed to be measurable, Eq. (9) represents the system's output equation. In the model proposed in the following, the resistance is described by different state equations for the filtration and the backwashing phase.

Filtration phase During filtration, the membrane resistance R_f can be described by

$$\frac{dR_f}{dt} = m{J_f}^{\alpha}{u_c}^{\beta},\ R_f\left(t_{f,0}\right) = R_f^0. \tag{10}$$

R_f^0 is the initial membrane resistance. Assuming that the filtration flux J_f and the cross-flow velocity u_c are constant, a linear increase of membrane resistance results. It describes the cake layer formation, which is the dominating effect on this timescale and which strongly depends on the flux and on the cross-flow. m, α, and β are parameters to adapt the model to a particular process.

Backwashing phase While often a linear increase of membrane resistance can be observed during filtration, its decrease during backwashing takes rather an exponential form, which converges to an irreversible resistance R_b^∞:

$$\frac{dR_b}{dt} = -\frac{n{J_b}^{\gamma}}{\tau_b {J_b}^{\delta}}\Delta R e^{-\frac{t-t_{f,e}}{\tau_b {J_b}^{\delta}}} \tag{11}$$

$$R_b\left(t_{b,0}\right) = n{J_b}^{\gamma}\Delta R + R_b^\infty,$$

$$\Delta R = R_f\left(t_{f,e}\right) - R_b^\infty. \tag{12}$$

Eqs. (11) and (12) are formulated such that a simple analytical expression for R_b can be obtained (Section 3). R describes the reversible resistance. The initial resistance $R_b\left(t_{b,0}\right)$ is the sum of the irreversible and the reversible resistance, but just like the resistance R_b it depends on the ux J_b due to unmodeled e ects. n, τ_b, γ, and are parameters.

Cost function Finally, those operating cost are described that can be in uenced by the process control system. They consist of the cost for electrical energy to provide the TMP and the cross- ow and the cost for membrane replacement. The rst two are given by

$$\frac{dE_E}{dt} = \frac{| \quad p_j J_j A|}{\eta_P} + e_c,\ E_E\left(t_0\right) = 0, \tag{13}$$

where A is the membrane area, η_P is an e ciency factor of the permeate pump, and e_c is the necessary power to provide the cross- ow. The cost for membrane replacement E_R cannot be described as straightforwardly as the energy cost. In fact, there is no quantitative insight to describe the in uence of the manipulated variables on the membrane lifetime. Depending on the ltration system under consideration, di erent models for E_R have to be developed. For MF/UF membranes in wastewater applications, it has been observed in practice that a strong increase of the resistance within a ltration cycle indicates an overstraining of the membrane. Therefore, its gradient is penalized:

$$E_R = \xi \frac{dR_f}{dt} = \xi m {J_f}^{\alpha} {u_{c,f}}^{\beta}, \tag{14}$$

where ξ is a parameter that needs to be speci ed for each application based on process experience. The overall objective function ϕ comprising the power consumption and the penalty term E_R is

$$\phi\left(t_e\right) = \frac{E_E\left(t_e\right)}{t_e \quad t_0} + E_R. \tag{15}$$

2.2 Run-to-run controller

In this section, the run-to-run controller is designed. First the estimation problem is considered, then the optimal control problem, and nally the control algorithm itself.

Estimation In industrial practice, only the TMP across the membrane is measured. In order to make the proposed approach widely applicable, it is therefore assumed that only this TMP is available as measurement. Since the uxes J_f and J_b and the cross- ow u_c are set to constant values for each phase, α, , γ, and cannot be estimated on a horizon of one cycle due to the missing excitation. This is referred to as the *dual control problem* (Wittenmark, 1995). The concerned parameters are estimated offline using historical data from several cycles and then set constant in the run-to-run control scheme.

