A DATA-BASED MEASURE FOR INTERACTIONS IN MULTIVARIATE SYSTEMS

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Abstract: This article focusses on the control loop performance diagnosis of a multivariate system with emphasis on the presence of interactions and poor performance of control loops. The paper provides a data-driven technique to determine if a decentralized PI(D)$^+$ controller will suffice or if an advanced controller (*e.g.*, MPC) is necessary to handle the control interactions and improve the loop performance. Two different techniques are proposed: the first one, based on the Power Spectrum of the error, analyzes interactions in the frequency domain, while the second one, based on the evaluation of a modified IAE (Integral of Absolute Error), analyzes interactions in the time domain. A performance index for the controller is also proposed for the case of set-point tracking. Simulation and experimental case studies are presented to highlight the applicability of the proposed techniques.

Keywords: Interactions, MIMO systems, frequency domain, time domain

1. INTRODUCTION

Over the last two decades, monitoring control loop performance has been addressed in several ways and several performance indices have been proposed (see Hoo *et al.* (2003) for a good survey). Different causes for low loop performance such as improper controller tuning, sensor faults, valve non-linearities have been identified (Bialkowski, 1993; Kozub, 1997). An important cause that demands attention in addition to these causes is the presence of interactions among loops. A key impact of the interaction on the loop performance is the propagation of the effects of other causes that deteriorate the loop performance, thereby corrupting other loops.

The schematic of a multivariate (MIMO) system under discussion in this sequel is shown in figure 1: the process **P**, not necessarily square; the controller **C** initially considered as a decentralized $PI(D)^+$ type. A disturbance through **Pd** and white noise passing through a first order filter **F** are included for completeness. In a routine operation, the set point array **r**, the control action array **u** and the controlled variables array **y** are measured quantities.

Diagonal elements of the matrix **P** represent the process transfer functions, while the off-diagonal elements (P_{ij}, $i \neq j$) represent the interaction transfer functions. When an excitation affects a loop *i*, some effect is also present on another loop *j* depending on the interaction transfer function P_{ij}.

The Relative Gain Array (RGA) is often used to describe the level of interaction among loops, for instance in (Persechini *et al.*, 2004). However, it has two key limitations: (*i*) a model of the process must be known and consequentially the RGA measure depends on the model uncertainty (Chen and Seborg, 2002) and (*ii*) RGA gives only a measure of stability once loops are closed and no indication on the real interaction among them.

Therefore, a novel approach is proposed, which does not use an explicit process model, but instead di-

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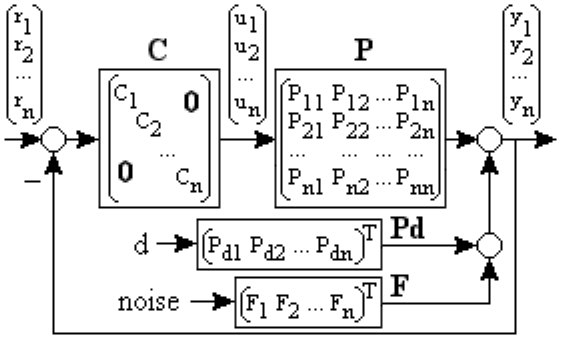

Fig. 1. The reference setup of a MIMO system

rectly uses routine operating data. Once the loops that are suspected to interact are selected, the proposed method can be used to assess the level of interaction.

Among all types of excitations, the set-point excitation is most preferred since it allows us to obtain **y** as a function only of **P** and **C**. The effect of the disturbance transfer function **Pd**, which may cloud the interaction measure, is thus avoided.

The outline of the paper is as follows. Two interaction measures are described in section 2, followed by an analysis of the controller performance in section 3. Application of the techniques to simulated, industrial and experimental setups are presented in Sections 4 and 5 respectively. The paper ends with a few concluding remarks in Section 6.

