OPTIMAL MULTIPERIOD DESIGN AND OPERATION OF MULTIPRODUCT BATCH PLANTS

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Abstract: New alternatives for the multiperiod design and operation planning of multiproduct batch plants are presented. Unlike previous works, this approach configurates the plant in every period considering the assignment of parallel units of different sizes operating either in or out-of-phase. The objective function maximizes the net profit considering incomes, investment costs, and both product and raw material inventory costs. The model takes into account batch units available in discrete sizes, and both raw material and product inventories accounting for seasonal variations for supplies and demands. Nonlinearities have been eliminated by an efficient scheme in order to get a MILP model to guarantee global optimality.

Keywords: Optimization, Batch, Design and Planning, Parallel Equipment, Integer Programming

1. INTRODUCTION

Continuous growth in complexity, competitivity, and uncertainty of the marketing environment of high added value chemicals and foodstuffs with a short life cycle has renewed the interest in batch operations and the development of optimization models. The main attraction of batch plants in this context is their inherent flexibility in utilizing the various resources available for the manufacture of relatively small amounts of several different products within the same facilities. Several excellent papers on designing and production planning of multiproduct batch plants have already been published (Grossmann, and Sargent, 1979; Ravermark, 1995). The goal is to determine the size and the number of batch units so they can meet production requirements in the provided time horizon.

Since such products usually have demand patterns that vary over time due to market or seasonal changes, multiperiod optimization models have been the object of a great deal of research effort in this area (Birewar and Grossmann, 1990; Voudouris and Grossmann, 1993; Varvarezos et al. 1992; Van den Heever and Grossmann, 1999). Multiperiod models for the design and operation planning in chemical plants involve designing plants that operate under variations in the model parameters along the time horizon. In general, this kind of problems is represented by mixed integer nonlinear programming (MINLP) models.

Multiproduct batch plants manufacture a set of products using the same equipment operating in the same sequence. Since products differ from one another, each unit is shared by all products but they do not use their total capacity for all of them. The unit with the minimum capacity limits the batch size while the limiting cycle time is fixed by stage with the longest processing time.

In order to reduce the investment cost, several alternatives are possible (Ravermark, 1995). The first one is the introduction of parallel units out of phase to reduce the cycle time if the unit has the longest operating time. Another option is to add a parallel unit in phase to increase the operating capacity of the stage.

The above referenced approaches for solving the multiperiod MINLP problem were restricted to equal periods in the design horizon (Voudouris, and Grossmann, 1993) or they did not include the design into the formulation (Birewar, and Grossmann, 1990). Also, no previous design work has considered the addition of parallel units in and out of phase, which can take different sizes, and has not addressed the issue of raw materials inventory.

This work is an attempt to expand the scope of multiperiod models for the design and production planning problems in multiproduct batch plants. In this paper, an optimization mixed integer linear programming (MILP) model which can handle seasonal changes of prices, costs and demands, discrete sizes of units, and inventories of both final products and raw materials will be proposed. Moreover, this multiperiod approach considers different period lengths and the possible alternatives to add parallel units that can have different sizes in every stage by using the concept of group introduced by Yoo, *et. al.* (1999) and extended by Montagna (2003). Also, this model takes into account flexible plant configurations where available units can be arranged in different structures for each product. In contrast to previous models that consider continuous sizes for the units, this model determines the optimal design selecting from available discrete sizes which corresponds to the real procurement of equipments. The major significance of the MILP model presented in this work is that it corresponds to a realistic design case that can be solved to global optimality with reasonable computation effort.

The remainder of this paper is structured as follows. The next section presents the problem description. In the subsequent section, a multiperiod model which incorporates all the elements of the design and planning problem is formulated. The non linear terms in the formulation are transformed into linear ones in order to obtain a MILP model by using a reformulation strategy. The application of this formulation to a specific example is illustrated for a plant that produces Oleoresins. Finally, the conclusions are presented in the last section.

2. PROBLEM STATEMENT

In a multiperiod scenario a multiproduct batch plant processes I products (i = 1, 2, …, I). Every product follows the same production sequence through all the J batch processing stages (j = 1, 2, …, J) of the plant. Each stage j may consist of one or more units k, which can have different sizes, operating either in phase to increase capacity, or out of phase to decrease the cycle time. The size of unit k at stage j is V_{jk} (k = 1, 2, …, K_j), where K_j is the maximum number of units that can be added at stage j. Also, the volume of each unit k at stage j is available in discrete sizes.

