PERFORMANCE ASSESSMENT OF MODEL PREDICTIVE CONTROL SYSTEMS

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Abstract: This paper aims at to propose a benchmark MPC controller to be used in the performance assessment of existing industrial MPC systems. The basic questions are how the performance could be evaluated in a realistic basis and how to judge the performance of a controller that is already in operation by comparing it with another controller that could be really implemented in the same system. Here, it is assumed that the ideal controller will inherit the structure, input constraints and tuning parameters of the controller whose performance is to be evaluated. This means that the design of the ideal controller is standard and there is no need to tune the performance assessment algorithm. It is proposed a controller that preserves closed loop stability for any adopted tuning parameters. This is requisite for any performance evaluation procedure that is expected to operate in an on-line scheme. The proposed controller is compared by simulation with other benchmark controllers proposed in the control literature.

Keywords: Performance assessment, Model predictive control, Controller performance monitoring, Constrained control systems, Industrial process control.

1. INTRODUCTION

Model predictive control (MPC) strategies, such as the ones based on dynamic matrix control (DMC), have become the standard control alternative for advanced control applications in the process industries (Qin and Badgwell, 2003). It is a market, which is growing at a compound annual rate of approximately 18% (Automation Research Corporation, 2000), and substantial benefits are generated directly from the ability of MPC to ensure that the plant operates at its most profitable constraints. But, as most control algorithms, after some operation time, MPC is seldom performing as when it was commissioned. It is common to find MPC applications delivering only 50% of the expected benefit when the assessment is made 2-3 years after commissioning (Treiber et al., 2003). MPC controller design and tuning involve many uncertainties related to approximate process models, estimation of disturbances and assumptions about operation conditions. A surprising high percentage of the implemented MPC controllers suffers degradation in terms of the achieved performance as a result of changes in process dynamics, sensor/actuator failure, estimator bias, equipment fouling, feedstock variability, changes in product specifications, etc. Therefore, to sustain the benefits of MPC systems over a considerable period of time, the performance needs to be monitored and assessed on a constant basis. This task has proven to be a much greater challenge (Hugo, 2000; Shah et al., 2001) than initially expected and it requires the presence of effective tools to establish the root causes of the poor control quality and to define the need to retune if necessary.

Practical applications of controller performance assessment (CPA) have triggered an increasing interest of academia and industry in the development of a benchmark MPC controller. Several CPA techniques have been proposed in the literature during the last years, some of them being incorporated into commercial software packages. But, in general, all the CPA techniques explicitly or implicitly involve comparison of the current controller quality with a theoretical benchmark, i.e. an ideal controller that could never be implemented.

CPA techniques can be divided into two major categories (Qin, 1998): stochastic and deterministic methods. Stochastic CPA methods evaluate the closed-loop performance for zero-mean changes, such as random disturbances, measurement noise, etc. These techniques utilize stochastic measures such as variance to evaluate the performance of the controller. In this area, the most notable work is by Harris (1989) that proposed the use of the minimum variance controller (MVC) as a benchmark to assess the performance of SISO feedback controllers. On the other hand, deterministic CPA methods are concerned with non-zero mean changes in the set-point or load disturbances and utilize deterministic measures such as settling time, integral square error (ISE), rise time, etc. Aström (1991) discussed some alternatives to evaluate the performance of PID controllers. Although MVC benchmarks bring up important aspects of the controller performance, deterministic methods are more informative and present a more practical way of assessing controller performance. Results from statistic and deterministic CPA methods usually cannot be best achieved simultaneously (Qin, 1998).

In this paper it is proposed a systematic CPA framework for MPC systems with focus on set-point tracking. The methodology utilizes the available process model to determine the ideal control system performance under constraints. The work is motivated by the fact that the major disturbances in chemical engineering processes are not stochastic but deterministic such as set-point moves and sudden load changes on the system (MacGregor et al., 1984) and for the demand for new methodologies to evaluate MPC performance (Patwardhan et al., 2002). The paper is organized as follows. In Section 2, we review the most relevant CPA techniques for MPC. Section 3, presents our proposed CPA method. Section 4 shows a case study based on the Shell standard control problem. Finally, conclusions and future directions are pointed in Section 5.