The estimation problems for the ltration and the backwashing phase are coupled through Eq. (12). In order to simplify the problem and decrease the computational demand, they are, however, solved sequentially. Since the model structure is simple enough, the di erential equations are solved analytically. The discretized estimation problem for cycle k using the measurement data from cycle $k \quad 1$ for the ltration phase is then

$$\min_{m,R_f^0} \sum_{l=1}^{n_{f,l}} \frac{1}{2}\left(\quad \tilde{p}_{f,l} \qquad p_{f,l}\right)^2 \tag{P2}$$

$$\text{s.t.} \quad p_{f,l} = J_f \eta R_{f,l}, \tag{16}$$

$$R_{f,l} = R_f^0 + m J_f^{\alpha} u_c^{\beta} t_l, \tag{17}$$

where $\tilde{p}_{f,l}$ are discrete measurements at the sampling points t_l, $l \in \{1, ..., n_{f,l}\}$, in cycle $k \quad 1$, and $p_{f,l}$ are the corresponding simulated TMP samples.

The parameters of the backwashing model are estimated from

$$\min_{n,\tau_b,R_b^\infty} \sum_{l=1}^{n_{b,l}} \frac{1}{2}\left(\quad \tilde{p}_{b,l} \qquad p_{b,l}\right)^2 \tag{P3}$$

$$\text{s.t.} \quad p_{b,l} = J_b \eta R_{b,l}, \tag{18}$$

$$R_{b,l} = R_b^\infty + \quad R n J_b^{\gamma} e^{\frac{t_l \quad t_{n_{f,l}}}{\tau_b J_b^{\delta}}}, \tag{19}$$

$$R = R_f \quad t_{n_{f,l}}) \quad R_b^\infty. \tag{20}$$

Optimal Control The control problem, which is solved based upon the updated parameters, is

$$\min_{J_f,J_b,u_c,t_{f,e},t_{b,e}} \phi \tag{P4}$$

$$\text{s.t.} \quad p_j = J_j \eta R_j, \tag{21}$$

$$R_f = R_f^0 + m J_f^{\alpha} u_c^{\beta} t, \tag{22}$$

$$R_b = R_b^\infty + \quad R n J_b^{\gamma} e^{\frac{t \quad t_{f,e}}{\tau_b J_b^{\delta}}}, \tag{23}$$

$$R = R_f\left(t_{f,e}\right) \quad R_b^\infty, \tag{24}$$

$$J_{\text{net}} = \frac{J_f\left(t_{f,e} \quad t_0\right) \quad J_b \quad \left(t_e \quad t_{b,0}\right)}{t_e \quad t_0}, \tag{25}$$

$$R_b\left(t_{b,e}\right) \le \nu R_b^\infty,\ \nu \ge 1, \tag{26}$$

$$J_f \le J_b, \tag{27}$$

$$p_{\min} \le \quad p \le \quad p_{\max}, \tag{28}$$

$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}, \tag{29}$$

$$t_0 = t_{f,0} \le t_{f,e} = t_{b,0} \le t_{b,e} = t_e, \tag{30}$$

$$t \in [t_0, t_e],\ j = \begin{cases} f & \text{for } t \in [t_{f,0}, t_{f,e}], \\ b & \text{for } t \in [t_{b,0}, t_{b,e}]. \end{cases} \tag{31}$$

The net ux J_{net} is considered a set-point speci ed by the operator or an upper level controller. Eq. (26) forces the nal resistance $R_b\left(t_{b,e}\right)$ to be close to the irreversible resistance R_b^∞ at the end of the cycle. The backwashing ux J_b is forced to be at least equal to the ltration ux J_f (Eq. (27)), which is a safety measure to limit pore blocking. Eqs. (28) and (29) give bounds on the TMP and on the manipulated variables J_f, J_b, $t_{f,e}$, $t_{b,e}$, and u_c. ϕ is de ned as in Section 2.1.

Algorithm Ideally, the model identi cation and optimization takes place between two cycles k 1 and k, and the optimized values for the manipulated variables are applied at the beginning of the new cycle k. This would require zero calculation time. Hence, a delay in the implementation of the new set-points is inevitable. The reader is referred to Findeisen and Allgöwer (2003) for a rigorous discussion of possible stability problems due to computational delay in NMPC applications.

