Robust MPC with Output Feedback and Realigned Model

J. M. Perez[1] and D. Odloak[2]

[1]PETROBRAS – Cenpes
e-mail: perez@petrobras.com.br
[2]Chemical Engineering Department, University of São Paulo.
C.P. 61548, 05424-970, São Paulo, Brazil.
e-mail: odloak@usp.br

Abstract: In this work, it is presented a contribution to the design of the robust MPC with output feedback and input constraints. This work extends existing approaches by considering a particular non-minimal state space model, which transforms the output feedback strategy into a state feedback strategy. The controller is developed to the case in which the system inputs may become saturated. We follow a two stages approach. In the off-line stage, a series of unconstrained robust MPCs is obtained by including in the control optimization problem, inequality constraints that force the state of the closed-loop system to contract along the time. Each of these controllers is associated to particular sets of manipulated inputs and controlled outputs. In the existing version of the method, the closed loop system involves a state observer that makes the solution to the robust MPC optimization problem a time consuming step. In the on-line step of the controller design proposed procedure, a sub optimal control law is obtained by combining control configurations that correspond to particular subsets of available manipulated inputs. The method is illustrated with a simulation example of the process industry.

Keyword: Model Predictive Control, Robust stability, Output feedback, Input constraints.

1. INTRODUCTION

Model predictive control has achieved a remarkable popularity in the process industry with thousands of practical applications (Qin and Badgwell, 2001). One of the reasons of this industrial acceptance is the ability of MPC to incorporate constraints in the inputs. However, one additional desirable characteristic still not attended by commercial MPC packages is closed loop stability in the presence of model uncertainty. When model uncertainty is considered in the synthesis of the predictive controller, the existing algorithms usually demand a computer effort that is prohibitive for practical implementation (Lee and Cooley, 2000). We could not find in the control literature a satisfactory solution, from the application viewpoint, to the robust MPC problem with output feedback and input constraints. Rodrigues and Odloak (2000) presented a formulation to the robust unconstrained MPC with output feedback where stability is achieved through the explicit inclusion of a Lyapunov inequality into the control optimization problem. Recently, Rodrigues & Odloak (2005) extended the method to include the switching of active input constraints during transient conditions. The controller is designed in two steps. The first step is developed off-line and results in a set of linear unconstrained MPCs each one corresponding to a controllable subsystem with previously defined inputs and outputs. The second step of the controller design procedure is performed on-line and involves the solution to an optimization problem that has the same objective function as the conventional MPC, but the control actions are computed as a convex combination of the linear controllers obtained in the first stage of design procedure. The main objective of this work is to improve the method of Rodrigues and Odloak (2005) by proposing an alternative solution to the off-line step of the synthesis of the robust MPC. The method proposed here is based on a non-minimal state space model formulation presented by Maciejowsky (2002) and designated by that author as realigned model. It will be shown here that the approach leads to a significant simplification in the off-line step of the design procedure. In the next section, the approach of Rodrigues and Odloak (2005) is briefly summarized. Then, the realigned model in the incremental form is introduced and the optimization problems of the off-line and on-line steps of the controller synthesis are presented. Next, a simulation example is used to illustrate the application potential of the new method and finally the paper is concluded.

2 A ROBUST UNCONSTRAINED MPC WITH OUTPUT FEEDBACK

We consider the following discrete time invariant model that is written in the incremental form:

$$[\tilde{x}]_{k+1|k} = \tilde{A}[\tilde{x}]_{k|k} + \tilde{B}\,\Delta u(k) \tag{1}$$

$$[\tilde{y}]_{k|k} = \tilde{C}[\tilde{x}]_{k|k} \tag{2}$$

where

$\tilde{x} \in \mathbb{R}^{nx}$ is the state of the predicting model, k is the present sampling instant, $u \in \mathbb{R}^{nu}$ is the input, $\Delta u(k) = u(k) - u(k-1)$ is the input increment, $y \in \mathbb{R}^{ny}$ is the output. $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ are matrices of appropriate dimensions.