The configuration of the batch units must be determined at each stage for every product. K_j units of stage j can be grouped in different ways for each product i (Yoo, et al., 1999). It is possible to have groups in which all units operate in parallel and in phase. The different groups at the stage operate in parallel and out of phase.

Since this is a multiperiod problem, the time horizon H is discretized into T (t = 1, 2, …, T) specified time periods H_t not necessarily of the same length. Bounds on products demands, costs and availability of raw materials vary from period to period. It will be assumed that the plant operates in single product campaign (SPC) mode under zero wait (ZW) policy in each time period.

The objective is to maximize the benefit of the plant considering incomes from inventory and product sales, and capital costs. In the design of this plant, the problem lies in deciding the convenience of adding units at any time period, and selects the size of batch units V_{jk} among available discrete sizes v_{js}. At each time period t the model determines the number of groups and which of the existing units in a period are assigned to each of them. Moreover, it decides the amount of product to be produced q_{it}, the number of batches n_{it} and the total time T_{it} to produce product i.

Inventory considerations are an important aspect of plant operation. Actually, in practice, a plant may be faced with product demands and raw materials supplies that vary seasonally. So, products and raw materials can be maintained in stock until needed.

At the end of every period t, the levels of both final product IP_{it} and raw material inventories IM_{it} are obtained. Moreover, the total sales QS_{it}, the amount of purchased raw material C_{it}, and the raw material to be used for the production RM_{it} of product i in each period t are determined with this formulation. Semicontinuous and intermediate storage are not considered.

3. MATHEMATICAL FORMULATION

3.1 Assignment Constraints

Several variables are introduced to determine the plant structure. Since the units can be added at any time period, a binary variable w_{jkt} is used. The value of this variable is 1 if unit k is included in the plant structure at stage j in the period t; otherwise the value is zero. Each unit k at stage j can be added only in one period:

$$\sum_{t}^{T} w_{jkt} \leq 1 \qquad \forall\, i, j \tag{1}$$

The units are included in a sequential manner in order to avoid alternative optimal solutions with the same value for the objective function:

$$\sum_{\tau=1}^{t} w_{j,k,t} \geq \sum_{\tau=1}^{t} w_{j,k+1,t} \qquad \forall\, i, k = 1,\ldots, K_j - 1, t \tag{2}$$

Since the units can be grouped in different ways at each stage in every period, the binary variable y_{ijgt} is introduced. The value of this variable is equal to 1 if group g is generated for product i at stage j in time period t; otherwise, the value is zero. Group g is generated if at least one unit is assigned to it. Binary variable y_{ijkgt} is 1 if unit k of stage j is assigned to group g for product i at period t; otherwise the variable is equal to zero (Montagna, 2003).

$$\sum_{g=1}^{G_j} y_{ijkgt} \leq 1 \qquad \forall\, i, j, k, t \tag{3}$$

G_j is the maximum number of groups allowed at stage j. Group g exists at stage j in period t only if at least one unit is assigned to the group in that period:

$$y_{ijgt} \leq \sum_{k=1}^{K_j} y_{ijkgt} \qquad \forall\, i, j, g, t \tag{4}$$

If unit k is assigned to the group in period t, the group must exist:

$$y_{ijkgt} \leq y_{ijgt} \qquad \forall\, i, j, k, g, t \tag{5}$$

If the unit is assigned to group g at stage j for product i in period t, the unit must exist in that period:

$$y_{ijkgt} \leq \sum_{\tau=1}^{t} w_{jkt} \qquad \forall\, i, j, k, g, t \tag{6}$$

If the unit exists at stage j in the period t, it must be included in a group:

$$\sum_{\tau=1}^{t} w_{jkt} = \sum_{g=1}^{G_j} y_{ijkgt} \qquad \forall\, i, j, k, t \tag{7}$$

Redundant assignation to a group with the same value for the objective function is avoided by the following constraint (Yoo, et al., 1999):

$$\sum_{k=1}^{K_j} 2^{K_j-k}\, y_{ijkgt} \geq \sum_{k=1}^{K_j} 2^{K_j-k}\, y_{ijk,g+1,t} \qquad \forall\, i, j, g = 1,\ldots,G_j - 1, t \tag{8}$$

This constraint order the different groups through a weight 2^{K_j-k} assigned to each unit k. The order of the group is obtained by adding the weights of all units in the group.

