ADAPTIVE ROBUST CONTROL FOR A CLASS OF UNCERTAIN TIME–DELAY SYSTEMS VIA OUTPUT FEEDBACK

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Abstract: The problem of robust stabilization is considered for a class of systems with the delayed state perturbations, uncertainties, and external disturbances. It is assumed that the upper bounds of the delayed state perturbations, uncertainties, and external disturbances, are unknown. An improved adaptation law with σ–modification is first introduced to estimate these unknown bounds. Then, by making use of the updated values of the unknown bounds, a class of adaptive robust output feedback controllers is proposed. On the basis of the strictly positive realness of the nominal system, it is also shown from the Kalman–Yakubovitch lemma that the solutions of the resulting adaptive closed–loop time-delay system can be guaranteed to be uniformly bounded, and the states decreases uniformly asymptotically to zero. Finally, a numerical example is given to demonstrate the validity of the results.

Keywords: Adaptive robust control, time–delay systems, asymptotic stability, output feedback.

1. INTRODUCTION

Many practical control problems, such as those arising in chemical processes, hydraulics and rolling mills, involve time–delay systems, connected with measurement of system variables, physical properties of the equipment, signal transmission, and so on. The existence of delay is frequently a source of instability.

On the other hand, it is well known that some uncertain parameters and disturbances are often included in practical control systems with time–delay due to modeling errors, measurement errors, linearization approximations, and so on. Therefore, the problem of robust stabilization of uncertain dynamical systems with time–delay has received considerable attention of many researchers (see, e.g., (Cheres *et al.*, 1989), (Wu and Mizukami, 1994), (Wu and Mizukami, 1996), and the references therein).

In the control literature, for dynamical systems with the delayed state perturbations, uncertainties, and external disturbances, where the system state vector is available, the upper bounds of the vector norms on the delayed state perturbations, uncertainties, and external disturbances, are generally supposed to be known, and such bounds are employed to construct some types of stabilizing state feedback controllers (see, e.g., (Cheres *et al.*, 1989), (Wu and Mizukami, 1996) for time–delay systems). However, in a number of practical control problems, such bounds may be unknown, or be partially known. Therefore, for such a class of uncertain time–delay systems whose uncertainty bounds are partially known, adaptive control schemes should be introduced to update these unknown bounds (see, e.g., (Wu, 2000), (Wu, 2002), (Wu, 2004) for time–delay systems). On the other hand, in many practical control problems, the states of the systems to be controlled may also be unknown or cannot be measured. Therefore, an output feedback controller should be designed to control such a class of dynamical systems.

In this paper, the problem of robust stabilization is considered for a class of systems with the delayed state perturbations, uncertainties, and external disturbances. It is assumed that the upper bounds of the delayed state perturbations, uncertainties, and external disturbances, are unknown, and that the states of the systems to be controlled are not measured. The purpose of the paper is to develop a stabilizing adaptive robust output feedback controller. For this, an improved adaptation law with σ–modification is employed to estimate the unknown bounds of the delayed state perturbations, uncertainties, and external disturbances. Then, by making use of the updated values of these unknown bounds, a class of output feedback controllers is constructed . On the basis of the strictly positive realness of the nominal system, it is shown from the Kalman–Yakubovitch lemma that by using the proposed adaptive robust output feedback controller, the solutions of the resulting adaptive closed–loop time–delay system can be guaranteed to be uniformly bounded, and the states decreases uniformly asymptotically to zero.

2. PROBLEM FORMULATION

Consider a class of uncertain time–delay systems described by

$$\frac{dx(t)}{dt} = \Big[A + \Delta A(\upsilon,t)\Big]x(t) + \sum_{j=1}^{r} \Delta E_j(\zeta,t)x(t - h_j) + \Big[B + \Delta B(\xi,t)\Big]u(t) + q(\nu,t) \quad \text{(1a)}$$

$$y(t) = Cx(t) \quad \text{(1b)}$$

where $t \quad R$ is the “time”, $x(t) \quad R^n$ is the current value of the state, $u(t) \quad R^m$ is the control input, $y(t) \quad R^p$ is the output vector, A, B, C, are constant matrices of appropriate dimensions, $\Delta A(\cdot)$, $\Delta B(\cdot)$, $\Delta E_j(\cdot)$, $j = 1, 2, \ldots, r$, represent the system uncertainties and are assumed to be continuous in all their arguments, and the vector $q(\cdot)$ is the external disturbance, which is also assumed to be continuous in all their arguments. Moreover, the uncertain parameters $(\upsilon, \xi, \zeta, \nu)$ $\Psi \quad R^L$ are Lebesgue measurable and take values in a known compact bounding set Ω. In addition, the time delays h_j, $j = 1, 2, \ldots, r$, are assumed to be any positive constants which are not required to be known for the system designer.