As the state is not usually measured, they need to be updated with the output measurement as follows:

$$[\tilde{x}]_{k+1|k+1} = \tilde{A}[\tilde{x}]_{k|k} + \tilde{B}\,\Delta u(k) + + K_F\{[y_p]_{k+1|k} - \tilde{C}[\tilde{x}]_{k+1|k}\} \quad (3)$$

where K_F is the gain of the observer that estimates the model states and y_p is the output of the true plant which is represented by a similar model as the one represented in Eqs (1) and (2) but with different coefficient matrices:

$$[x_p]_{k+1|k} = A_p[x_p]_{k|k} + B_p\Delta u(k) \quad (4)$$

$$[y_p]_{k+1|k} = C_p[x_p]_{k+1|k} \quad (5)$$

Assume now that related to the output reference vector y^{sp}, we can define a state reference v^{sp} for the plant and predicting model. Consequently, we can define errors for these states: $x_p^e = v^{sp} - x_p$ and $\tilde{x}^e = v^{sp} - \tilde{x}$.

Now, combining Eqs (3), (4) and (5) we obtain the plant plus observer model:

$$[x^e]_{k+1|k+1} = A[x^e]_{k|k} + B\,\Delta u(k) \quad (6)$$

where

$$x^e = \begin{bmatrix} \tilde{x}^e \\ x_p^e \end{bmatrix},\ x^e \in \mathbb{R}^{2nx},$$

$$A = \begin{bmatrix} (I - K_F\tilde{C})\tilde{A} & K_F C_p A_p \\ 0 & A_p \end{bmatrix},\ A \in \mathbb{R}^{2nx\times 2nx},$$

$$B = -\begin{bmatrix} (I - K_F\tilde{C})\tilde{B} + K_F C_p B_p \\ B_p \end{bmatrix},\ B \in \mathbb{C}^{2nx\times nu}$$

The cost function of the MPC considered here is defined as follows:

$$J_k = (\bar{A}[\tilde{x}^e]_{k|k} - \bar{B}\,\Delta u)^T Q(\bar{A}[\tilde{x}^e]_{k|k} - \bar{B}\Delta u) + \Delta u^T\, R\,\Delta u \quad (7)$$

where Q and R are weighting matrices and

$$\Delta u = \begin{bmatrix} \Delta u(k) \\ \vdots \\ \Delta u(k+m-1) \end{bmatrix},\quad \bar{A} = \begin{bmatrix} \tilde{C} \\ \tilde{C}\tilde{A} \\ \vdots \\ \tilde{C}\tilde{A}^p \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ \tilde{C}\tilde{B} & 0 & 0 & \cdots & 0 \\ \tilde{C}\tilde{A}\tilde{B} & \tilde{C}\tilde{B} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{C}\tilde{A}^{p-1}\tilde{B} & \tilde{C}\tilde{A}^{p-2}\tilde{B} & & \cdots & \tilde{C}\tilde{A}^{p-m}\tilde{B} \end{bmatrix}$$

m and p are the input and output horizons.

The vector of future unconstrained control actions can be related to the error on the state of the prediction model as follows:

$$\Delta u = K_{MPC}\,[\tilde{x}^e]_{k|k},\quad K_{MPC} \in \mathbb{R}^{m.nu\times nx} \quad (8)$$

Substituting (8) in (6) produces

$$\left[x^e\right]_{k+1|k+1} = (A + B\tilde{K}_{MPC}\,N)\left[x^e\right]_{k|k} \quad (9)$$

where

$\tilde{K}_{MPC} = C_K K_{MPC}$, $C_K = [I_{nu}\quad 0]$, $N = [I_{nx}\quad 0]$

Consider also the following Lyapunov inequality:

$$\left(A + B\tilde{K}_{MPC}\,N\right)^T P^{-1}\left(A + B\tilde{K}_{MPC}N\right) - P^{-1} < 0 \quad (10)$$

with $P = P^T > 0$

Applying the Schur's complement to (10), results:

$$\begin{bmatrix} P & P\left(A + B\tilde{K}_{MPC}N\right) \\ \left(A + B\tilde{K}_{MPC}N\right)^T P^T & P \end{bmatrix} > 0 \quad (11)$$

A stable MPC would result from the minimization of the objective defined in (7) subject to the constraint defined in (11). However, to guarantee the contraction of the close-loop error vector, K_{MPC} and P should be fixed and so they need to be computed off-line and considering a fixed value for the error vector. For instance, they can be computed for an error $\tilde{x}_0^e$ that corresponds to a step change on the desired value of the output. The result is the following optimization problem:

