ADAPTIVE CONTROL OF THREE – TANK - SYSTEM: POLYNOMIAL APPROACH

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Abstract: Control of a three – tank - system laboratory model as a two inputs – two outputs system is presented. The objective laboratory model is a multivariable nonlinear system. It is based on experience with authentic industrial control applications. Two control algorithms utilizing polynomial theory and pole – placement are proposed. Particular controllers are based on various configuration of the closed loop. The algorithms in adaptive version are then used for control of the model. The results of the real-time experiments are also included. Quality of control achieved by both methods is compared and discussed.

Key words: Polynomial methods, Nonlinear system, Self-tuning control, Multivariable control, Real-time control.

1. INTRODUCTION

Many technological processes require a simultaneous control of several variables related to one system. Each input may influence all system outputs. The three – tank - system in Fig. 1 is a typical multivariable nonlinear system with significant cross – coupling. The design of a controller able to cope with such a system must be quite sophisticated. There are many different methods of controlling MIMO (multi input – multi output) systems. Several of these use decentralized PID controllers (Luyben, 1986), others apply single input-single-output (SISO) methods extended to cover multiple inputs (Chien *et al.*, 1987).

Here polynomial theory approach (Kučera 1980, Kučera, 1991, Skogestad and Postlethwaite, 1996) is used for the design of multivariable controllers. Two controllers are presented. The first one is based on traditional 1DOF (one degree of freedom) configuration of the closed loop, the second one applies 2DOF configuration proposed in (Ortega and Kelly, 1984) for SISO control loop. Application of the designed methods for adaptive control of the three – tank - system is then presented. The algorithms were applied as self – tuning controllers. It was assumed, that the dynamic behaviour of the system could be described in the neighbourhood of a steady state by a discrete linear model. The recursive least squares method with the directional forgetting was used for the identification part of the self – tuning controllers.

This paper is organised as follows: Section 2 contains description of the three – tank - system; Section 3 presents a mathematical model of the system which was used for the controllers design; Sections 4 and 5 describes designs of the 1DOF and 2DOF controllers; Section 6 describes the system identification method; Section 7 contains the experimental results; finally, Section 8 concludes the paper.

2. THREE – TANK – SYSTEM

The experiments were carried out with an experimental laboratory model three – tank – system. Such a system can be viewed as a prototype of many industrial applications in process industry, such as chemical and petrochemical plants, oil and gas systems. The typical control issue involved in the system is how to keep the desired liquid level in each tank. The principle scheme of the model is shown in Fig 1. The basic apparatus consists of three plexiglass tanks numbered from left to right as T1, T3 and T2. These are connected serially with each other by cylindrical pipes. Liquid, which is collected in a reservoir, is pumped into the first and the third tanks to maintain their levels. The level in the tank T3 is a response which is uncontrollable. It affects the level in the two end tanks. Each tank is equipped with a static pressure sensor, which gives a voltage output proportional to the level of liquid in the tank. Hmax denotes the highest possible liquid level. In case the liquid level of T1 and T2 exceeds this value the corresponding pump will be switched off automatically. Q_1 and Q_2 are the flow rates of the pumps 1 and 2. Two variable speed pumps driven by DC motor are used in this apparatus. These pumps are designed to give an accurate well defined flow per rotation. Thus, the flow rate provided by each pump is proportional to the voltage applied to its DC motor.

There are six manual valves v1, v2...v6 that can be used to vary the configuration of the process or to introduce disturbances or faults.

The pump flow rates Q_1 and Q_2 denote the input signals, the liquid levels of T1 and T2 are the output signals.