The initial condition for system (1) is given by

$$x(t) = \chi(t), \qquad t \quad [t_0 - \bar{h},\, t_0] \quad (2)$$

where $\chi(t)$ is a continuous function on $[t_0 - \bar{h},\, t_0]$, and $\bar{h} := \max\{h_j,\, j = 1, 2, \ldots, r\}$.

Furthermore, a nominal system is described by

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t), \quad y(t) = Cx(t) \quad (3)$$

For the uncertain time–delay systems described above, an output feedback controller $u(t)$ is introduced as follows.

$$u(t) = p(y(t),\, t) \quad (4)$$

where $p(\cdot) : R^p \times R \quad R^m$ is a continuous function.

Now, the main objective of this paper is to synthesize an output feedback controller $u(t)$ that can guarantee the stability of system (1) in the presence of the delayed state perturbations, uncertainties, and external disturbances.

Assumption 2.1. The pair $\{A,\, B\}$ given in (1) is completely controllable.

Assumption 2.2. For all $(\upsilon,\, \xi,\, \zeta,\, \nu) \quad \Psi$, there exist some continuous and bounded matrix functions $H(\cdot)$, $H_j(\cdot)$, $E(\cdot)$, $w(\cdot)$, of appropriate dimensions such that

$$\begin{aligned}
\Delta A(\upsilon,t) &= B(t)H(\upsilon,t) \\
\Delta E_j(\zeta,t) &= B(t)H_j(\zeta,t), \quad j = 1, 2, \ldots, r \\
\Delta B(\xi,t) &= B(t)E(\xi,t) \\
q(\nu,t) &= B(t)w(\nu,t)
\end{aligned}$$

For convenience, the following notations are introduced which represent the bounds of the uncertainties and external disturbances.

$$\begin{aligned}
\rho(t) &:= \max_{\upsilon} \big\|H(\upsilon,t)\big\| \\
\rho_j(t) &:= \max_{\zeta} \big\|H_j(\zeta,t)\big\|, \quad j = 1, 2, \ldots, r \\
\mu(t) &:= \min_{\xi} \left[\frac{1}{2}\lambda_{\min}\left(E(\xi,t) + E^{\top}(\xi,t)\right) \right] \\
\rho_q(t) &:= \max_{\nu} \big\|w(\nu,t)\big\|
\end{aligned}$$

In this paper, the functions $\rho(t)$, $\rho_j(t)$, $\mu(t)$, $\rho_q(t)$ assumed to be unknown. Moreover, the uncertain $\rho(t)$, $\rho_j(t)$, $\mu(t)$, $\rho_q(t)$ are also assumed, without loss of generality, to be uniformly continuous and bounded for any $t \quad R^+$.

By employing the notations given above, for system (1), the following assumption is introduced.

Assumption 2.3. For every $t \quad t_0$, $\mu(t) > -1$.

Remark 2.1. It is well known that *Assumption 2.1* is standard and denotes the internally stabilizability of the nominal system, i.e., the system in the absence of uncertainties and external disturbances. *Assumption 2.2* defines the matching condition about the uncertainties and external disturbances, and is a rather standard assumption for robust control problem (see, e.g., (Cheres *et al.*, 1989), (Wu, 2000), (Wu and Mizukami, 1996), (Wu, 2002), (Choi and Kim, 1993)). On the other hand, Assumption 2.3 is also standard, and can be regarded as a necessary condition for robust stability of uncertain dynamical systems (see, e.g., (Cheres *et al.*, 1989), (Wu, 2000), (Choi and Kim, 1993), and the references relative to robust stabilization of uncertain systems).

Moreover, for the nominal systems described by (3), the following assumption is also introduced.

Assumption 2.4. For the nominal system described by (3), there exists a matrix $F \quad R^{m\times p}$ such that the transfer function matrix

$$T(s) \;=\; FC\big(sI - A\big)^{-1}B$$

is strictly feedback positive real (SFPR). Therefore, it follows from the Kalman–Yakubovitch lemma (see, e.g., (Narendra and Taylor, 1973), (Narendra and Annaswamy, 1989), (Khalil, 1996)) that there exist the matrices $P \quad R^{n\times n}$, $Q \quad R^{n\times n}$, $K \quad R^{m\times n}$, $P = P^\top > 0$, $Q = Q^\top > 0$ satisfying

$$(A + BK)^\top P + P\,(A + BK) \;=\; -\,Q \tag{5}$$

$$\mathrm{Re}\Big[\,\lambda\,(A + BK)\,\Big] \;<\; 0 \tag{6}$$

such that

$$FC \;=\; B^\top P \tag{7}$$

Remark 2.2. Assumption 2.4 means that the nominal system is output feedback stabilizable. Indeed, in order to guarantee the stability of an uncertain system by the output feedback controllers, it is necessary that its nominal system can be stabilized by using uncorrupted output signal.