2. INTERACTION MEASURES

Depending on the nature of process excitation, interaction can be analyzed in either time- or a frequency-domain as may be deemed appropriate. For instance, in rotary machines, it is necessary to evaluate the interaction in a defined range of frequencies by exciting loops with oscillatory set-points. A frequency domain analysis, named in the sequel *Power Spectrum Analysis*, is more suited to such situations. On the other hand, oscillatory set-point changes (steps, ramps, etc.) are not a commonplace in chemical industries and therefore, a time domain technique, named in the sequel as *IAE technique*, may be chosen.

For both cases a comparison between controlled variables belonging to different loops has to be performed. For this reason a normalization factor is chosen so as to make the task independent of the measuring scale. Denoting as $CR_{UP,i}$ and $CR_{LW,i}$ the upper and lower limit of the control range for the loop *i* respectively, the normalization factor NF_i can be evaluated as described in equation 1:

$$NF_i = min\{CR_{UP,i} - \overline{r_i}; \quad \overline{r_i} - CR_{LW,i}\} \quad (1)$$

where $\overline{r_i}$ is the mean value of the set-point of loop *i*. All controlled variables are divided by their respective normalization factors for subsequent analysis.

2.1 *Power Spectrum Analysis*

Detection of interacting loop is performed by the use of Power Spectral Correlation Index (PSCI) (Tangirala *et al.*, 2005). Its application allows one to exclude loops characterized by different frequencies due to other oscillating sources. The PSCI between loop *i* and *j* is calculated as:

$$PSCI_{i,j} = \frac{\sum_{\omega} PS_{y_i}(\omega) \cdot PS_{y_j}(\omega)}{\sqrt{\sum_{\omega} PS_{y_i}(\omega)^2 \cdot \sum_{\omega} PS_{y_j}(\omega)^2}} \quad (2)$$

where $PS_{y_i}(\omega)$ is the raw power spectrum of the controlled variable of the loop *i* evaluated at the frequency ω. This index (Tangirala *et al.*, 2005), lies in the range [0 1]: with similar shapes of power spectra its value is near one, indicating the presence of interaction.

Once an interacting loop is detected, the amount of the interaction is calculated as:

$$IFD_{i,j} = 1 - \frac{\sqrt{max(PS_{SP})}}{\sqrt{max(PS_{SP})} + \sqrt{max(PS_I)}} \quad (3)$$

where SP and I indicate respectively the loop affected by the set point change and the interacting loop. IFD lies in the range $[0 \;\; 1]$, the larger the interaction, the higher is the index.

Equations 2 and 3 can be used in combination to assess the interaction in the frequency domain.

2.2 *IAE technique*

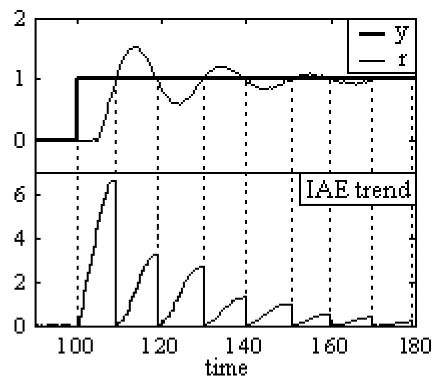

Fig. 2. Example of IAE trend for a set-point change

For the time domain analysis, the error signal **e=r-y** is considered: if the error does not change its sign from the sample $k-1$ to the sample k a modified IAE (Integral of Absolute Error) is evaluated as given in equation 4:

$$IAE(k) = IAE(k-1) + |e(k)| \cdot h \quad (4)$$

where h is the sampling interval. If a change in the error sign occurs, $IAE(k)$ is reset to zero (Hägglund, 1995). The trend of IAE is composed of peaks that coincide with the zero crossing of the error signal as shown in figure 2.