Problem P1

$$\min_{\gamma, K_{MPC}, P} \gamma$$

subject to (11) and

$$\begin{bmatrix} (\bar{B}^T Q\bar{B} + R)^{-1} & K_{MPC}(\tilde{x}_0^e) \\ (\tilde{x}_0^e)^T K_{MPC}^T & \begin{bmatrix} \gamma - (\tilde{x}_0^e)^T \bar{A}^T Q\bar{A}\tilde{x}_0^e + \\ +(\tilde{x}_0^e)^T \bar{A}^T Q\bar{B}K_{MPC}\tilde{x}_0^e + \\ +(\tilde{x}_0^e)^T K_{MPC}^T \bar{B}^T Q\bar{A}\tilde{x}_0^e \end{bmatrix} \end{bmatrix} > 0 \quad (12)$$

where γ is a cost bound such $J_k \le \gamma$.

For the operating condition in which none of the input constraints becomes active, the control law obtained as the solution to Problem P1 results optimal and stability of the closed loop system is assured by the following theorem (Rodrigues and Odloak, 2000):

Theorem 1: Suppose that Problem P1 has a feasible solution. The resulting control law applied to the system defined in Eqs (4) and (5) will be asymptotic stable, as long as the system inputs do not become saturated.

Remark 1

As Inequality (11) is bilinear in the unknown variables P and K_{MPC}, Problem P1 has to be solved via an iterative algorithm that may be highly computer demanding for on-line implementation. Rodrigues and Odloak (2005) propose a two stages strategy to design and implement the stable MPC with output feedback and input saturation. The off-line synthesis is based on the solution to Problem P1. The on-line step is based on an optimization problem where the objective function is the same as the control objective used in the off-line step. The on-line controller is assumed to be a linear combination of the controllers obtained in the off-line step. The objective function of the on-line step is the true control objective and the input constraints are included in the optimization problem.

To consider model uncertainty in the controller resulting from the solution to Problem P1, observe that in matrices A and B defined in Eq. (6), matrices A_p and B_p of the true plant are not usually known and they are not the same as matrices $\tilde{A}$ and $\tilde{B}$ of the predicting model. Note also that matrices A and B are affine in A_p and B_p respectively. The consequence of this observation is that, if model uncertainty concentrates on A_p and B_p, Problem P1 can be extended to produce a controller, which is robust to model uncertainty. For this purpose, we assume that state matrix A_p and input matrix B_p of the plant model defined in Eq. (4) are known to lie in the polytope defined in (13).

$$\left(A_p, B_p\right) = \sum_{i=1}^{L} \lambda_i \left(A_{p,i}, B_{p,i}\right),\ \sum_{i=1}^{L} \lambda_i = 1,\ \lambda_i \geq 0,\quad i = 1, \ldots, L \tag{13}$$

Also, let us define

$$A_i = \begin{bmatrix} (I - K_F \tilde{C})\tilde{A} & K_F C_p A_{p,i} \\ 0 & A_{p,i} \end{bmatrix}$$

$$B_i = -\begin{bmatrix} (I - K_F \tilde{C})\tilde{B} + K_F C_p B_{p,i} \\ B_{p,i} \end{bmatrix}$$

Next, consider the following off-line optimization problem

Problem P2

$$\min_{\gamma, K_{MPC}, P} \gamma$$

subject to (12) written for the nominal model and the following constraints

$$\begin{bmatrix} P & P\left(A_i + B_i C_K K_{MPC} N\right) \\ \left(A_i + B_i C_K K_{MPC} N\right)^T P^T & P \end{bmatrix} > 0,\quad i = 1, \ldots, L \tag{14}$$

In the above problem, each of the inequalities represented in Inequality (14) corresponds to one of the vertices of the polytope defined in (13). Stability of the uncertain closed-loop system with the output feedback controller defined by Problem P2 is ensured by the theorem below (Rodrigues and Odloak, 2000):

Theorem 2 Consider the system defined in Eqs (4) and (5), in which the true plant model matrices are unknown but defined by a polytope as in Eq. (13). Then, the closed loop system, with the control law obtained from the solution to Problem P2, will be asymptotically stable, as long as the system inputs do not become saturated.

Observe that in order to Problem P2 to have a feasible solution for the uncertain system, it is necessary that the system be observable and controllable. When one or more of the inputs become saturated, this condition may not be attained. Then, for the development that follows, it is assumed that the unstable and integrating modes of the system remains controllable after the saturation of one or more inputs.