3.2 Design and Planning and Constraints

Unit k at each stage j can be configured in a different way for every product i manufactured in the plant. B_{it} is the batch size of product i in time period t. When B_{it} gets into a group of units, that is, units operating in phase, B_{it} is divided between the units that belong to the group. Thus, the sum of the units sizes included in the group g in every period t must be large enough to produce a batch of product i.

$$\sum_{k\in g} V_{jkt} \geq S_{ijt} \cdot B_{it} \qquad \forall\, i, j, g, t \tag{9}$$

where S_{ijt} is the size factor at stage j for product i, that can vary in each period taking into account seasonal effects.

In this work, the unit sizes V_{jk} are considered available in discrete sizes v_{js} which correspond to the real commercial procurement of equipments. To rigorously tackle this situation, the binary variable z_{jks} is introduced. It is one if unit k at stage j has size s; otherwise, it is zero. The variable V_{jk} is restricted to take values from the set $SV_j = \{v_{j1}, v_{j2}, \ldots, v_{jn_j}\}$, where n_j is the number of discrete sizes available for each stage. Using the previous definition, V_{jk} can be expressed in terms of discrete variables as:

$$V_{jk} = \sum_{s} v_{js} \cdot z_{jks} \qquad \forall\, j, k \tag{10}$$

If the unit k at stage j is added in some period t, it must take a size s for the volume from the available sizes at that stage:

$$\sum_{s} z_{jks} = \sum_{t=1}^{T} w_{jkt} \qquad \forall\, j, k \tag{11}$$

Only one of the available sizes at stage j must be selected if unit k at stage j exists:

$$\sum_{s} z_{jks} \leq 1 \qquad \forall\, j, k \tag{12}$$

The amount of product i produced in time period t is

$$q_{it} = B_{it} \cdot n_{it} \qquad \forall\, i, t \tag{13}$$

where n_{it} is the batch number of product i in period t.

By combining Eq. (9) and Eq. (13) the constraints take the following form:

$$q_{it} \leq \sum_{k\in g} V_{jkt} \cdot \frac{n_{it}}{S_{ijt}} + M_{ij} \cdot (1 - y_{ijgt}) \qquad \forall\, i, j, g, t \tag{14}$$

Eq. (14) is a Big-M constraint that guarantees that batches can be processed if group g exists; otherwise the constraint is redundant because of the large value of M_{ij}. The value of M_{ij} can be calculated by:

$$M_{ij} = K_j \cdot \max(s, v_{js}) \cdot \max(t, n_{it}^{U} / S_{ijt}) \qquad \forall\, i, j \tag{15}$$

In order to obtain the volumes that belong to each group, it is necessary to multiply the volume V_{jk} by the binary variable y_{ijkgt} which produces the equation:

$$q_{it} \leq \sum_{k=1}^{K_j} \left(V_{jk} \cdot y_{ijkgt}\right) \cdot \frac{n_{it}}{S_{ijt}} + M_{ij} \cdot (1 - y_{ijgt}) \qquad \forall\, i, j, g, t \tag{16}$$

By substituting Eq. (12) into Eq. (16) new constraints can be formulated that restrict the volumes to discrete sizes:

$$q_{it} \leq \sum_{k=1}^{K_j} \sum_{s} \left(\frac{v_{js}}{S_{ijt}} \cdot z_{jks} \cdot y_{ijkgt} \cdot n_{it} \right) + M_{ij} \cdot (1 - y_{ijgt}) \qquad \forall\, i, j, g, t \tag{17}$$

Constraint (17) is nonlinear because of the product of binary variables. In order to reformulate these constraints as linear ones, the cross product $n_{it}\, z_{jks}\, y_{ijkgt}$ can be eliminated by introducing the continuous variable h_{ijkgst} that is equal to n_{it} if z_{jks} and y_{ijkgt} are one; otherwise the variable is equal to zero.

$$q_{it} \le \sum_{k=1}^{K_j} \sum_{s} \left(\frac{\nu_{js}}{S_{ijt}} \right) \cdot h_{ijkgst} + M_{ij} \cdot (1 - y_{ijgt}) \quad \forall\, i, j, g, t \tag{18}$$

$$\sum_{s} h_{ijkgst} \le n_{it}^{U} \cdot y_{ijkgt} + M_{ij} \cdot (1 - y_{ijgt}) \quad \forall\, i, j, k, g, t \tag{19}$$

$$h_{ijkgst} \le n_{it}^{U} \cdot z_{jks} + M_{ij} \cdot (1 - y_{ijgt}) \quad \forall\, i, j, k, g, t \tag{20}$$

$$\sum_{g} \sum_{s} h_{ijkgst} = n_{it} \quad \forall\, i, j, k, t \tag{21}$$

where n_{it}^{U} is the upper bound for n_{it}. The summation over the groups in Eq. (21) is performed in order to reduce the number of generated constraints because only one of the values is equal to n_{it}.