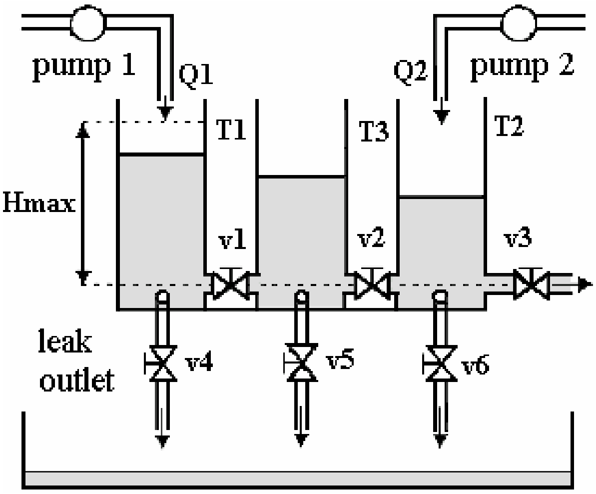

Fig. 1. Principal scheme of three – tank - system

3. MATHEMATICAL MODEL OF THE APPARATUS

An analytical model of the three – tank - system based on physics and the equipment construction is presented in (AMIRA, 1996). All the parameters in this model have a particular physical denotation. The apparatus is a nonlinear system, as it was mentioned above. A possible method for control of nonlinear systems is using of self – tuning controllers. A suitable model for adaptive control of the real object is an input – output model ("black box model"). This is a standard approach in self tuning controller area. Instead of often tedious construction of a model from first principles and then calculating its parameters from plant dimensions and physical constants, general type of model is chosen (here it is in fact transfer function (1)) and its parameters are identified from data. It is a model of the system behaviour and its parameters do not have a particular physical denotation. Of course, not all properties of the plant can be extracted from the data in this way but as a rule dominant properties are modelled, which is sufficient for a controller design. Advantages of this kind of model are its simplicity and accuracy in an operational range in which the input – output dependence is measured. In the framework of adaptive controllers it was chosen this kind of model. It was necessary to determine its structure in advance. The aim here was to find experimentally as simple structure of the model as possible, as it is mentioned bellow. The parameters are identified during the process of the recursive identification in virtue of the measured input and output signals.

A general transfer matrix of a two inputs – two outputs system with cross coupling is expressed as

$$\boldsymbol{G}(z) = \begin{bmatrix} G_{11}(z) & G_{12}(z) \\ G_{21}(z) & G_{22}(z) \end{bmatrix} \tag{1}$$

$$\boldsymbol{Y}(z) = \boldsymbol{G}(z)\boldsymbol{U}(z) \tag{2}$$

Where $\boldsymbol{U}(z)$ and $\boldsymbol{Y}(z)$ are vectors of the manipulated variables (flow rates of liquid into tanks T1 and T2) and the controlled variables (liquid levels of T1 and T2).

$$\boldsymbol{U}(z) = [u_1(z), u_2(z)]^T \qquad \boldsymbol{Y}(z) = [y_1(z), y_2(z)]^T \tag{3}$$

It is possible to assume that the dynamic behaviour of the system can be described in the neighbourhood of a steady state by a discrete linear model in the following form of the matrix fraction

$$\boldsymbol{G}(z) = \boldsymbol{A}^{-1}(z^{-1})\boldsymbol{B}(z^{-1}) = \boldsymbol{B}_1(z^{-1})\boldsymbol{A}_1^{-1}(z^{-1}) \tag{4}$$

Where polynomial matrices $\boldsymbol{A} \in R_{22}\lfloor z^{-1} \rfloor, \boldsymbol{B} \in R_{22}\lfloor z^{-1} \rfloor$ are the left coprime factorization of matrix $\boldsymbol{G}(z)$ and matrices $\boldsymbol{A}_1 \in R_{22}\lfloor z^{-1} \rfloor, \boldsymbol{B}_1 \in R_{22}\lfloor z^{-1} \rfloor$ are the right coprime factorization of $\boldsymbol{G}(z)$.

At first, the algorithms described bellow were designed for a model with polynomials of the first order. This model proved to be unsuitable for the process and the control algorithms failed. Consequently, the polynomial orders were increased and the algorithms were designed for a model with second order polynomials. This model proved to be effective. The model has sixteen parameters:

$$\boldsymbol{A}(z^{-1}) = \begin{bmatrix} 1 + a_1 z^{-1} + a_2 z^{-2} & a_3 z^{-1} + a_4 z^{-2} \\ a_5 z^{-1} + a_6 z^{-2} & 1 + a_7 z^{-1} + a_8 z^{-2} \end{bmatrix} \tag{5}$$

$$\boldsymbol{B}(z^{-1}) = \begin{bmatrix} b_1 z^{-1} + b_2 z^{-2} & b_3 z^{-1} + b_4 z^{-2} \\ b_5 z^{-1} + b_6 z^{-2} & b_7 z^{-1} + b_8 z^{-2} \end{bmatrix} \tag{6}$$