2. REVIEW OF CPA TECHNIQUES FOR MPC

In the literature on CPA of feedback control systems, the MVC benchmark has been used as a measure of performance in the first level of the control structure where SISO controllers are to be evaluated. This benchmark is reasonable because the objective of most SISO controllers is to keep the process output at their set-point. However, MPC controllers have much more sophisticated objectives than merely keeping outputs at their set-points. They are usually implemented as part of a hierarchical control structure, where in an upper layer an optimization algorithm continuously updates a set of optimal economic reference values and passes this set to the MPC (Qin and Badgwell, 2003). The MPC must move the plant from one reference point to another subject to operational constraints. Of course, MPC turns to be essentially a nonlinear controller, especially when operating at the constraints. In this case, the use of MVC or a linear controller benchmark is not well suitable as some inherent limitations imposed by constraints are neglected and minimum variance performance will be unachievable by the MPC (Zhang and Henson, 1999). Examples highlighting the limitations of the MVC benchmark to assess the DMC controller are shown by Hugo (1999).

Patwardhan et al. (1998) discussed the use of the best historical values of objective function as a practical benchmarking technique. This approach requires *a priori* knowledge of an example case where the performance was good during a certain time period of time according to some expert assessment. Huang and Shah (1999) proposed the linear quadratic Gaussian (LQG) benchmark as an alternative to the MVC. The LQG is more general than the MVC and it can be designed, using the available process model, in a similar form as the MPC. This benchmark is translated in a tradeoff curve that displays the minimal achievable performance in terms of the input and output variances. However, the LQG cannot handle constraints and it still represents an unattainable standard for commercial MPC algorithms. Zhang and Henson (1999) suggested the use of the on-line comparison between expected and actual process performance. The expected performance is obtained when the MPC controller is applied to the process model instead of the actual plant and it is neglected the effects of unmeasured disturbances. Ko and Edgar (2001) presented a benchmark based on the constrained finite-horizon MVC controller, which is obtained using the knowledge of the process and noise models. The main utility of this approach lies in quantifying the effects of constraints on the MPC performance. When the constraints become inactive, the proposed method naturally becomes the unconstrained MVC.

Patwardhan et al. (2002) suggested the design case as a benchmark to evaluate the statistical performance of MPC. The methodology is very straightforward to be implemented on-line. The cost function used in the design of the CPA can be obtained from the MPC controller. The achieved cost function can be computed with little effort through appropriate weighting of the measured input and output data. The technique can explicitly handle constraints and it is a true index that represents whether the controller is performing as it was designed or not. However its application is limited, as most of the commercial MPC algorithms do not return the design value of the cost function. Grimble (2003) presented a multistep linear quadratic Gaussian predictive control (LQGPC) cost function as benchmark to evaluate MPC. The cost function involves the unconditional expected value of the tracking error and weighted control signal components at present and future time steps, whose values are obtained from the solution of appropriate Riccati and Lyapunov equations. The results highlight the relationship between MPC and LQG and the way that the performance of MPC should be assessed. Julien et al. (2004) proposed a MPC benchmark for assessing univariate MPC controllers. By using routine operating data and knowledge of the process time-delay, two performance curves are constructed. One represents the operation of the installed MPC, while the other corresponds to the operation of a hypothetical MPC. If the gap between these operating curves is significant, it may indicate that a re-design of the MPC is necessary.

Schäfer and Cinar (2004) presented an integrated methodology for CPA and diagnosis of MPC systems. They use a LQG benchmark to evaluate performance and a ratio between design and achieved costs for diagnosis of causes of poor performance. Finally, Huang and Georgakis (2005) proposed the minimum (settling) time optimal control (MTOC) benchmark. MTCO-FB is an ideal benchmark for unmeasurable disturbance regulation, while MTCO-FF is an ideal benchmark for set-point tracking. This last, serves also as a reference to determine whether extra sensors and feedforward will yield significant control improvement.

3. PROPOSED TECHNIQUE FOR CPA OF MPC SYSTEMS

Following Zhang and Henson (1999), we propose a CPA technique that involves an on-line performance comparison between expected and actual MPC. But, in this case, the expected performance is obtained with a particular MPC, called here "ideal MPC", that is used to control the nominal process model. The Proposed benchmark is represented schematically in figure 1.

From figure 1, we observe that the optimal set-point (y_{sp}) provided by the upper optimization layer is applied to both MPC systems, actual and benchmark. Estimated disturbances ($\hat{d}$) are used to correct model prediction to asymptotically remove offset. The performance of these systems is measured using adequate indeces, which are compared to determine the performance status of the actual MPC.

