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Real time thermal tracking of ladle furnaces: an analytical approach

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Abstract

This work presents a new analytical approach for solving unsteady diffusion problems. In the proposed method, a formal solution is converted in closed form ones, which are obtaining by performing a straightforward procedure that starts with a classical split. The low time processing required to obtain the exact solutions allows performing online control of ladle furnaces. Numerical results are reported.

1 - Introduction

It is widely felt that the significant reduction of time processing due to the recent advances in numerical and analytical methods can make possible to proceed the online control based on direct simulation for some important applications in Engineering, such as in environmental problems (Zabadal, 2005), neutron scattering in nuclear reactors (Bogado, 2004) and casting of steel alloys (Zabadal, 2004). Specifically, for steel casting processes, the simulation of ladle furnaces is particularly advantageous from the operational point of view, because the calculated steel temperature agrees with experimental data even when nonlinear effects are ignored. The major aim of the online control in casting of steel alloys is to ensure that the temperature of the liquid steel which flows from the furnace does not fall out of an interval of roughly 10^{o}C around a mean value about 1600^{o}C (which varies from one specific steel alloy to another) . This control prevents failures in the lattice, which occurs when the temperature is low, and unsuitable flow conditions, when the temperature is higher than a certain value.
The main limitation of the use of numerical schemes to proceed the simulation of casting processes occurs for some scenarios where the ladle remains out of operation for long time intervals (about 24h after the last batch). In these cases the ladle must suffers a slow heating process (during about 10h), and the simulation becomes a very difficult task. The online control based on numerical simulation results unfeasible, due to the large time processing required.
In this work a new analytical method for simulating the heating process is presented. This method, based on the formal solutions of partial differential equations, is employed to overcome the limitations of the methods which were originally conceived to carry out the online control of ladle furnaces. The most important features of the proposed method are the high processing speed and the analytical character of the solutions obtained.

2 – General formulation

The partial differential equation given by:

$$Lf = 0 \quad , \tag{1}$$

where L is a linear operator can be decomposed as

$$Af = Bf \quad , \tag{2}$$

in which A and B are also linear operators. Applying A^{-1} on both sides of (2) we obtain:

$$f = A^{-1}Bf + h_A \ . \tag{3}$$

In this expression h_A stands for the null space of A. Rearranging terms it yields

$$[I - A^{-1}B]f = h_A \ . \tag{4}$$

Solving equation (4) for f it results:

$$f = [I - A^{-1}B]^{-1} h_A \ . \tag{5}$$

The inverse operator appearing in equation (5) can be written as a geometrical series:

$$[I - A^{-1}B]^{-1} = \sum_{k=0}^{k=\infty} \left(A^{-1}B \right)^k \quad . \tag{6}$$

Disregarding the restrictions about the norm of the operator $A^{-1}B$, the solution is then readily obtained in the form:

$$f = \sum_{k=0}^{k=\infty} \left(A^{-1}B \right)^k h_A \quad . \tag{7}$$

In order to obtain a particular solution for equation (1), it becomes necessary to choose a function $f_0 \in N(A)$. In practice, f_0 can be chosen as a function belonging to the intersection of the null spaces of $(A^{-1}B)^n$ and A, in order to convert the series solution into a finite sum. In what follows it will be showed that the closed-form solutions achieved by means of the described method generate high performance algorithms for online control.

3 – Application in online control of ladle furnaces

The online control of ladle furnaces can be carried out by solving the heat equation in the form

$$\frac{\partial f}{\partial t} = \alpha \left(\frac{\partial^2 f}{\partial r^2} + \frac{1}{r} . \frac{\partial f}{\partial r} \right) \quad , \tag{8}$$

in a hollow cylinder, subjected to the following boundary conditions:

$$\left. \frac{\partial f}{\partial r} \right|_{r=o} = 0 \tag{9}$$

and

$$f(l,t) = f_l(\mathrm{t}) \quad . \tag{10}$$

In these equations f represents the temperature, r is the radial coordinate, α is the thermal diffusivity, f_l is the temperature at r=l, and t stands for the time. This model describes a process in which the ladle is heated inside by a flame before receiving liquid steel from the main furnace. The first boundary condition, given by equation (9), states that the external wall does not exchanges energy with the air at room temperature during the process. Although the external wall being not really insulated, equation (9) is a reasonable approximation for thick walls composed of materials whose thermal diffusivity is low, and hence possesses a high thermal inertia. The second boundary condition informs that inner wall is at the flame temperature, which is time dependent.