Polynomial matrices of the right matrix fraction of the system are defined in the following form

$$\boldsymbol{A}_1(z^{-1}) = \begin{bmatrix} 1 + a_9 z^{-1} + a_{10} z^{-2} & a_{11} z^{-1} + a_{12} z^{-2} \\ a_{13} z^{-1} + a_{14} z^{-2} & 1 + a_{15} z^{-1} + a_{16} z^{-2} \end{bmatrix} \tag{7}$$

$$\boldsymbol{B}_1\left(z^{-1}\right)=\begin{bmatrix} b_9 z^{-1}+b_{10}z^{-2} & b_{11}z^{-1}+b_{12}z^{-2} \\ b_{13}z^{-1}+b_{14}z^{-2} & b_{15}z^{-1}+b_{16}z^{-2} \end{bmatrix} \tag{8}$$

The coefficients of the matrices are given by solving the matrix equation

$$\boldsymbol{B}\left(z^{-1}\right)\boldsymbol{A}_1\left(z^{-1}\right)-\boldsymbol{A}\left(z^{-1}\right)\boldsymbol{B}_1\left(z^{-1}\right)=0 \tag{9}$$

4. DESIGN OF 1DOF CONTROLLER

The 1DOF control configuration is depicted in Fig. 2.

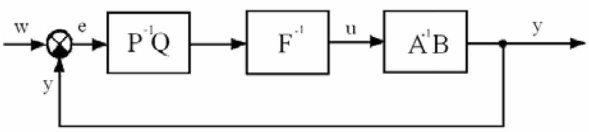

Fig. 2. Block diagram of 1DOF configuration

Similarly as it was for the controlled system, the transfer matrix of the controller takes the form of the following matrix fraction

$$\boldsymbol{G}_R(z)=\boldsymbol{P}^{-1}\left(z^{-1}\right)\boldsymbol{Q}\left(z^{-1}\right)=\boldsymbol{Q}_1\left(z^{-1}\right)\boldsymbol{P}_1^{-1}\left(z^{-1}\right) \tag{10}$$

Generally, the vector $\boldsymbol{W}\left(z^{-1}\right)$ of input reference signals is specified as

$$\boldsymbol{W}(z)=\boldsymbol{F}_w^{-1}\left(z^{-1}\right)\boldsymbol{h}\left(z^{-1}\right) \tag{11}$$

In case of control of the three – tank - system, the reference signals were considered as a class of step functions. In this case $\boldsymbol{h}\left(z^{-1}\right)$ is a vector of constants and $\boldsymbol{F}_W\left(z^{-1}\right)$ is expressed as

$$\boldsymbol{F}_w\left(z^{-1}\right)=\begin{bmatrix} 1-z^{-1} & 0 \\ 0 & 1-z^{-1} \end{bmatrix} \tag{12}$$

The compensator $\boldsymbol{F}\left(z^{-1}\right)$ is a component formally separated from the controller. It has to be included in the controller to fulfil the requirement on the asymptotic tracking. If the reference signals are step functions, then $\boldsymbol{F}\left(z^{-1}\right)$ is an integrator.

The control law in the block diagram in Fig. 2 (operator z^{-1} will be omitted from some operations for the purpose of simplification) is defined as

$$\boldsymbol{U}=\boldsymbol{F}^{-1}\boldsymbol{Q}_1\boldsymbol{P}_1^{-1}\boldsymbol{E} \tag{13}$$

where $\boldsymbol{E}$ is a vector of control errors. Using matrix operations it is possible to modify this vector as

$$\boldsymbol{E}=\boldsymbol{W}-\boldsymbol{Y}=\boldsymbol{P}_1\left(\boldsymbol{AFP}_1+\boldsymbol{BQ}_1\right)^{-1}\boldsymbol{AW} \tag{14}$$

Asymptotic tracking of the reference signals is then fulfilled if $\boldsymbol{FP}_1$ is divisible by $\boldsymbol{F}_w$.