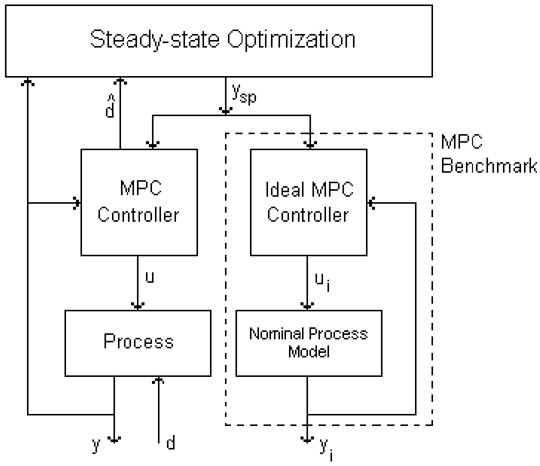

Fig. 1. Scheme of the proposed CPA for MPC.

In the sequel, our ideal MPC controller and performance index will be discussed.

3.1 "*Ideal MPC*" *Controller*

For a more realistic comparison, an "ideal MPC" should preserve some characteristics of the implemented MPC, i.e. full utilization of the available process model, incorporation of constraints and computation based on the receding horizon control philosophy. Also, as the ideal controller may utilize tuning parameters that are different of the tuning parameters of the implemented controller, we require that the ideal controller be at least nominally stable. A MPC controller with nominal stability and that tolerates input saturation was proposed by Rodrigues and Odloak (2005). It is assumed that input saturation can occur in the transition from one set-point to another and that the system remains stabilizable during the time the input remains saturated. Consideration of input saturation is usually necessary in a process that operates near the optimal economic conditions.

It can be shown that an unconstrained MPC law can be formulated as:

$$\Delta u_{(m\cdot nu \mathrm{x} 1)}(k) = K_{MPC} E^{o}_{(p\cdot ny \mathrm{x} 1)}(k) \tag{1}$$

where $\Delta u(k) = u(k) - u(k-1)$ is the vector of future increment control actions, $K_{MPC} \in \Re^{(m\cdot nu)\mathrm{x}(p\cdot ny)}$ is the time-invariant feedback control gain matrix, $E^{o}(k)$ is the vector of predicted unforced errors, k is the sampling instant, m is the control horizon, p is the prediction horizon, and nu and ny ($nu > ny$) are the number of manipulated and controlled variables, respectively. The development of our "ideal MPC" is to follow a two-step procedure (Rodrigues and Odloak, 2005):

1) *Off-line step*. Compute a bank of stable unconstrained MPC controllers (K_{MPC}),

corresponding to all possible configurations of manipulated inputs and stabilizable outputs. Let nc be the number of configurations and let us designate as $K_{MPC,j}$ the gain of the controller corresponding to configuration j (j=1,…,nc).

2) *On-line step*. At each sampling period, compute the predicted unforced error (E^o) and find the solution of the following optimization problem:

$$\min_{\beta_0,\cdots,\beta_{nc}} \quad J_k = \sum_{i=1}^{p} e^T(k+i)Qe(k+i) + \sum_{i=0}^{m-1} \Delta u^T(k+i)R\Delta u(k+i) \tag{2}$$

Subject to:

$$\Delta u(k) = \left[\beta_0 K_{MPC} + \cdots + \beta_{nc} K_{MPC,nc}\right] E^o(k) \tag{3}$$

$$\sum_{j=0}^{nc} \beta_j = 1 \tag{4}$$

$$\beta_j \geq 0, \qquad j = 0,1,\cdots,nc \tag{5}$$

$$u_{\min} \leq u(k+j) \leq u_{\max}, \qquad j = 0,1,\cdots,m-1 \tag{6}$$

where Q is the output error weighting matrix and R is the input increment weighting matrix. Note that the input increment constraints are not included in the above problem. Only the first component of the computed Δu is used. The successive application of this control law produces an asymptotically stable closed-loop system.