Remark 2.3. In a recent paper (Wu, 2000), a memoryless adaptive robust state feedback controller is proposed for a class of uncertain time–delay systems. It should be pointed out that the systems considered in (Wu, 2000) do not involve the uncertainty of input gain, and external disturbances, and the adaptive robust state feedback controllers proposed in (Wu, 2000) stabilize the systems only in the sense of uniform ultimate boundedness. In this paper, the purpose of the paper is to propose a class of adaptive robust output feedback controllers for system (1). It will be also shown that by employing the proposed controllers, one can guarantee the asymptotic stability, instead of the ultimate boundedness, of the considered systems.

On the other hand, it follows from Assumption 2.1 that there exists always a matrix K such that (6) holds. In particular, if one chooses K as

$$K \;=\; -\,\big(1\,/\,2\big)\;\eta\,B^\top P$$

where η is any positive constant, then the Lyapunov equation, described by (5), is reduced to the algebraic Riccati equation of the form

$$A^\top P + PA - \eta PBB^\top P \;=\; -\,Q \tag{8}$$

It is obvious from Assumption 2.1 that for any positive definite matrix $Q \quad R^{n\times n}$, there exists an unique positive definite matrix $P \quad R^{n\times n}$ as the solution of (8).

3. MAIN RESULTS

In this section, since the bounds $\rho(t)$, $\rho_j(t)$, $\mu(t)$, $\rho_q(t)$ have been assumed to be uniformly continuous and bounded for any $t \quad R^+$, it can be supposed that there exist some positive constants ρ^*, ρ_j^*, μ^*, ρ_q^*, which are defined by

$$\rho^* := \max\big\{\,\rho(t)\;:\;t \quad R^+\,\big\} \tag{9a}$$

$$\rho_j^* := \max\big\{\,\rho_j(t)\;:\;t \quad R^+\,\big\} \tag{9b}$$

$$\mu^* := \min\big\{\,\mu(t)\;:\;t \quad R^+\,\big\} \;>\; -\,1 \tag{9c}$$

$$\rho_q^* := \max\big\{\,\rho_q(t)\;:\;t \quad R^+\,\big\} \tag{9d}$$

Here, it is worth pointing out that the constants ρ^*, ρ_j^*, μ^*, ρ_q^*, are still unknown. Therefore, such unknown bounds can not be directly employed to construct stabilizing output feedback controllers.

Without loss of generality, the following definition is also introduced :

$$\psi^* := \frac{1}{1+\mu^*}\left(1+\eta^{-1}\alpha\,(\rho^*)^2+\sum_{j=1}^{r}\eta^{-1}\alpha\,\big(\rho_j^*\big)^2\right) \tag{10a}$$

$$\phi^* := \frac{\rho_q^*}{1+\mu^*} \tag{10b}$$

where η are α are any positive constants. It is obvious from (10) that ψ^* and ϕ^* are two unknown positive constants.

Now, the following adaptive robust output feedback controller is proposed :

$$u(t) \;=\; p_1(y(t),\,t) + p_2(y(t),\,t) \tag{11a}$$

where $p_1(\,\cdot\,)$ and $p_2(\,\cdot\,)$ are given by

$$p_1(y(t), t) = -\,\frac{1}{2}\,\eta\hat{\psi}(t)Fy(t) \tag{11b}$$

$$p_2(y(t), t) = -\,\frac{\hat{\phi}^2(t)Fy(t)}{Fy(t)\quad\hat{\phi}(t)\;+\;\sigma(t)} \tag{11c}$$

and where $\sigma(t) \quad R^+$ is any positive uniform continuous and bounded function which satisfies

$$\lim_{t\to\infty}\int_{t_0}^{t}\sigma(\tau)d\tau \qquad \bar{\sigma}\;< \tag{11d}$$

where $\bar{\sigma}$ is any constant. In addition, $F \quad R^{m\times p}$ is the output feedback gain matrix.

In particular, $\hat{\psi}(t)$ and $\hat{\phi}(t)$ in (11) are, respectively, the estimates of the unknown ψ^* and ϕ^*, which are, respectively, updated by the following adaptive laws:

$$\frac{d\hat{\psi}(t)}{dt} = -\,\gamma_1\,\sigma(t)\hat{\psi}(t) + \gamma_1\eta\quad Fy(t)\quad^2 \tag{12a}$$

$$\frac{d\hat{\phi}(t)}{dt} = -\,\gamma_2\,\sigma(t)\hat{\phi}(t) + 2\gamma_2\quad Fy(t) \tag{12b}$$

where γ_1 and γ_2 are any positive constants, and the initial conditions $\hat{\psi}(t_0)$, $\hat{\phi}(t_0)$ are finite.