After the heating process, the ladle receives liquid steel, whose temperature is also time depending. The evolution of the inner wall temperature along the time is fitted using a standard least square procedure available in MapleV. Finally, the initial condition imposes the final profile of the former batch to the next one at t=0.

In equation (8), the operators A, B and A^{-1} are promptly identified as

$$A = \frac{\partial}{\partial t} \quad , \tag{11}$$

$$B = \alpha \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} . \frac{\partial}{\partial r} \right) \tag{12}$$

and

$$A^{-1} = \int (.) dt \quad , \tag{13}$$

whereas in the boundary condition (10) the subscript l refers to the coordinate r = l, where l denotes the thickness of the ladle wall (a typical value for l is about 0,35m). Once T_l is time dependent, the desired solution shall follow the time evolution of the boundary condition given by (10).

The simpler choice of $f_0 \in N(A^{-1}B)^k \cap N(A)$ is given by:

$$f_0 = r^2 \tag{14}$$

Applying operator $A^{-1}B$ it came out

$$f_1 = 4\,\alpha\, t \tag{15}$$

Applying the same operator over f_1 it yields:

$$A^{-1}Bf_1 = 0 \tag{16}$$

Therefore $f_0 = r^2$, which belongs simultaneously to the nullspaces of A and $(A^{-1}B)^2$, produces a closed form solution which contains only two terms:

$$f(r,t) = f_0 + f_1 = r^2 + 4\alpha t \quad . \tag{17}$$

Analogously, another particular solution can be easily obtained by setting $f_0 = r^4$. In this case, the closed form solution is expressed as

$$f(r,t) = f_0 + f_1 + f_2 = r^4 + 16\alpha r^2 t + 32\alpha^2 t^2 \quad , \tag{18}$$

and $f_0 = r^4$ belongs to the nullspaces of A and $(A^{-1}B)^3$. Each even power of r, namely r^{2n}, generates a closed form solution belonging to the nullspace of $(A^{-1}B)^{n+1}$. Other examples of solutions obtained from even powers of r are given below:

$$f_0 = r^6 \rightarrow f(r,t) = r^6 + 36\alpha r^4 t + 288\alpha^2 r^2 t^2 + $$
$$+384\alpha^3 t^3 \quad , \qquad (19)$$

$$f_0 = r^8 \rightarrow f(r,t) = r^8 + 64\alpha r^6 t + 1152\alpha^2 r^4 t^2 + $$
$$+6144\alpha^3 r^2 t^3 + 6144\alpha^4 t^4 \quad , \qquad (20)$$

and

$$f_0 = r^{10} \rightarrow f(r,t) = r^{10} + 100\alpha r^8 t + 3200\alpha^2 r^6 t^2 + $$
$$+38400\alpha^3 r^4 t^3 + 153600\alpha^4 r^2 t^4 + 122880\alpha^5 t^5 \ . \qquad (21)$$

Reminding that the desired solution shall contain some arbitrary parameters in order to fulfill the boundary condition at r = 0, as well as to follow the time evolution of the boundary condition at r = l, a linear combination of the above solutions must be employed in order to simulate the physical scenario. The boundary condition at r = 0 is automatically satisfied because the solution is an even function of r. Hence, all the numerical coefficients in the linear combination are specified in order to fit the boundary condition at r=l. This task is accomplished by means of a conventional curve fitting procedure.
At this point, one may ask why use such a scheme to obtain closed-form solutions, once the analytical one is yet available in literature. The foremost reason is the need to expand the analytical solution in a basis set containing the Bessel functions J_v and Y_v . The oscillations associated with the J_v functions requires a large number of terms in the expansion in order to smooth out the "wigglyness" appearing due to the contributions of the eigenfunctions related to the lowest eigenvalues. Since the definition of both Bessel functions involves the evaluation of the gamma function, which is expressed as a product, a summation or a high degree polynomial, a large number of floating point operations is demanded in order to produce numerical results.