It is possible to derive the following equation for the system output

$$\boldsymbol{Y}=\boldsymbol{A}^{-1}\boldsymbol{BF}^{-1}\boldsymbol{P}^{-1}\boldsymbol{QE}=\boldsymbol{A}^{-1}\boldsymbol{BF}^{-1}\boldsymbol{P}^{-1}\boldsymbol{Q}(\boldsymbol{W}-\boldsymbol{Y}) \tag{15}$$

and this can be modified

$$\boldsymbol{Y}=\boldsymbol{P}_1\left(\boldsymbol{AFP}_1+\boldsymbol{BQ}_1\right)^{-1}\boldsymbol{BQ}_1\boldsymbol{P}_1^{-1}\boldsymbol{W} \tag{16}$$

It is apparent, that the elements of the vector of the output signal have in their denominators the determinant of the matrix $\boldsymbol{AFP}_1+\boldsymbol{BQ}_1$. This determinant is the characteristic polynomial of a MIMO system. The roots of this polynomial matrix are the ruling factors for the behaviour of a closed loop system. The roots must be inside the unit circle (of the Gauss complex plain), in order for the system to be stable. Conditions of BIBO (bounded input bounded output) stability can be defined by the following diophantine equation

$$\boldsymbol{AF}\ \boldsymbol{P}_1+\boldsymbol{BQ}_1=\boldsymbol{M} \tag{17}$$

Where $\boldsymbol{M}\in R_{22}\left[z^{-1}\right]$ is a stable diagonal polynomial matrix.

$$\boldsymbol{M}\left(z^{-1}\right)=\begin{bmatrix} 1+m_1z^{-1}+m_2z^{-2}+ \\ +m_3z^{-3}+m_4z^{-4} & 0 \\ 0 & 1+m_1z^{-1}+m_2z^{-2}+ \\ & +m_3z^{-3}+m_4z^{-4} \end{bmatrix} \tag{18}$$

The degree of the controller polynomial matrices depends on the internal properness of the closed loop. The structure of matrices $\boldsymbol{P}_1$ and $\boldsymbol{Q}_1$ was chosen so that the number of unknown controller parameters equals the number of algebraic equations resulting from the solution of the diophantine equation. The method of the uncertain coefficients was used to solve the diophantine equation.

$$\boldsymbol{P}_1\left(z^{-1}\right)=\begin{bmatrix} 1+p_1z^{-1} & p_2z^{-1} \\ p_3z^{-1} & 1+p_4z^{-1} \end{bmatrix} \tag{19}$$

$$\boldsymbol{Q}_1\left(z^{-1}\right)=\begin{bmatrix} q_1+q_2z^{-1}+q_3z^{-2} & q_4+q_5z^{-1}+q_6z^{-2} \\ q_7+q_8z^{-1}+q_9z^{-2} & q_{10}+q_{11}z^{-1}+q_{12}z^{-2} \end{bmatrix} \tag{20}$$

The solution of the diophantine equation results in a set of sixteen algebraic equations with unknown controller parameters. Using matrix notation the algebraic equations can be expressed in the following form

$$\begin{bmatrix} -a_2 & -a_4 & 0 & 0 & b_2 & 0 & 0 & b_4 \\ a_2-a_1 & a_4-a_3 & 0 & b_2 & b_1 & 0 & b_4 & b_3 \\ a_1-1 & a_3 & b_2 & b_1 & 0 & b_4 & b_3 & 0 \\ 1 & 0 & b_1 & 0 & 0 & b_3 & 0 & 0 \\ -a_6 & -a_8 & 0 & 0 & b_6 & 0 & 0 & b_8 \\ a_6-a_5 & a_8-a_7 & 0 & b_6 & b_5 & 0 & b_8 & b_7 \\ a_5 & a_7-1 & b_6 & b_5 & 0 & b_8 & b_7 & 0 \\ 0 & 1 & b_5 & 0 & 0 & b_7 & 0 & 0 \end{bmatrix}\begin{bmatrix} p_1 \\ p_3 \\ q_1 \\ q_2 \\ q_3 \\ q_7 \\ q_8 \\ q_9 \end{bmatrix}=\begin{bmatrix} m_4 \\ m_3+a_2 \\ m_2-a_2+a_1 \\ m_1-a_1+1 \\ 0 \\ a_6 \\ a_5-a_6 \\ -a_5 \end{bmatrix}$$