The inventory of final product i at the end of a period t, IP_{it}, depends on the inventory that is left from the previous interval, $IP_{i,t-1}$, the quantity produced and the total sales, QS_{it}.

$$IP_{it} = IP_{it-1} + q_{it} - QS_{it} \quad \forall\, i, t \tag{22}$$

In the same way, the inventory of raw material is:

$$IM_{it} = DE_{i,t-1} \cdot IM_{it-1} + C_{it} - RM_{it} \quad \forall\, i, t \tag{23}$$

The amount of raw material in the inventory IM_{i0} for each product at the beginning of the time horizon is assumed to be given. Idem for the initial product inventory, IP_{i0}.

The amount assigned to sales must be less than the amount of product in inventory plus the quantity produced during a period:

$$QS_{it} \le IP_{it-1} + q_{it} \quad \forall\, i, t \tag{24}$$

The raw material necessary for the production of the product i is obtained from a mass balance:

$$RM_{it} = F_{it} \cdot q_{it} \quad \forall\, i, t \tag{25}$$

where F_{it} is a parameter that accounts for the process conversion, e.g. ratio of solvent to solids, time of contact etc. In this presentation only one raw material is considered. However this condition can be easily extended in order to accounting for several raw materials.

The limiting cycle time is the maximum time between two successive batches of product i. It can be calculated by the division between processing time t_{ijt} and the number of groups out of phase for product i at stage j in every period:

$$TL_{it} \ge \frac{t_{ijt}}{\sum_{g=1}^{G_j} y_{ijgt}} \quad \forall\, i, j, t \tag{26}$$

The total time for producing product i in time period t is defined as:

$$T_{it} = TL_{it} \cdot n_{it} \quad \forall\, i, t \tag{27}$$

By multiplying Eq.(28) by the number of batches, the expression takes the form:

$$T_{it} \ge \frac{t_{ijt} \cdot n_{it}}{\sum_{g=1}^{G_j} y_{ijgt}} \quad \forall\, i, j, t \tag{28}$$

Equation (28), however, is nonlinear. In order to obtain a linear expression, the following constraints are introduced:

$$\sum_{g=1}^{G_j} y_{ijgt} = \sum_{g=1}^{G_j} g \cdot u_{ijgt} \quad \forall\, i, j, t \tag{29}$$

$$\sum_{g=1}^{G_j} u_{ijgt} = 1 \quad \forall\, i, j, t \tag{30}$$

where the variable binary u_{ijgt} is 1 if at stage j there are g groups operating out of phase. Substituting y_{ijgt} for u_{ijgt} in Eq. (28), the expression gets the following form

$$T_{it} \ge \sum_{g=1}^{G_j} \left(\frac{t_{ijt} \cdot n_{it}}{g} \right) \cdot u_{ijgt} \quad \forall\, i, j, t \tag{31}$$

This constraint is also nonlinear. To eliminate bilinear terms $n_{it}\, u_{ijgt}$, a new nonnegative continuous variable e_{ijgt} is defined to represent this cross product (Voudouris and Grossmann, 1992). Then the following linear constraints are obtained:

$$T_{it} \ge \sum_{g=1}^{G_j} \left(\frac{t_{ijt}}{g} \right) \cdot e_{ijgt} \quad \forall\, i, j, t \tag{32}$$

$$\sum_{g=1}^{G_j} e_{ijgt} = n_{it} \quad \forall\, i, j, t \tag{33}$$

$$e_{ijgt} \le n_{it}^{U} \cdot u_{ijgt} \quad \forall\, i, j, g, t \tag{34}$$

where n_{it}^{U} is the upper bound for n_{it}.