3.2 Measure Performance Index

Various dimensionless performance indices have been proposed in the literature. In this work, the controller performance is represented by:

$$J(k) = \sum_{j=1}^{N} \left(y_{sp}(k-j) - y(k-j)\right)^T Q \;\mathrm{x}\; \left(y_{sp}(k-j) - y(k-j)\right) \tag{7}$$

where $y_{sp}(k)$ is the set-point, $y(k)$ is the value of the controlled variable and N is the length of the past data operation window. The performance measure index $\eta(k)$, which is selected, to bear some similarity with the one proposed by Harris (1989), is the ratio of the performance provided by the "ideal MPC" to the actual performance provided by the present MPC system:

$$\eta(k) = 1 - \frac{J_{\text{ideal}}(k)}{J_{\text{act}}(k)} \tag{8}$$

The index defined in Eq. (8) gives numerical bounds for controller performance $0 \leq \eta \leq 1$, where $\eta = 0$ indicates excellent performance and $\eta = 1$ indicates poor performance. In the subsequent section, the proposed approach is applied to evaluate the MPC performance for a simulated industrial process.

4. CASE-STUDY

4.1 The Shell Standard Control Problem

The Shell standard control problem (SSCP) is a well-known process control problem developed with the intention of providing a standardized simulation protocol for the evaluation of control systems. This system is an industrial heavy oil fractionator process, as shown in figure 2 (Prett and Morari, 1987).

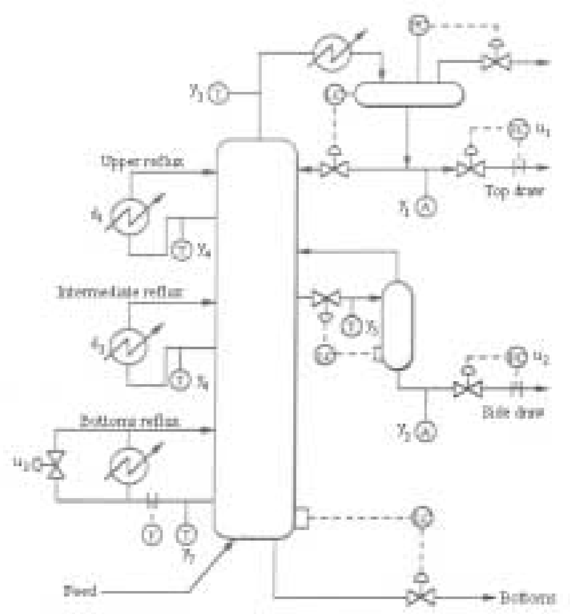

Fig. 2. Layout of the heavy oil fractionator process

The SSCP embodies a number of scenarios that can occur in controlling the process unit. It is represented by a 5x7 MIMO system, which is highly constrained, with very strong interactions, unmeasured disturbances, mixed fast and slow responses, severe uncertainties, large time-delays and simultaneous and conflicting control and economic objectives. The process input/output relations are modeled by transfer functions of first-order plus time-delay. The full process model and its associated uncertainty can be found in Prett and Morari (1987) and Maciejowski (2002).

4.2 MPC Control Design

Let us consider only a part of the SSCP and study the servo problem of the subsystem in which the controlled variables are the top draw composition (y_1) and side draw composition (y_2), and the manipulated variables are the top draw (u_1), side draw (u_2) and bottom reflux duty (u_3). The transfer function of this subsystem is:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{(4.05+2.11\delta_1)e^{-27s}}{50s+1} & \frac{(1.77+0.39\delta_2)e^{-28s}}{60s+1} & \frac{(5.88+0.59\delta_3)e^{-27s}}{50s+1} \\ \frac{(5.39+3.29\delta_1)e^{-18s}}{50s+1} & \frac{(5.72+0.57\delta_2)e^{-14s}}{60s+1} & \frac{(6.90+0.89\delta_3)e^{-15s}}{40s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (9)$$

where δ_1, δ_2 and δ_3 represent uncertainties in the gain parameters and they can vary between –1 and +1. Here, they are assumed to be $\delta_1 = \delta_3 = 0.5$, $\delta_2 = -0.5$.