$$\begin{bmatrix} -a_2 & -a_4 & 0 & 0 & b_2 & 0 & 0 & b_4 \\ a_2-a_1 & a_4-a_3 & 0 & b_2 & b_1 & 0 & b_4 & b_3 \\ a_1-1 & a_3 & b_2 & b_1 & 0 & b_4 & b_3 & 0 \\ 1 & 0 & b_1 & 0 & 0 & b_3 & 0 & 0 \\ -a_6 & -a_8 & 0 & 0 & b_6 & 0 & 0 & b_8 \\ a_6-a_5 & a_8-a_7 & 0 & b_6 & b_5 & 0 & b_8 & b_7 \\ a_5 & a_7-1 & b_6 & b_5 & 0 & b_8 & b_7 & 0 \\ 0 & 1 & b_5 & 0 & 0 & b_7 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_2 \\ p_4 \\ q_5 \\ q_6 \\ q_7 \\ q_{10} \\ q_{11} \\ q_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ a_4 \\ a_3-a_4 \\ -a_3 \\ m_4 \\ m_3+m_4 \\ m_2-a_4+a_7 \\ m_1-a_3+1 \end{bmatrix} \quad (21)$$

The controller parameters are obtained by solving these equations.

5. DESIGN OF 2DOF CONTROLLER

The configuration of the closed loop, shown in Fig. 3, was presented in (Ortega and Kelly, 1984) for SISO control loop.

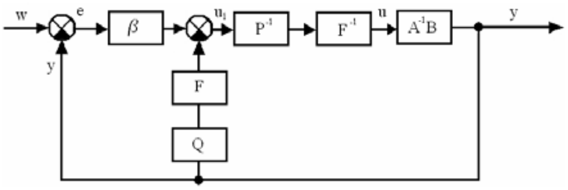

Fig. 3. Block diagram of 2DOF configuration

It is possible to derive the following equation for the system output

$$\boldsymbol{Y} = \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{U} = \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{U}_1 \quad (22)$$

Where

$$\boldsymbol{U}_1 = \boldsymbol{\beta}(\boldsymbol{W}-\boldsymbol{Y}) - \boldsymbol{Q}\boldsymbol{F}\boldsymbol{Y} \quad (23)$$

The corresponding equation for the controller's output, as shown in the block diagram in Fig. 3, follows as

$$\boldsymbol{U} = \boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{U}_1 \quad (24)$$

The substitution of $\boldsymbol{U}_1$ and $\boldsymbol{Y}$ results in

$$\boldsymbol{U} = \boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\left[\boldsymbol{\beta}\left(\boldsymbol{W}-\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{U}\right) - \boldsymbol{Q}\boldsymbol{F}\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{U}\right] \quad (25)$$

The equation (25) can be modified using the right matrix fraction of the controlled system into the form

$$\boldsymbol{U} = \boldsymbol{A}_1\left[\boldsymbol{P}\boldsymbol{F}\boldsymbol{A}_1 + (\boldsymbol{\beta}+\boldsymbol{F}\boldsymbol{Q})\boldsymbol{B}_1\right]\boldsymbol{\beta}\boldsymbol{W} \quad (26)$$

The closed loop system is stable when the following diophantine equation is satisfied

$$\boldsymbol{P}\boldsymbol{F}\boldsymbol{A}_1 + (\boldsymbol{\beta}+\boldsymbol{F}\boldsymbol{Q})\boldsymbol{B}_1 = \boldsymbol{M} \quad (27)$$

Where $\boldsymbol{M}$ is defined by expression (18) and the structure of the matrices $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{\beta}$ were chosen according to the same rules that are presented in the previous section. The matrices $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{\beta}$ are the matrices of the controller.

$$\boldsymbol{P}(z^{-1}) = \begin{bmatrix} 1+p_1 z^{-1} & p_2 z^{-1} \\ p_3 z^{-1} & 1+p_4 z^{-1} \end{bmatrix} \quad (28)$$

$$\boldsymbol{Q}(z^{-1}) = \begin{bmatrix} q_1+q_2 z^{-1} & q_3+q_4 z^{-1} \\ q_5+q_6 z^{-1} & q_7+q_8 z^{-1} \end{bmatrix} \quad (29)$$

$$\boldsymbol{\beta}(z^{-1}) = \begin{bmatrix} \beta_1 & \beta_2 \\ \beta_3 & \beta_4 \end{bmatrix} \quad (30)$$