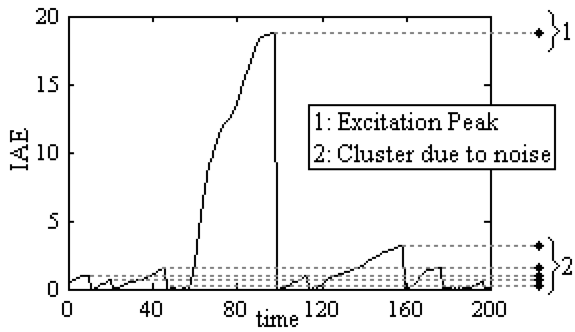

Fig. 3. Example of Excitation Peak and Cluster of Noise peaks

The use of this index allows one to magnify the difference between excitations (bigger peaks) and noise (smaller peaks) taking into account both the amplitude of the error and the duration between two consecutive zero crossings. Furthermore, the comparison of IAE peaks for different variables can be directly used to evaluate the amount of interaction between loops. To analyze only peaks due to excitations, a technique for the detection of outliers is applied (Daszykowski *et al.*, 2001). This technique analyzes maxima of each peak: maxima of noise peaks generate a cluster from which maxima of excitation peaks are excluded (figure 3).

In the presence of interactions, a set-point change will generate a peak in IAE trends in the two examined loops almost at the same time. Considering the time delay of the interaction as unknown, it is impossible to establish the exact gap which occurs between the two peaks. To overcome this problem a time window is chosen according to the duration of the source peak: defining t_0 the time in which the set point change starts and t_1 the time in which the time trend reaches its maximum, the time horizon t_h for the time window can then be evaluated as:

$$t_h = t_0 + a \cdot (t_1 - t_0) \qquad (5)$$

where the parameter a is adjustable and set to 4 in this work. This setting of a provides an over-estimate of the interaction delay θ_I). A few remarks follow:

- $t_1 - t_0$, the time gap in which the controlled variable reaches the set-point value for the first time, is an overestimation of θ_P.
- Under the hypothesis of similar values of θ_I and θ_P, the choice $a = 4$ allows to obtain for most cases a time horizon bigger than θ_I.

However the value of a can be changed easily by the operator to analyze the effect of the time window horizon on the interaction measure.

An interaction index Interaction in Time Domain (ITD) is thus proposed based on the IAE of the windowed trend:

$$ITD_{i,j} = 1 - \frac{\sum_{t_0}^{t_h} IAE_{SP}}{\sum_{t_0}^{t_h} IAE_{SP} + \sum_{t_0}^{t_h} IAE_I} \qquad (6)$$

with the same formalism used in equation 3. Similar to IFD, ITD lies in $[0\ 1]$ and a strong interaction is associated with a high value.

A heuristic interpretation of the proposed interaction indices is given in table 1. Of particular importance is the limit of 0.5, over which the value implies that the set-point change in a loop *i* affect more other loops than loop *i* itself.

Table 1. Interpretation of the index values

ITD (/IFD)	Interpretation
$[0 \quad 0.125]$	No Interaction
$[0.125 \quad 0.25]$	Low Interaction
$[0.25 \quad 0.375]$	Medium Interaction
$[0.375 \quad 0.5]$	High Interaction
$[0.5 \quad 1]$	Very High Interaction

It is remarked that a limitation of this method is that it can not correctly estimate the interaction when set-point activity in a loop and a disturbance in another loop coincide. However, the presence of other set-point changes in the data set can help overcome this limitation to a large extent.

3. CONTROLLER PERFORMANCE INDEX

Information from the interaction measure can be used to establish if a retuning is sufficient to improve the performance or if an advanced controller is required. For this purpose, a new Controller Performance Index (CPI) is defined.