The control objective is set-point tracking and, for this purpose, a conventional QDMC (quadratic dynamic matrix control) is proposed. This controller is designed based on the nominal process model (i.e., $\delta_1 = \delta_2 = \delta_3 = 0$) and on the minimization of the following cost function:

$$\min_{\Delta u(k),\cdots,\Delta u(k+m-1)} \quad J_k = \sum_{i=1}^{p} e^T(k+i)Qe(k+i) + \sum_{i=0}^{m-1} \Delta u^T(k+i)R\Delta u(k+i) \quad (10)$$

Subject to:

$$\begin{aligned} -0.5 \le u_i \le 0.5, \quad & i = 1,2,3 \\ |\Delta u_i| \le 0.1, \quad & i = 1,2,3 \end{aligned} \quad (11)$$

The tuning parameters used in the QDMC and in the "ideal MPC" are: $p = 75$, $m = 10$, $Q = \text{diag}(1,1)$, $R = \text{diag}(1.5, 0.15, 1.5)$ and sampling time $T = 4\,\text{min}$. The controllers are implemented in the hierarchical control structure as shown in figure 1. However, in this study, it is considered that the optimization layer can be ignored in the simulations, i.e. the set-points are assumed to be known.

4.3 MPC Performance Assessment

The hierarchical control structure of the MPC system is not included in the problem considered by the performance assessment method, since it does not really matter if the set-point for the MPC controllers comes from an operator or a computer program.

Thus, in our example performance assessment is carried out for the set-point changes shown in figure 3. It is also shown the responses of the system outputs for the QDMC based on the nominal model when controlling the true system that contains uncertainties as described before. In figure 3 are also shown the output responses for three different benchmark controllers. QDMC$_2$ is the benchmark corresponding to the same QDMC controller of the previous case but controlling the nominal model. This scenario corresponds to the design case. The second benchmark controller corresponds to the LQG (Dorato et al., 1995) optimally tuned to control the nominal process. Finally, the third benchmark controller is the proposed controller with the same tuning parameters as the QDMC except the constraint in the control moves that is not included in the control problem. Figure 4 illustrates the responses for the inputs of the system.

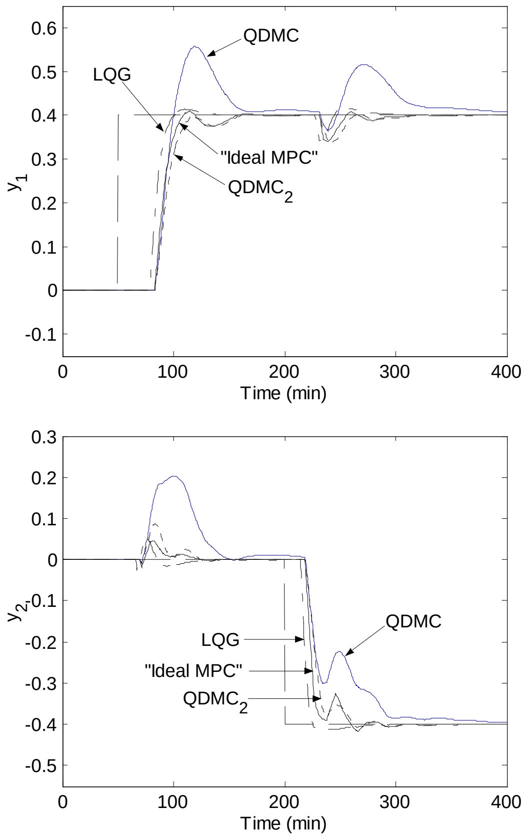

Fig. 3. Output responses of the Shell system

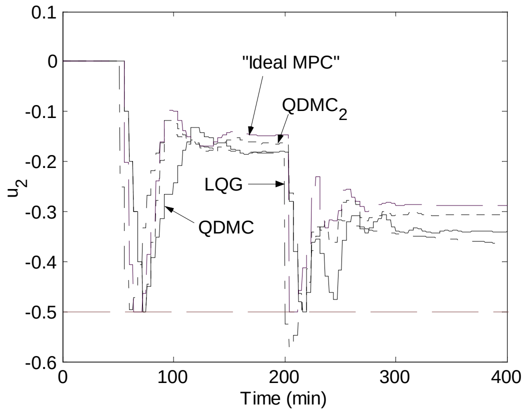

Fig. 4 Input u_2 responses of the Shell system

Table 1 shows the numerical values of the controller performance defined by Eq. (7) and also shows the performance indices defined by Eq. (8) for the QDMC controller applied to the uncertain system in terms of each of the benchmark controllers described before. From figure 3, it is clear that the LQG produces the best nominal performance but it is not realistic as it does not satisfy the input constraints as shown in figure 4, where the minimum bound of input u_2 is not satisfied during part of the simulation time. Table 1 shows that $QDMC_2$ produces a more conservative index that the proposed benchmark which gives a better indication of the performance of the implemented controller.