A set of algebraic equations, that is used to obtain the unknown controller parameters, is defined by solving the diophantine equation (27). The algebraic equations in the matrix form are specified by the following expressions:

$$\begin{bmatrix} 1 & 0 & b_9 & 0 & b_{13} & 0 & b_9 & b_{13} \\ a_9-1 & a_{13} & b_{10}-b_9 & b_9 & b_{14}-b_{13} & b_{13} & b_{10} & b_{14} \\ a_{10}-a_9 & a_{14}-a_{13} & -b_{10} & b_{10}-b_9 & -b_{14} & b_{14}-b_{13} & 0 & 0 \\ -a_{10} & -a_{14} & 0 & -b_{10} & 0 & -b_{14} & 0 & 0 \\ 0 & 1 & b_{11} & 0 & b_{15} & 0 & b_{11} & b_{15} \\ a_{11} & a_{15}-1 & b_{12}-b_{11} & b_{11} & b_{16}-b_{15} & b_{15} & b_{12} & b_{16} \\ a_{12}-a_{11} & a_{16}-a_{15} & -b_{12} & b_{12}-b_{11} & -b_{16} & b_{16}-b_{15} & 0 & 0 \\ -a_{12} & -a_{16} & 0 & -b_{12} & 0 & -b_{16} & 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} m_1-a_9+1 \\ m_2+a_9-a_{10} \\ m_3+a_{10} \\ m_4 \\ -a_{11} \\ a_{11}-a_{12} \\ a_{12} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & b_9 & 0 & b_{13} & 0 & b_9 & b_{13} \\ a_9-1 & a_{13} & b_{10}-b_9 & b_9 & b_{14}-b_{13} & b_{13} & b_{10} & b_{14} \\ a_{10}-a_9 & a_{14}-a_{13} & -b_{10} & b_{10}-b_9 & -b_{14} & b_{14}-b_{13} & 0 & 0 \\ -a_{10} & -a_{14} & 0 & -b_{10} & 0 & -b_{14} & 0 & 0 \\ 0 & 1 & b_{11} & 0 & b_{15} & 0 & b_{11} & b_{15} \\ a_{11} & a_{15}-1 & b_{12}-b_{11} & b_{11} & b_{16}-b_{15} & b_{15} & b_{12} & b_{16} \\ a_{12}-a_{11} & a_{16}-a_{15} & -b_{12} & b_{12}-b_{11} & -b_{16} & b_{16}-b_{15} & 0 & 0 \\ -a_{12} & -a_{16} & 0 & -b_{12} & 0 & -b_{16} & 0 & 0 \end{bmatrix} \begin{bmatrix} p_3 \\ p_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ \beta_3 \\ \beta_4 \end{bmatrix} = \begin{bmatrix} -a_{13} \\ a_{13}-a_{14} \\ a_{14} \\ 0 \\ m_5-a_{15}+1 \\ m_6+a_{15}-a_{16} \\ m_7+a_{16} \\ m_8 \end{bmatrix}$$

(31)

The controller parameters are derived by solving these equations. The control law apparent from the block diagram is defined as

$$\boldsymbol{F}\boldsymbol{P}\boldsymbol{U} = \boldsymbol{\beta}\boldsymbol{E} - \boldsymbol{F}\boldsymbol{Q}\boldsymbol{Y} \quad (32)$$

6. SYSTEM IDENTIFICATION

For control of the three – tank – system, the control algorithms were applied as self tuning controllers. They were incorporated into an adaptive control system with recursive identification. The recursive least square method proved to be effective for self-tuning controllers (Kulhavý, 1987; Bittanti *et al.*, 1990) and was used as the basis for our algorithm. For our two-variable example it was considered the disintegration of the identification into two independent parts.

Difference equations describing the models in a vector form are as follows:

$$\begin{aligned} y_1(k) &= \boldsymbol{\Theta}_1^T(k)\boldsymbol{\varphi}_1(k-1) + n_1(k) \\ y_2(k) &= \boldsymbol{\Theta}_2^T(k)\boldsymbol{\varphi}_2(k-1) + n_2(k) \end{aligned} \quad (33)$$

where $n_1(k)$, $n_2(k)$ are unmeasurable random signals.

The parameter vectors are specified as shown below:

$$\begin{aligned} \boldsymbol{\Theta}_1^T(k) &= [a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4] \\ \boldsymbol{\Theta}_2^T(k) &= [a_5, a_6, a_7, a_8, b_5, b_6, b_7, b_8] \end{aligned} \quad (34)$$

The data vector is

$$\boldsymbol{\varphi}_{1,2}^T(k-1) = [-y_1(k-1), -y_1(k-2), -y_2(k-1), -y_2(k-2), u_1(k-1), u_1(k-2), u_2(k-1), u_2(k-2)] \quad (35)$$

The parameter estimates are updated using the recursive least squares method with adaptive directional forgetting.