The CPI is proposed on the basis of the response to a set-point change. Given a set-point change, under *minimum variance control*, after $\theta_P + t_0$, the error immediately reaches zero. Suppose a minimum error e_{min} is associated with this case. Otherwise a residual error is still present until the controlled variable reaches the settling time. Denote the error in such a case by e_{tot}. The CPI is then defined as,

$$\text{CPI} = \frac{e_{tot} - e_{min}}{e_{tot} + e_{min}} \qquad (7)$$

If e_{tot} is near to the minimum achievable, the controller has a good performance and CPI is near zero. If $e_{tot} \gg e_{min}$, the controller has a poor performance and CPI is near to one. Given the fact that the minimum variance controller is an idealistic case and of little practical use (Huang and Shah, 1999) and considering the presence of interaction, a threshold value of $CPI = 0.5$ is chosen. Below this value of CPI, retuning would be practically of little benefit. Furthermore, a high value of the CPI with a high value of ITD/IFD implies that the present controller configuration yields good performance but unable to handle interactions. Therefore, a structural change may be necessary.

To evaluate the CPI, the time delay of the process θ_P must be estimated. The recorded response y to the set-point change in closed loop can be approximated by a open loop response to a step-test $\tilde{y}$. Choosing a second order model $\tilde{P}$ and varying its parameters, it is possible to find the best approximation in the least square sense. The obtained model will not have any physical meaning: it is used only to generate a good estimate of the time-delay (for the same reason the order of the model is not critical). Assuming a fixed value for the time delay $q = \theta_P/h$ with h sampling time and defining n the length of the data set, it is possible to generate the best approximation of y in the least square sense:

$$y(z^{-1}) = \frac{b_1 z^{-1-q} + b_2 z^{-2-q}}{a_1 z^{-1} + a_2 z^{-2} + 1} \cdot r(z^{-1}) \quad (8)$$

$$y_k = b_1 r_{k-1-q} + b_2 r_{k-2-q} - a_1 y_{k-1} - a_2 y_{k-2} \quad (9)$$

$$\underbrace{\begin{bmatrix} y_{q+3} \\ y_{q+4} \\ \vdots \\ y_n \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} -y_{q+2} & -y_{q+1} & r_2 & r_1 \\ -y_{q+3} & -y_{q+2} & r_3 & r_2 \\ \vdots & \vdots & \vdots & \vdots \\ -y_{n-1} & -y_{n-2} & r_{n-1-q} & r_{n-2-q} \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix}}_{\mathbf{p}} \quad (10)$$

$$\mathbf{p} = (\mathbf{M}^T \mathbf{M})^{-1} \mathbf{M}^T \cdot \mathbf{y} \quad (11)$$

Among all models $\tilde{P} = f(q, \mathbf{p})$, the one that generates the lowest error in the least square sense is associated with the best estimation of θ_P.

4. CASE STUDIES

This section presents two of the several MIMO systems that were successfully analyzed with the proposed techniques. The first system is the *Marlin Column*, known to contain a high level of interaction. The process transfer functions are reported below:

$$\mathbf{y} = \begin{bmatrix} \frac{0.0747e^{-3s}}{12s+1} & \frac{-0.0667e^{-2s}}{15s+1} \\ \frac{0.1173e^{-3.3s}}{11.7s+1} & \frac{-0.1253e^{-2s}}{10.2s+1} \end{bmatrix} \cdot \mathbf{u} \quad (12)$$

It can be observed that the gains, time constant and time delay for diagonal and off-diagonal elements are similar, indicating a strongly interacting system. A more detailed description of the process together with the definition of a decentralized PI controlled are reported in Marlin (2000). Pre-specified set-point changes were performed to analyze the presence of interaction as depicted in figure 4a); the corresponding IAE trends are reported in 4b). The presence of interaction is indicated by the high values of $\text{ITD}_{1,2} = 0.47$ and $\text{ITD}_{2,1} = 0.43$, which confirms with the earlier discussion. The CPI is over 0.9 for both the controllers indicating that a retuning will improve the performance but, considering the high values of ITD in this case, a different structure is suggested for the controller (e.g. MPC).