3. CASE STUDY - SUBMERGED MF/UF IN WASTEWATER TREATMENT

In order to evaluate the proposed model and control algorithm, it is tested against simulated data from a rigorous membrane ltration model, which describes MF/UF with submerged membranes in a wastewater treatment plant. The uid feed consists of water, in which a variety of organic and inorganic particles and dissolved substances are present. Organic fouling, biofouling, and pore blocking are therefore the dominating fouling e ects. Usually hollow bre membranes or plate modules with nominal pore sizes around 1 m are employed. The cross- ow is realized with air bubbles that are periodically injected at the bottom of the modules.

In order to study the highly complex process, a rigorous model has been developed, which is discussed in detail by Broeckmann *et al.* (2005) and Cruse (2006). It has been shown to adequately represent real plant behavior, and is used as a reference model in the following.

The model proposed in Section 2.1 is adapted to re ect the speci c characteristics of the given process. The cross- ow velocity u_c is usually not explicitely available as manipulated variable, since air is injected with a constant, yet intermitted volume ow Q. The periodically changing intervals with and without aeration have the lengths t_{on} and t_{off}. u_c is then heuristically described as

$$u_c = Q\frac{t_{\mathrm{on}}}{t_{\mathrm{on}} + t_{\mathrm{off}}}, \qquad (32)$$

and t_{off} is chosen as manipulated variable. The power for the aeration e_c is expressed as

$$e_c = \frac{QTR_g\gamma_{\mathrm{a}}\left[(1+p_{\mathrm{a}})^{\frac{\gamma_{\mathrm{a}}}{\gamma_{\mathrm{a}}\ \ 1}} \ \ 1\right]t_{\mathrm{on}}}{v_{\mathrm{a}}\left(\gamma_{\mathrm{a}} \ \ 1\right)\left(t_{\mathrm{on}} + t_{\mathrm{off}}\right)\eta_A}, \qquad (33)$$

assuming that the compression is a polytropic process. T is the ambient air temperature, v_{a} is the molar volume of air, R_g is the gas constant, $\gamma_{\mathrm{a}} = 1.4$ is the polytropic coe cient, p_{a} is the pressure di erence across the compressor (in bar), and η_A is an e ciency factor.

3.1 Results

Three aspects are analyzed in the following: the quality of the TMP prediction, the adaptation to process changes, and the predicted optimal solutions. The simulation results based on the reference model will be referred to as *measurements.*

TMP prediction Fig. 2 depicts a snap-shot of the simulated controlled process. It shows the measured and the predicted TMP for cycles k and $k + 1$, between which the ux is increased. Each cycle comprises a ltration (positive TMP) and a backwashing phase (negative TMP). The parameters α, , γ, and , which are not estimated online, have been tted a priori. During ltration the predicted and the measured TMP are almost identical, and only small errors are observed during backwashing. The relative deviation is below 1%. The same results are achieved with respect to changing backwashing uxes, ltration and backwashing durations, and cross- ow intensities. This shows the excellent prediction capability of the model.

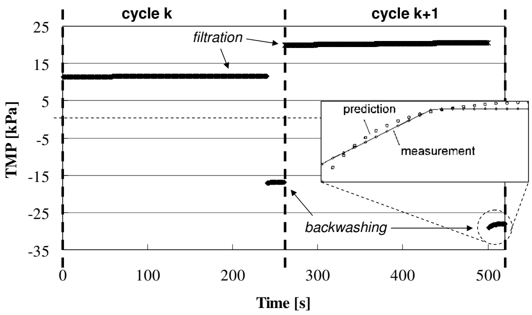

Fig. 2. TMP measurement and prediction

Model adaptation Next, the performance in the presence of unforeseen process changes is evaluated. In Fig. 3, the ltration ux after cycle k is reduced by 20%. This could be caused by an unexpected problem with a pump. The TMP prediction is false in cycle $k+1$, but the controller adapts by solving the estimation problems (P2) and (P3) with data from cycle $k + 1$. Reliable predictions are provided from cycle $k + 2$ on.