Considering the case of SPC-ZW policy in the period t, all productions must be completed within the corresponding production horizon H_t:

$$\sum_{i}^{I} n_{it} \cdot TL_{it} \le H_t \quad \forall\, t \tag{35}$$

Taking into account Eq. (27) the following expression is obtained:

$$\sum_{i}^{I} T_{it} \le H_t \quad \forall\, t \tag{36}$$

3.3 Objective Function

The strategic objective in this formulation is to maximize the operating profit of the plant,

$$\Phi = \sum_t \sum_i p_{it} \cdot QS_{it} - \sum_t \sum_i \kappa_{it} \cdot C_{it} - CEQ -$$
$$\sum_t \sum_i \varepsilon_{it} \cdot \left(\frac{IM_{it-1} + IM_{it}}{2} \right) \cdot H_t - \qquad (37)$$
$$\sum_t \sum_i \sigma_{it} \cdot \left(\frac{IP_{it-1} + IP_{it}}{2} \right) \cdot H_t$$

The first term of the objective function is the income corresponding to the product sales where the parameter p_{it} is the price of product i in each period. The second term is the cost of purchases with κ_{it} the price of raw material. The last two terms correspond to raw material and final product inventory costs, where ε_{it} and σ_{it} are inventory cost coefficients (Birewar, and Grossmann, 1990). Finally, the third term is the investment cost of the batch units and is obtained through the following equations:

$$CEQ = \sum_t^T \sum_j^J \sum_k^{K_j} \alpha_{jt} \cdot V_{jk}^{\ \beta_{jt}} \cdot w_{jkt} \qquad (38)$$

where α_{jt} and β_{jt} are specific cost coefficients for each stage j in every period t. Eq.(10) is introduced into this expression to get:

$$CEQ = \sum_t^T \sum_j^J \sum_k^{K_j} \sum_s \alpha_{jt} \cdot v_{js}^{\ \beta_{jt}} \cdot z_{jks} \cdot w_{jkt} \qquad (39)$$

$$CEQ = \sum_t^T \sum_j^J \sum_k^{K_j} \sum_s c_{jst} \cdot r_{jkst} \qquad (40)$$

where the terms $c_{jst} = \alpha_{jt} \cdot v_{js}^{\beta_{jt}}$ represent the cost of standard batch vessels, and new variables r_{jkst} are introduced to eliminate the product of binary variables $z_{jks}\, w_{jkt}$ through the constraints:

$$r_{jkst} \geq z_{jks} + w_{jkt} - 1 \qquad \forall j,k,s,t \qquad (41)$$

$$0 \leq r_{jkst} \leq 1 \qquad (42)$$

4. MODEL RESOLUTION

To sum up, the multiperiod model of a multiproduct batch plant is defined by maximizing the objective function represented by Eq. (37) using Eq. (40) as the term of investment cost and subject to constraints Eqs. (1) – (8), (11), (12), (15), (18) – (25), (29), (30), (32) – (34), (36), (41), (42) plus the upper bounds that may apply. Bilinear terms have been eliminated through an efficient method in order to generate a MILP model which can be solved to global optimality.

5. EXAMPLES

5.1 Example 1

To illustrate the use of the MILP formulation presented in the previous section, let us consider optimizing the production of five oleoresins, sweet bay (A), oregano (B), pepper (C), rosemary (D), and thyme (E) oleoresins, manufactured in a multiproduct batch plant.

This plant consists of the following stages: 1) extraction in a four-stage countercurrent arrangement (2) expression, (3) evaporation, and (4) blending. All of these stages can be duplicated up to four units, so, the maximum number of groups that can exist at a stage is four, too.

In order to obtain parameter F_{it} necessary for Eq. (25), the following equations are used:

$$x_i^{n+1} \cdot \left[1 + E_i \cdot (1 - \eta_i)\right] = x_i^n \cdot (1 + E_i - \eta_i) + \eta_i \cdot x_i^1 \qquad (43)$$

$$F_{it} = \frac{1}{\left(x_i^{n+1} - x_i^1\right)} \qquad (44)$$

where E_i is the extraction factor, η_i is the extent of the extraction, and x_i is the product concentration in the vegetable solid feed. Index n is the number of each stage for the n-staged countercurrent extraction.

A global horizon time of one year (6000 h working) has been considered, and it was divided into a set of equal time periods, namely from 1 to 6. Demands, costs, and prices differ from period to period. Table 1 contains some data for this example.