3. ROBUST MPC WITH REALIGNED MODEL

Maciejowski (2002) shows that a suitable state space model to be used in the model predictive controller is the realigned model. Apart from the disadvantage of being a non-minimal representation of the system, this model form has the advantage that the state is composed of the inputs and outputs of the system at present and past time instants. Consequently, with this model form, the assumption that the state is perfectly known is always true. A consequence of this property of the model is that it is not necessary to include both the prediction model and the plant model in the closed loop representation of the system. To present the structure of the model as employed here, assume that the system is represented by the following equation:

$$y(k) + \sum_{i=1}^{na} a_i y(k-i) = \sum_{i=1}^{nb} b_i u(k-i)$$

It can be shown that this system can also be represented in the following state space form:

$$x(k+1) = \begin{bmatrix} I-a_1 & -a_2+a_1 & -a_3+a_2 & -a_4+a_3 & \ldots & -a_{na}+a_{na-1} & a_{na} & b_2 & b_3 & b_4 & \ldots & b_{nb} \\ I & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\ 0 & I & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & I & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \cdots & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & I & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & I & 0 \end{bmatrix} x(k) + \begin{bmatrix} b_1 \\ 0 \\ 0 \\ 0 \\ \ldots \\ 0 \\ 0 \\ I \\ 0 \\ 0 \\ \ldots \\ 0 \end{bmatrix} \Delta u(k)$$

where

$$x(k) = \left[y(k)^T \ \cdots \ y(k-na)^T \quad \Delta u(k-1)^T \ \cdots \ \Delta u(k-nb+1)^T \right]^T$$

The above model corresponds to the following general state space model form:

$$[x]_{k+1/k} = A\,[x]_{k/k} + B\,\Delta u(k) \tag{15}$$

$$[y]_{k/k} = C\,[x]_{k/k} \qquad C = \begin{bmatrix} I_{ny} & 0 & \cdots & 0 \end{bmatrix}$$

In the objective function defined in (7), the state error takes the following form

$$x^e = x^{sp}(k) - x(k) = \begin{bmatrix} y^{sp} - y(k) \\ \vdots \\ y^{sp} - y(k-na) \\ -\Delta u(k-1) \\ \vdots \\ -\Delta u(k-nb+1) \end{bmatrix}$$

Consider now the Lyapunov inequality presented in (10). In the realigned model case, matrix N becomes an identity matrix and (10) assumes the form:

$$\left(A + B\tilde{K}_{MPC}\right)^T P^{-1} \left(A + B\tilde{K}_{MPC}\right) - P^{-1} < 0$$

with $P = P^T > 0$
or

$$\begin{bmatrix} P & \left(A + B\tilde{K}_{MPC}\right)P \\ P^T\left(A + B\tilde{K}_{MPC}\right)^T & P \end{bmatrix} > 0 \quad (16)$$

Inequality (16) is still not a LMI as both $\tilde{K}_{MPC}$ and P are variables of the MPC optimization problem. However, in this case we can define a new variable $Y = \tilde{K}_{MPC}P$ and (16) is transformed into the following LMI:

$$\begin{bmatrix} P & AP + BY \\ P^T A^T + Y^T B^T & P \end{bmatrix} > 0$$

Observe that, as P is not singular, if Y and P are both known then $\tilde{K}_{MPC}$ can be computed by $\tilde{K}_{MPC} = Y P^{-1}$.

With these new variables, Problem P1 can be re-written as follows

Problem P3

$$\min_{\gamma, K_{MPC}, P, Y} \gamma$$

subject to

$$\begin{bmatrix} \left(\bar{B}^T Q \bar{B} + R\right)^{-1} & K_{MPC} x_0^e(k) \\ x_0^e(k)^T K_{MPC}^T & \begin{bmatrix} \gamma + x_0^e(k)^T \bar{A}^T Q \bar{B} K_{MPC} x_0^e(k) + \\ + x_0^e(k)^T K_{MPC}^T \bar{B}^T Q \bar{A} x_0^e(k) - \\ - x_0^e(k)^T \bar{A}^T Q \bar{A}\, x_0^e(k) \end{bmatrix} \end{bmatrix} > 0$$

$$\begin{bmatrix} P & AP + BY \\ P^T A^T + Y^T B^T & P \end{bmatrix} > 0$$

where

$$\tilde{K}_{MPC} = Y P^{-1}, \quad \tilde{K}_{MPC} = C_K\ K_{MPC} \qquad (17)$$

Problem P3 is not linear because of Eq. (17) and consequently, we cannot use the existing LMI packages to solve this problem. Thus, we propose a sub optimal solution that is based on the solution to two LMI sub problems:

Problem P3a

$$\max_{\alpha, P, Y} \alpha$$

subject to

$$\begin{bmatrix} P & AP + BY \\ P^T A^T + Y^T B^T & P - \alpha \end{bmatrix} > 0$$

$$\alpha \geq 0$$

Let us call the solution to this problem as α^*, Q^* and Y^*, and we can obtain the gain of the MPC controller as $\tilde{K}_{MPC}{}^* = Y^*(P^*)^{-1}$.