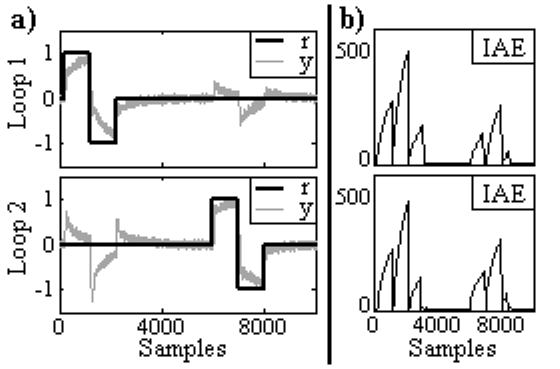

Fig. 4. Marlin Column: a) set-point (r) and controlled variable (y) values; b) IAE trends

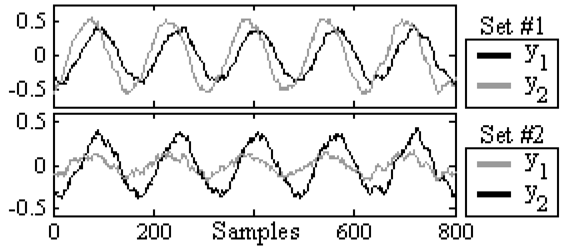

Fig. 5. Marlin Column: controlled variables for the loop affected by the oscillatory set-point (black) and the interacting loop (gray)

A frequency-domain analysis was also performed using oscillatory set-points. In figure 5 the controlled variable for the loop affected by the set-point change (black) and the controlled variable for the interacting loop (gray) are shown. The value of PSCI is over 0.98 for both the set of data indicating that the interaction is present: for this case the interaction from loop 1 to loop 2 is higher (IFD = 0.57) than the one from loop 2 to loop 1 (IFD = 0.26). It is important to recall that this analysis is suited to set-point changes that are well-localized in frequency while the time domain analysis, on the contrary, is well-suited to set-points that contain a range of frequencies.

The second system under study is the *Shell problem*; the process transfer functions are reported below:

$$\mathbf{y} = \begin{bmatrix} \frac{4.5e^{-27s}}{50s+1} & \frac{1.77e^{-28s}}{60s+1} & \frac{5.88e^{-27s}}{50s+1} \\ \frac{5.39e^{-18s}}{50s+1} & \frac{5.62e^{-14s}}{60s+1} & \frac{6.9e^{-15s}}{50s+1} \\ \frac{4.38e^{-20s}}{33s+1} & \frac{4.42e^{-22s}}{44s+1} & \frac{7.2}{19s+1} \end{bmatrix} \cdot \mathbf{u} \quad (13)$$

For this problem two solutions have been analyzed: firstly a decentralized PI controller has been implemented and secondly it has been compared with the MPC proposed in (Patwardhan and Shah, 2004). It is noted that, as explained in (Patwardhan and Shah, 2004), y_3 can be considered as a “slack” variable. The response for the two cases to the same set-point changes are shown in figure 6 and figure 7 respectively. The two different situations are well explained by the values of ITD measure:

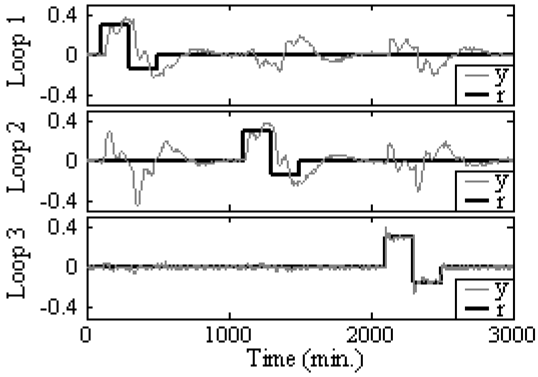

Fig. 6. Shell Problem with decentralized PI controllers; set-points (black) and controlled variables (gray)