Control In the following an updated model is assumed to be available, and the optimization for the next cycle is carried out for di erent

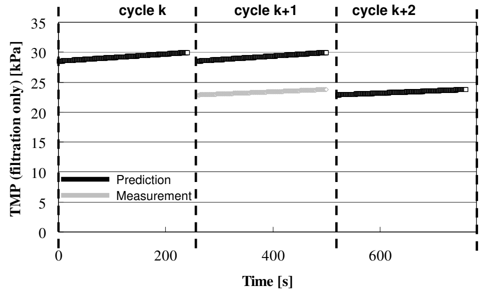

Fig. 3. Adaptation to a sudden process change

required net uxes. Fig. 4 presents the results for the ltration ux and the aeration pause. The ltration ux increases almost linearly with higher net uxes. The ltration time is at its upper bound of 600s. The backwashing ux always equals the ltration ux, and together with the minimum backwashing time of 15s, the constraint on the minimum resistance removal (Eq. (26)) is always met. The aeration pause becomes smaller with increasing ux.

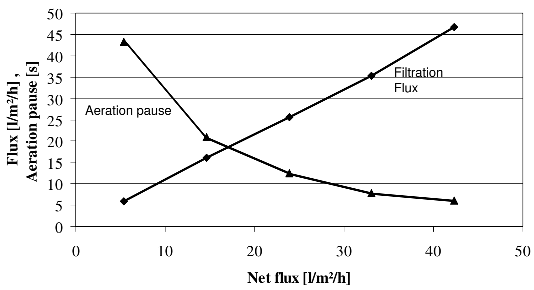

Fig. 4. Variation of the net ux

Finally, the performance of the proposed controller is compared against manual operation with xed set-points. A typical choice in an industrial installation is e.g. $J_f = 40\frac{\mathrm{l}}{\mathrm{m^2h}}$, $J_b = 50\frac{\mathrm{l}}{\mathrm{m^2h}}$, $t_f = 240$s, $t_b = 20$s, and $t_{\mathrm{off}} = 6$s, which gives a net ux of $J_{\mathrm{net}} = 33.1\frac{\mathrm{l}}{\mathrm{m^2h}}$. For the same net ux, the optimized solution depicted in Fig. 4 requires 20% less energy despite employing a 14% higher aeration.

3.2 Discussion

Assuming a decent choice and adaptation of the parameters α, , γ, and in the ltration models, an excellent prediction of the TMP is achieved. Furthermore, the controller quickly adapts to unexpected changes in the process.

The interpretation of the optimization results is straightforward. Low uxes with long ltration times are preferred over small ltration periods with high uxes. This is in line with current observations in MBR installations. Instead of placing an upper bound on the ltration time, a more sophisticated approach could be designed, which e.g. establishes a link between the upper bound and the ux, if according process knowledge is available. The shortening of the aeration pauses with increasing uxes is clearly due to the penalty term, which prevents long-term damage to the membranes. In the case study, the backwashing intensity is at its lower bounds, yet this depends on the cleaning e ciency of the process, which is detected in the parameter estimation step. The remaining tuning parameter ξ (Eq. (14)) re ects the balance between short-term (energy) and long-term (replacement) cost.

Finally, employment of the approach does not only promise substantial economical bene t, but also implies continuous adaptation of the membrane's operation to process drifts and changes. This enables not only optimal, but also safe process operation.

4. CONCLUSIONS

A methodology for the model-based control of membrane ltration processes is proposed. It is based on run-to-run control concepts and employs a newly developed process model. The proposed controller is tested in a simulation scenario describing submerged membrane ltration in wastewater applications. It is shown to achieve excellent results concerning the prediction quality, the adaptation to process changes, and the process optimization with respect to power consumption and membrane replacement cost. Its performance is currently experimentally veri ed in an industrial pilot plant.

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