7. EXPERIMENTAL EXAMPLES

The model was connected with a PC equipped with a control and measurement PC card. The Matlab and the Real Time Toolbox were used to control the system.

For the experiments presented in this paper, the three – tank – system was configured in such a way that the valves v3 and v5 were closed and the remaining valves were open.

The best sampling period T_0=5 s was found in virtue of many experiments. Another problem was finding of suitable poles of the characteristic polynomial. In comparison with controllers for SISO control loops, where it is often possible to assume influence of particular poles to behaviour of the closed loop, pole – placement of multivariable controllers is much more complicated. Pole – placement applicable for both controllers was obtained from a number of experiments as follows:

$$\boldsymbol{M}\left(z^{-1}\right)=\begin{bmatrix} 1-0{,}9z^{-1}+0{,}19z^{-2}- & 0 \\ -0{,}009z^{-3}-0{,}002z^{-4} & \\ & 1-0{,}9z^{-1}+0{,}19z^{-2}- \\ 0 & -0{,}009z^{-3}-0{,}002z^{-4} \end{bmatrix} \quad (36)$$

In Fig. 4 and Fig. 5 are shown time responses of the control when the initial parameter estimates were chosen without any a-priori information:

$$\boldsymbol{\Theta}_1^T(0)=[0.1,0.2,0.3,0.4,0.1,0.2,0.3,0.4] \quad (37)$$
$$\boldsymbol{\Theta}_2^T(0)=[0.5,0.6,0.7,0.8,0.5,0.6,0.7,0.8]$$

The reference signals contain frequent step changes in the beginning of experiments to activate input and output signals and improve the identification. The controlled variables y_1 and y_2 are liquid levels of tanks T1 and T2. The manipulated variables u_1 and u_2 are flow rates of liquid into the tanks. As w_1 and w_2 are denoted desired liquid levels in particular tanks (reference signals).

Subsequent experiments were carried out in such a way that initial parameter estimates were set as the last parameter estimates obtained in the ends of the previous experiments. The initial conditions of the recursive identification were also modified by reducing of diagonal elements of the square covariance matrix from 1000 to 10. Because the system is nonlinear and the identified parameters were valid only for particular steady states, the reference signals were set to the same values as it was in the ends of the previous experiments. Time responses of these experiments are shown in Fig. 6 and Fig. 7.

Tables 1 and 2 contain values of control quality criterions. The criterions are sum of powers of tracking errors and sum of increments of manipulated variables. The table 1 contains values obtained from the experiments, when the initial parameter estimates were chosen without a-priori information. The table 2 relates to the experiments with steady parameters.

Fig. 4. Control of the laboratory model using 1DOF controller

Fig. 5. Control of the laboratory model using 2DOF controller

Fig. 6. Control of the laboratory model using 1DOF controller – experiment with steady parameters

[redacted]

Fig. 7. Control of the laboratory model using 2DOF controller – experiment with steady parameters

Table 1 Control quality criterions

Controller	$\sum e_1^2$	$\sum e_2^2$	$\sum \Delta u_1^2$	$\sum \Delta u_2^2$
1DOF	0,2181	0,1185	3,9846	4,6327
2DOF	0,2204	0,1262	1,7375	1,3647

Table 2 Control quality criterions – experiments with steady parameters

Controller	$\sum e_1^2$	$\sum e_2^2$	$\sum \Delta u_1^2$	$\sum \Delta u_2^2$
1DOF	0,4059	0,2195	5,5649	8,2888
2DOF	0,4472	0,2810	4,1135	4,1825

8. CONCLUSIONS

It is possible to derive several conclusions from the values of the control quality criterions obtained during experiments with particular controllers. If the first criterion (powers of tracking errors) is considered, comparable results were achieved with both controllers. According to the second criterion (increments of manipulated variables) the 2DOF controller performed significantly better. This fact is also evident from the courses in Figs. 4 – 7.

The control tests executed on the laboratory model provide very satisfactory results, despite of the fact, that the non–linear dynamics was described by a linear model. The objective laboratory model simulates technological processes, which frequently occur in industry. The laboratory tests proved that the examined methods could be implemented and used successfully to control such processes.

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