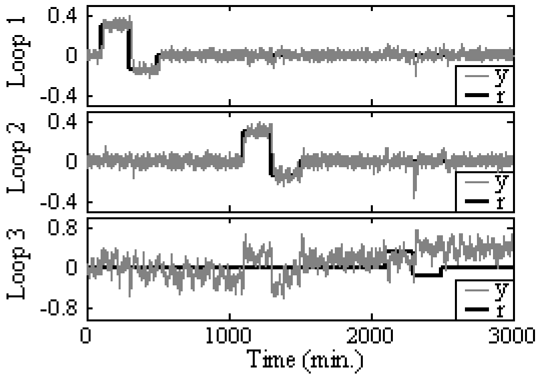

Fig. 7. Shell Problem with MPC; set-points (black) and controlled variables (gray)

$$\underbrace{ITD = \begin{bmatrix} 1 & .47 & .01 \\ .46 & 1 & .01 \\ .98 & .97 & 1 \end{bmatrix}}_{dec.PI} ; \quad \underbrace{ITD = \begin{bmatrix} 1 & .05 & .6 \\ .16 & 1 & .92 \\ .0 & .0 & 1 \end{bmatrix}}_{MPC} \qquad (14)$$

It is clear that the decentralized PI controllers do not yield a satisfactory performance; the first two loops are strongly interacting (ITD = 0.46 and 0.47); and loop 3 is affecting them (ITD = 0.97 and 0.98) without being affected (ITD < 0.1). The CPI values for the three PI controllers are respectively 0.39, 0.59 and 0.53 indicating that a new tuning cannot be expected to improve the performance. Therefore, an advanced control scheme such as MPC is required. With such a scheme, the first two loops are no more interacting because the third loop is absorbing all excitations; the only residual interaction from loop 2 to loop 1 ($ITD_{2,1} = 0.16$) has low importance. In both cases, the ITD measure is able to rightly explain the interacting behaviour.

5. EXPERIMENTAL SETUP

The IAE technique was applied to an experimental setup consisting of the four-tank system depicted in figure 8. Two combinations were considered - the first comprising tank #1 and tank #2 (this is a minimum phase system) and the second one comprising tank #3 and tank #4 (this is a non-minimum phase system) (for more details see Johansson (2000)). In the first case, pump #1 is feeding the left tank and pump #2 is feeding the right tank, while in the second case it is the converse. For each case, the target is the control of the levels in the two tanks by manipulating the inlet flowrates to the tanks. The set-point was changed for each of the levels and the interaction measure was evaluated for the two cases.

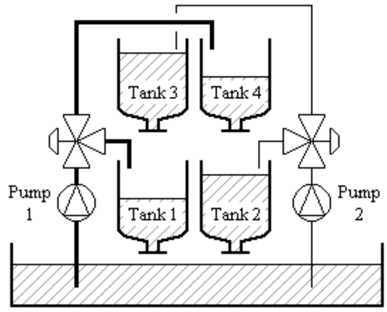

Fig. 8. Simple schematic for the four tank problem

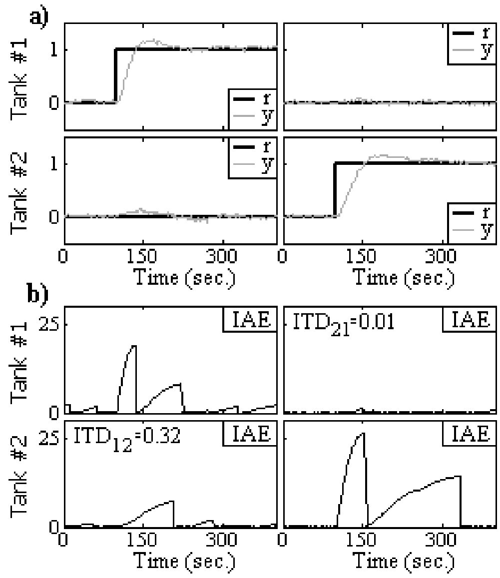

Fig. 9. Minimum Phase System: a) Set point (black) and controller variables (gray); b) IAE trends and interaction measures