Table 1. Control performances and indices

System	J	η
QDMC	3.4273	---------
$QDMC_2$	2.6710	0.2207
"Ideal MPC"	2.4845	0.2751
LQG	2.0597	0.3990

5. CONCLUSIONS

In this work, it is proposed a new MPC benchmark controller whose purpose is the realistic evaluation of the performance of MPC controllers implemented in industry. The proposed benchmarks has as main characteristics to consider input constraints and guaranteed nominal stability, which is usually not attended by other proposed benchmark controllers. The approach was tested by simulation in a typical system of the process industry with satisfactory results.

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REFERENCES

Aström, K.J. (1991). Assessment of achievable performance of simple feedback loops, *International Journal of Adaptive Control and Signal Processing*, 5(1), 3-19.

Automation Research Corporation (2000). *Real time process optimization and training worldwide outlook*, http://www.arcweb.com/.

Dorato, P.C., C. Abdallah, V. Cerone (1995). *Linear quadratic control: an introduction*, Prentice Hall.

Grimble, M.J. (2003). Performance assessment and benchmarking LQG predictive optimal controllers for discrete-time state-space systems, *Transactions of the Institute of Measurement and Control*, 25(3), 239-264.

Harris, T.J. (1989). Assessment of control loop performance, *Canadian Journal of Chemical Engineering*, 67(5), 856-861.

Huang, B., S.L. Shah (1999). *Performance assessment of control loops – theory and applications*, Springer-Verlag: London.

Huang, L., C. Georgakis (2005). On the assessment of controller performance for constrained systems, *7th Italian Conference on Chemical and Process Engineering* (ICheaP-7), Giardini di Naxos-Taormina.

Hugo, A. (1999). Performance assessment of DMC controllers, *1999 American Control Conference* (ACC'99), San Diego-CA.

Hugo, A. (2000). Limitations of model predictive controllers, *Hydrocarbon Processing*, 79(1), 83-88.

Julien, R.H., M.W. Foley, W.R. Cluett (2004). Performance assessment using a model predictive control benchmark, *Journal of Process Control*, 14(4), 441-456.

Ko, B.-S., T.F. Edgar (2001). Performance assessment of constrained model predictive control systems, *AIChE Journal*, 47(6), 1363-1371.

MacGregor, J.F., T.J. Harris, J.D. Wright (1984). Duality between the control of processes subject to randomly occurring deterministic disturbances and arima stochastic disturbances, *Technometrics*, 26(4), 389-397.

Maciejowski, J.M. (2002). *Predictive control with constraints*, Pearson Education Limited: England.

Patwardhan, R.S., S.L. Shah, K.Z. Qi (2002). Assessing the performance of model predictive controllers, *Canadian Journal of Chemical Engineering*, 80(5), 954-966.

Patwardhan, R.S., S.L. Shah, G. Emoto, H. Fujii (1998). Performance analysis of model-based predictive controllers: an industrial case study, *1998 AIChE Annual Meeting*, Miami-FL.

Prett, D.M., M. Morari (1986). *The Shell process control workshop*, Butterworths: Stoneham-MA.

Qin, S.J. (1998). Control performance monitoring – a review and assessment, *Computers and Chemical Engineering*, 23(2), 173-186.

Qin, S.J., T.A. Badgwell (2003). A survey of industrial model predictive control technology, *Control Engineering Practice*, 11(7), 733-764.

Rodrigues, M.A., D. Odloak (2005). Robust MPC for systems with output feedback and input saturation, *Journal of Process Control*, 15(7), 837-846.

Schäfer, J., A. Cinar (2004). Multivariable MPC system performance assessment, monitoring, and diagnosis, *Journal of Process Control*, 14(2), 113-129.

Shah, L.S., R.S. Patwardhan, B. Huang (2001). Multivariate controller performance analysis: methods, applications and challenges, *VI Chemical Process Control* (CPC-6), Tucson-AZ.

Treiber, S., R. Sttelmaier, M. Starling (2003). Sustaining benefits of process automation, *2003 AIChE Spring Nat. Meeting*, New Orleans-LA.

Zhang, Y., M.A. Henson (1999). A performance measure for constrained model predictive controllers, *5th European Control Conference* (ECC'99), Karlsruhe, Germany.