4 – Results and conclusion

The exact solution employed to simulate the heating process is a linear combination given by

$$f = \sum_{k=0}^{5} c_k p_{2k}(r,t) \qquad (22)$$

where $p_{2k}(r,t)$ are the polynomials defined by equations (17) to (21), and the coefficients c_0 to c_5 are given in table 1. These coefficients were obtained by fitting the data corresponding to the boundary condition at r=l, as mentioned earlier. The fitting generates a time evolution which reproduces the experimental data at r=l with a mean square deviation about 1°C (notice that c_0 was included in the linear combination, because a constant function is also an exact solution of the heat equation).

Table 1 - Numerical values of the coefficients

Coefficients	Values
∝	1e-7
C_0	102,5
C_1	198,6
C_2	2830
C_3	8116
C_4	235,4
C_5	28,74

Figure 1 shows the corresponding time evolution of the temperature profile along the heating process.

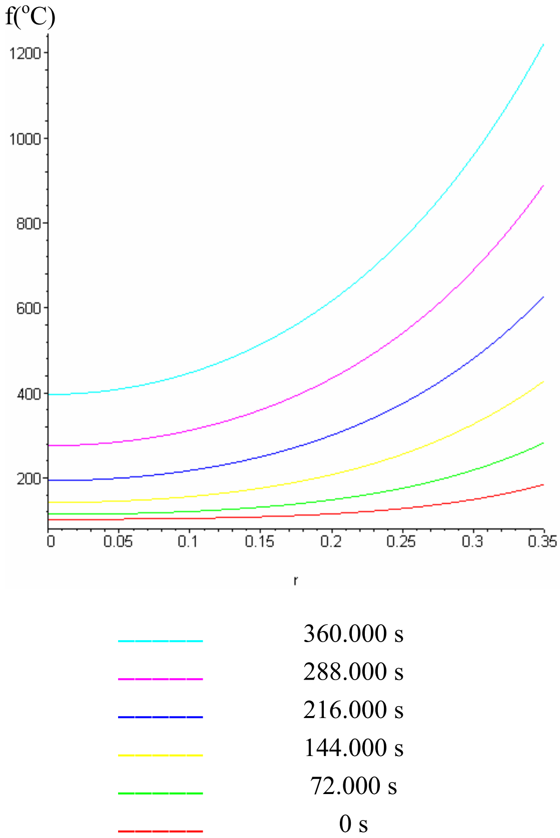

Figure 1 – Time evolution of the temperature profile (°C) during the heating process.

The mean square deviation between the predictions and the experimental data available at the "Aços Finos Piratini" steel casting facilities is about 0,16%, and satisfies the requirement that the temperature of the steel flowing from the furnace does not fall out of an interval of 10°C around the mean value in 79% of the cases (237 batches).
It is important to emphasize that the time required to obtain the temperature profiles are virtually negligible (less than 1s - Sempron 2.4 GHz, 512 Mb RAM, using

Maple V). Nevertheless, the curve fitting procedure must be carried out in advance. It means that, in practice, it becomes necessary to include a fitting routine in the computational code in order to proceed the online control.

Another important question about the method must be answered. Once the proposed method was developed to simulate the thermal behavior of the wall, how to predict the time evolution of the temperature profile at the bottom of the ladle? In this case there exists an exact solution for the corresponding Cartesian problem in the z coordinate, given by

$$f = \mathbf{b_0} + \mathbf{b_1} \mathbf{e}^{\mathbf{b_2 z + b_2^2 t}} \qquad (23)$$

Notwithstanding the solution in cartesian coordinates were obtained directly by inspection, it is also suitable for real time thermal tracking.

It is also important to remark that the solutions can be employed separately, which means that the thermal coupling between the wall and the bottom of the ladle furnace can be neglected without appreciable loss in accuracy.

Finally, it is convenient to emphasize that all the functions obtained through the iterative scheme, namely, equations (17) to (21), are exact solutions. Hence, equation (22) is not a truncated series which constitutes an approximation to the exact solution, but it is itself an exact solution to the heat equation. Once the functions obtained are conceived to belong to the nullspace of a finite power n of the operator $A^{-1}B$, the "truncated" series defined by

$$f = \sum_{k=0}^{k=n} \left(A^{-1} B \right)^k h_A \quad . \qquad (24)$$

is always an exact solution, provided that all the terms beyond n are automatically dropped out. Therefore, no questions about convergence arises along the development of the proposed formulation.

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