Table 1 Data for Example 1

	Size Factors (L/kg)				Processing Time (h)			
i	j_1	j_2	j_3	j_4	j_1	j_2	j_3	j_4
A	20	15	12	1.5	1.5	1	2.5	0.5
B	80	55	49	1.5	1.5	1	2	0.5
C	20	15	12	1.5	2.5	2	3	2
D	40	25	24	1.5	1.5	1	1.5	1
E	30	20	17	1.5	1.5	1	3	1
Sizes (liter, L) SV_j = {250, 500, 750, 1000, 1500}								

Table 2 shows the optimal unit assignment for this plant. To show the optimal solution, product B, the least convenient to produce, was chosen. The first diagram of Figure 1 shows that raw material for B is purchased during the two initial periods. In the second diagram, it can be noted that B is produced only during the first two periods, because the costs are lower mainly due the lower raw material price and the amount produced in these periods is stored as inventory for satisfying minimum demands in the subsequent intervals.

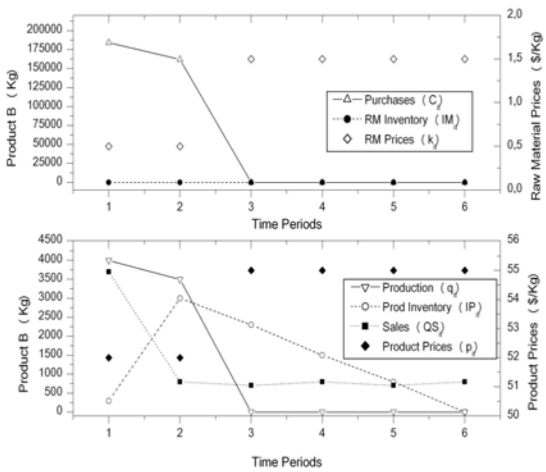

Figure 1. Results for Product B of Example 1

Table 2 Optimal unit assignment for Example 1

	Stage (L)			
Unit	j_1	j_2	j_3	j_4
k_1	1500	1000	750	250
k_2	-	-	750	-

5.2 Example 2

Consider now the production of 3 oleoresins (A, B, C) in 3 time periods where demands take higher values in subsequent periods to consider a possible market expansion. Table 3 shows the optimal unit assignment. Table 4 shows the different configuration for the units for each product in every period, respectively. In this table, units between parentheses are included in the same group. Also, it can be seen that all units are introduced in period 1 except units 2 and 3 that are added in stage 1 in period 2.

Table 3 Optimal unit assignment for Example 2

	Stage (L)			
Unit	j_1	j_2	j_3	j_4
k_1	5000	1500	5000	1500
k_2	5000	5000	5000	-
k_3	5000	5000	-	-

These problems were solved by using CPLEX through the modeling system GAMS on a Pentium IV Processor (3GHz). The total profit for example 1 and example 2 were \$1982822.84 and \$7766239.71 respectively. The information about the resolution of these examples is as follows. Example 1 has 12091 constraints, 8293 single variables, 1932 binary variables and the optimal solution was obtained after a CPU time of 111.70s. Example 2 has 3736 constraints, 2617 single variables, 636 binary variables and the optimal solution was obtained after a CPU time of 390.95s.
Due to the large amount of data for these examples, they are not presented in this paper. Readers interested in the data can contact the authors.

6. CONCLUSION

A new model for the optimal design and operation planning of multiproduct batch plants has been formulated as an MILP problem, which guarantees the global optimum solution.

This multiperiod MILP model involves discrete decisions for the structure selection and continuous decisions for the operation plan of the plant at each time period. Furthermore, this model allows considering all possible alternatives for the addition of equipments in parallel, which are available in discrete sizes.

Seasonal variations of products demands and raw materials availability are readily accounted for, and both raw materials and final product inventories are included in the formulation. The proposed model was applied to a plant that produces oleoresins.

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Table 4 Arrangement of units for each product in every period

	Period 1			Period 2			Period 3		
	A	B	C	A	B	C	A	B	C
q_{it}	2500	500	2000	17000	23000	18000	175000	0	104166
j_1	(k_1)	(k_1)	(k_1)	(k_1,k_3)-(k_2)	(k_1,k_2,k_3)	(k_1,k_2,k_3)	(k_1,k_2,k_3)		(k_1,k_2,k_3)
j_2	(k_1,k_2,k_3)	(k_1,k_2,k_3)	(k_1,k_3)-(k_2)	(k_1,k_2,k_3)	(k_1,k_2,k_3)	(k_1,k_2,k_3)	(k_1,k_2,k_3)		(k_1,k_2,k_3)
j_3	(k_1,k_2)	(k_1)-(k_2)	(k_1,k_2)	(k_1,k_2)	(k_1,k_2)	(k_1,k_2)	(k_1,k_2)		(k_1,k_2)
j_4	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)		(k_1)