Set-point changes and controller variables are shown in figure 9a while IAE trends and interaction measures are shown in figure 9b for the minimum phase system. Analyzing the trend of the controlled variables, it is very difficult to establish properly the level of interaction: an excitation with small amplitude is shown in tank #2 for a set-point change in tank #1, but it appears as a weak interaction. Compare this with the IAE of the trends which exhibits a significant peak similar to peaks showed in tank #1. On the other hand, a set point change in tank #2 does not generate excitations in tank #1. Both of these phenomena are captured by the corresponding interaction measures, $ITD_{1,2} = 0.32$ reveals the presence of a moderate interaction from tank #1 to tank #2; while $ITD_{2,1} = 0.01$ implies that tank #2 is not affecting tank #1.

For the second combination, the set-point changes and controller variables are shown in figure 10a) while the IAE trends and interaction measure are shown

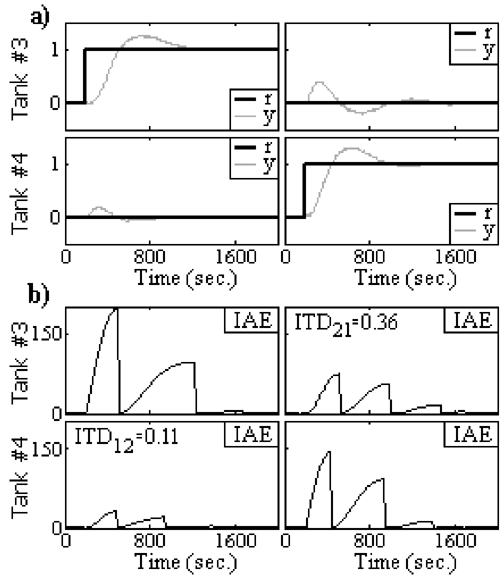

Fig. 10. Non-Minimum Phase System: a) Set point (black) and controller variables (gray); b) IAE trends and interaction measures

in figure 10b) for the non-minimum phase system. Again the analysis of IAE peaks reveals the presence of interaction which is stronger from tank #4 to tank #3: the higher ITD is now on the right tank while in the previous case it was on the left tank. These results are in agreement with the switch in the position of the feed for the two cases, once again indicating that the IAE technique is successful in highlighting the interaction among the loops. The level of interaction for this pair is larger than the earlier one due to the time delay.

The CPI values for the two cases are $[0.58 \;\; 0.71]$, indicating that the used controllers have a sufficiently good performance; the benefit obtained with a new tuning would be marginal. In retrospection, the IAE technique is able to capture the interaction, which otherwise appeared insignificant by visual inspection.

6. CONCLUSIONS

Two different techniques have been proposed to detect and quantify control loop interactions in MIMO processes. The IFD and PSCI measures used in the frequency domain are well-suited to analyze process excitations that are well-localized in the frequency domain; and the ITD, which analyzes data in the time-domain, is suited to process excitations that are spread over a range of frequencies. An important feature of these indices is that they can be computed from measured data, without any need for an explicit knowledge of the process model.

To avoid the effect of the disturbance transfer functions, which may cloud the interaction measures, set-point changes have been analyzed. In the presence of disturbance effects in the given data set, the reliability depends on the time coincidence of the process excitations caused by these two different sources.

The interaction measure in the time domain is completed by an analysis of the performance of the controller and an index CPI has been defined: it serves as an indicator to determine whether retuning of the controller is beneficial or if it is better to use an advanced MIMO (e.g. an MPC) controller.

The application of the proposed techniques have been demonstrated on two industrial simulation case studies and on an experimental setup comprising the four (interacting) tanks. In all cases, the proposed methods successfully revealed and quantified the interaction. which in one case appeared insignificant from a direct observation of the time-domain trends.

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