Problem P3b

$$\min_{K_{MPC}} J_k = x_0^e(k)^T K_{MPC}^T \left(\bar{B}^T Q \bar{B} + R\right) K_{MPC} x_0^e(k) - \\ - x_0^e(k)^T K_{MPC}^T \bar{B}^T Q \bar{A} x_0^e(k) - \\ - x_0^e(k)^T \bar{A}^T Q \bar{B} K_{MPC} x_0^e(k) + x_0^e(k)^T \bar{A}^T Q \bar{A} x_0^e(k)$$

subject to

$$\left(A + BC_K K_{MPC}\right)^T (P^*)^{-1} \left(A + BC_K K_{MPC}\right) - (P^*)^{-1} < 0 \qquad (18)$$

that can be transformed into the following LMI:

$$\begin{bmatrix} P^* & B\tilde{K}_{MPC} P^* \\ (P^*)^T \tilde{K}_{MPC}^T B^T & \begin{bmatrix} P^* + (P^*)^T A^T (P^*)^{-1} A P^* + \\ + (P^*)^T \tilde{K}_{MPC}^T B^T (P^*)^{-1} A P^* + \\ + (P^*)^T A^T (P^*)^{-1} B \tilde{K}_{MPC} P^* \end{bmatrix} \end{bmatrix} > 0$$

Problem P3a searches for a $\tilde{K}_{MPC}$ that maximizes α, which represents the distance the closed loop is from the stability limit. The purpose of Problem P3b is to improve the performance of the controller obtained in Problem P3a by minimizing the true objective function of the MPC while preserving stability, which is guaranteed by the presence of Inequality (18). It is easy to show that if Problem P3a is feasible then Problem P3b is also feasible as

$$K_{MPC} = \begin{bmatrix} \tilde{K}_{MPC}^{*\ T} & 0 & \cdots & 0 \end{bmatrix}^T$$

is a feasible solution to Problem P3b.

Assuming the same class of model uncertainty as the one defined in (13), Problems P3a and P3b can be extended to produce a new unconstrained robust MPC with output feedback, which is obtained from the solution of the following problems:

Problem P4a

$$\max_{\alpha, P, Y} \alpha$$

subject to

$$\begin{bmatrix} P & A_iP + B_iY \\ P^T A_i^T + Y^T B_i^T & P - \alpha \end{bmatrix} > 0\,, \quad i = 1, \ldots, L$$
$$\alpha > 0$$

Problem P4b

$$\min_{K_{MPC}} J_k = x_0^e(k)^T K_{MPC}^T \left(\bar{B}^T Q \bar{B} + R\right) K_{MPC} x_0^e(k) -$$
$$- x_0^e(k)^T K_{MPC}^T \bar{B}^T Q \bar{A} x_0^e(k) - x_0^e(k)^T \bar{A}^T Q \bar{B} K_{MPC} x_0^e(k)$$
$$+ x_0^e(k)^T \bar{A}^T Q \bar{A} x_0^e(k)$$

subject to

$$\begin{bmatrix} P^* & B_i \tilde{K}_{MPC} P^* \\ (P^*)^T \tilde{K}_{MPC}^T B_i^T & \begin{bmatrix} P^* + (P^*)^T A_i^T (P^*)^{-1} A_i P^* + \\ +(P^*)^T \tilde{K}_{MPC}^T B_i^T (P^*)^{-1} A_i P^* + \\ +(P^*)^T A_i^T (P^*)^{-1} B_i \tilde{K}_{MPC} P^* \end{bmatrix} \end{bmatrix} > 0$$
$$i = 1, \ldots, L$$

Now, to obtain a new robust MPC with output feedback and input constraints, we propose the following algorithm:

Algorithm

Off-line step

Compute the output feedback gain K_{MPC} solving problems P4a and P4b considering the nc possible control configurations including the cases where one or more inputs become saturated. Each of these control configurations has a specific set of controlled outputs and unconstrained manipulated inputs. All the subsystems are assumed controllable. To stabilize all these subsystems that may result when one or more inputs become saturated, the following inequalities should be included in Problem P4a:

$$\begin{bmatrix} P & P\left(A_i + B_i \tilde{K}_{MPC}^j\right) \\ P^T \left(A_i + B_i \tilde{K}_{MPC}^j\right)^T & P \end{bmatrix} > 0$$
$$i = 1, \ldots, L\,, \ j = 1, \ldots, nc$$

where matrix $\tilde{K}_{MPC}^j$ is obtained from $\tilde{K}_{MPC}$ by zeroing the terms related to the saturated inputs

On-line step

At each sampling step k, with the real output measurement compute the error on the state of the predicting model $[x^e]_{k|k}$ and solve the following problem:

Problem P5

$$\min_{\beta_0, \beta_1, \ldots, \beta_{nc}} J_k$$

Subject to

$$J_k = (\bar{A}[x^e]_{k|k} - \bar{B}\,\Delta u)^T Q (\bar{A}[x^e]_{k|k} - \bar{B}\Delta u) + \Delta u^T R\,\Delta u$$
$$\sum_{j=0}^{nc} \beta_j = 1$$
$$0 \le \beta_j \le 1 \qquad j = 0, 1 \ldots, nc$$
$$\Delta u = \left[\beta_0 K_{MPC} + \beta_1 K_{MPC}^1 + \cdots \beta_{nc} K_{MPC}^{nc}\right][x^e]_{k|k}$$
$$u_{\min} \le u(k+j) \le u_{\max} \qquad j = 1, \ldots, m-1$$

The successive application of the control law provided by the solution to Problem P5, for the uncertain system defined in Eq. (13), produces an asymptotically stable closed-loop system as shown in Theorem 3 below. The proof of this theorem can be obtained by following the same steps as in Rodrigues and Odloak (2005).

Theorem 3: Consider an uncertain system as defined in Eq. (13) and assume that this system remains controllable when one or more manipulated inputs become saturated. The closed loop system with the control strategy obtained by solving Problem P5 will remain stable when the system is moved from a point where none of the inputs is saturated to another point where one or more inputs become saturated. Stability is preserved in the reverse direction, when an input becomes available to be manipulated by the MPC.

4. EXAMPLE

The proposed control strategy was tested with a system of the process industry. The system is part of a distillation column where isobutene is separated from n-butane in an oil refinery. The controlled outputs are the level of liquid in the overhead drum (y_1) and the contents of isobutene in the distillate (y_2). The manipulated variables are the reboiler heat duty (u_1) and the distillate flow rate (u_2). From practical tests the following two models were obtained for different operating conditions:

$$\mathrm{G}_1(s) = \begin{bmatrix} \dfrac{2.3}{s} & \dfrac{-0.7}{s} \\ \dfrac{-7e^{-7s}}{20s+1} & \dfrac{-1.8}{4s+1} \end{bmatrix}, \mathrm{G}_2(s) = \begin{bmatrix} \dfrac{3}{s} & \dfrac{-0.5}{s} \\ \dfrac{-5e^{-5s}}{15s+1} & \dfrac{-2.5}{10s+1} \end{bmatrix}$$

In the simulation performed here, we study the set-point tracking problem where the desired value of isobutene in the distillate is increased by 1% and the desired value of the liquid level is not modified. The tuning parameters that were used in the off-line stage of the controller synthesis are the following: T = 1; m = 3; p = 50; Q = diag[1 1]; R = $diag$[0.1 0.1]. Related to the values the variables at the initial steady state of the system, the input limits are $u_{\max} = [0.2 \quad 2]$, $u_{\min} = [-0.2 \quad -2]$. The nominal model is represented by model 1 and the true plant can be either model 1 or model 2. Figure 1 shows the system responses for the nominally stable controller defined by problems 3a and 3b for three different cases. The first case corresponds to the ideal case where the predicting model and the true plant are represented by model 1. In Case 2, we have the same system as in Case 1 but the minimum bound for input u_1 was modified to –0.1 in order to force this input to become active during part of the simulation time.

From Fig. 1, we see that the controller stabilizes the true model without any problem and the performance of the controller is acceptable. However, in Case 3 where the controller is based in model 1 and the true plant is represented by model 2, the closed loop system becomes unstable.

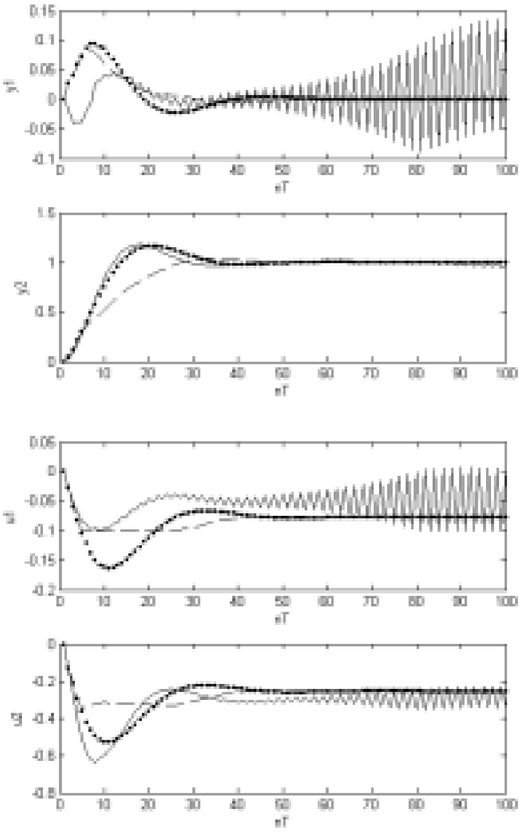

Figure 1. MPC with nominal model: Case 1 (….), Case 2 (- - - -) and Case 3 (——)

Fig.2 shows the closed loop responses with the robust MPC defined by Problems 4a and 4b considering models 1 and 2. In the two simulated cases plant is represented by model 2. In Case 4, the inputs do not saturate and the controller drives the two outputs to their desired values. The performance of the controller is slightly worse than in the ideal case as the output responses are slower than the responses for the nominal case. In Case 5, the minimum bound of input u_2 was modified to –0.2 in order to force this constraint to become active. We see from Fig. 2, that offset appears in both outputs because after saturation, the controller has only one manipulated input and two controlled outputs.

5. CONCLUSION

In this paper, it was developed a new version of the constrained robust MPC with output feedback. In the proposed method, assuming that controllability is preserved, stability is assured even when the system inputs become saturated at transient or equilibrium states. Polytopic model uncertainty is considered. Computer burden of the numerical methods involved in the practical synthesis of the controller is substantially reduced through the adoption of a simplified sub optimal solution to the control problem. On-line computation involves the search for the coefficients of a linear combination of previously defined MPC controllers. A simulation example shows that the implementation of the developed approach to real industrial systems may be achieved at least for systems of small to medium dimension.

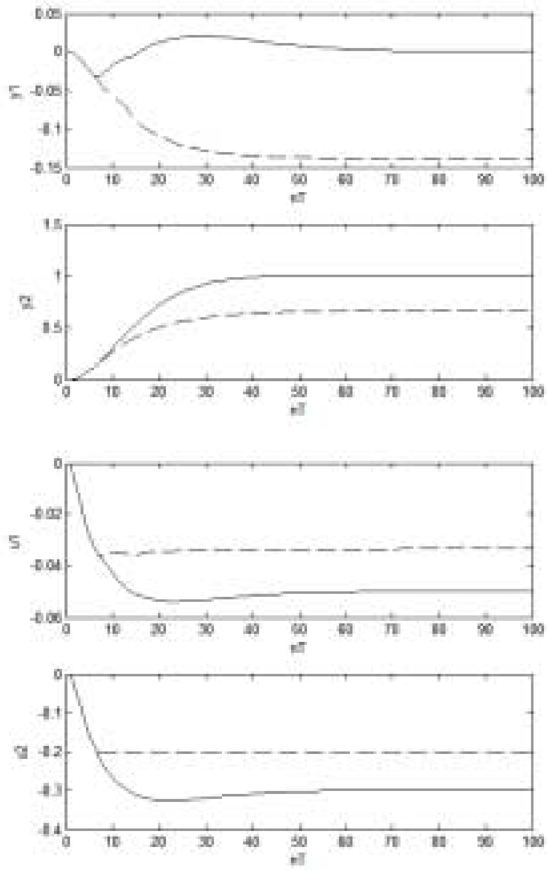

Fig.2. Robust MPC: Case 4 (——), Case 5 (- - - -)

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