Thus, applying the output feedback controller given in (11) to (1) yields an uncertain closed–loop time–delay system of the form:

$$\begin{aligned}\frac{dx(t)}{dt} \;&=\; [A + \Delta A(\upsilon, t)]\, x(t) \\ &+ \sum_{j=1}^{r} \Delta E_j(\zeta, t) x(t - h_j) \\ &+ \Big[B + \Delta B(\xi, t)\Big] p(y(t),\, t) + q(\nu, t) \qquad (13)\end{aligned}$$

where $p(\cdot)$ is given in (11).

On the other hand, letting

$$\tilde{\psi}(t) \;=\; \hat{\psi}(t) - \psi^*, \quad \tilde{\phi}(t) \;=\; \hat{\phi}(t) - \phi^*$$

one can rewrite (12) as the following error system

$$\begin{aligned}\frac{d\tilde{\psi}(t)}{dt} &= - \gamma_1\, \sigma(t)\tilde{\psi}(t) + \gamma_1 \eta \quad Fy(t) \quad ^2 \\ &\quad - \gamma_1\, \sigma(t)\psi^* \qquad (14a)\end{aligned}$$

$$\begin{aligned}\frac{d\tilde{\phi}(t)}{dt} &= - \gamma_2\, \sigma(t)\tilde{\phi}(t) + 2\gamma_2 \quad Fy(t) \\ &\quad - \gamma_2\, \sigma(t)\phi^* \qquad (14b)\end{aligned}$$

In the following, by $(x, \tilde{\psi}, \tilde{\phi})(t)$ one denote a solution of the uncertain closed–loop time–delay system and the error system. Then, one can have the following theorem.

Theorem 3.1. Consider the adaptive closed–loop time–delay dynamical system, described by (13) and (14). Suppose that *Assumptions 2.1* to *2.4* are satisfied. Then, the solutions $(x, \tilde{\psi}, \tilde{\phi})$ $(t;\, t_0, x(t_0), \tilde{\psi}(t_0), \tilde{\phi}(t_0))$ to the closed-loop time–delay system described by (13) and the error system described by (14) are uniformly bounded and

$$\lim_{t \to \infty} x(t;\, t_0, x(t_0)) \;=\; 0 \qquad (15)$$

Proof: For the adaptive closed–loop time–delay system described by (13) and (14), one first define a Lyapunov–Krasovskii functional candidate as follows.

$$\begin{aligned}V(x, \Psi) &= x^\top(t) P x(t) \\ &+ \sum_{j=1}^{r} \alpha^{-1} \int_{t-h_j}^{t} x^\top(\tau) x(\tau) d\tau \\ &+ \frac{1}{2}\big(1 + \mu^*\big) \Psi^\top(t) \Gamma^{-1} \Psi(t) \qquad (16)\end{aligned}$$

where P is the solution to (8), $\Psi(\cdot) := [\psi(\cdot) \;\; \phi(\cdot)]^\top$, and $\Gamma^{-1} := \mathrm{diag}\{\, \gamma_1^{-1},\;\, \gamma_2^{-1}\, \}$.

Let $(x(t), \Psi(t))$ be the solution to (13) and (14) for $t \quad t_0$. Then by taking the derivative of $V(\cdot)$ along the trajectories of (13) and (14) it is obtained that for $t \quad t_0$,

$$\begin{aligned}\frac{dV(x, \Psi)}{dt} \;&=\; x^\top(t) \left[A^\top P + PA\right] x(t) \\ &+ 2x^\top(t) P \sum_{j=1}^{r} \Delta E_j(\zeta, t) x(t - h_j) \\ &+ 2x^\top(t) P \Delta A(\upsilon, t) x(t) \\ &- \eta \hat{\psi}(t) x^\top(t) P \Big[B + \Delta B(\xi, t)\Big] Fy(t) \\ &- \frac{2\hat{\phi}^2(t) x^\top(t) P \Big[B + \Delta B(\xi, t)\Big] Fy(t)}{Fy(t) \quad \hat{\phi}(t) \; + \; \sigma(t)} \\ &+ 2x^\top(t) P q(\nu, t) \\ &+ \sum_{j=1}^{r} \alpha^{-1} \Big[x^\top(t) x(t) - x^\top(t - h_j) x(t - h_j)\Big] \\ &+ \big(1 + \mu^*\big) \Psi^\top(t) \Gamma^{-1} \frac{d\Psi(t)}{dt} \qquad (17)\end{aligned}$$

From Assumption 2.4 and (7) one can obtain that

$$Fy(t) \;=\; FCx(t) \;=\; B^\top P x(t) \qquad (18)$$

Then, from Assumption 2.2, (17), and (18), by making use of some manipulations, one can obtain that for any $t \quad t_0$,

$$\begin{aligned}\frac{dV(x, \Psi)}{dt} \quad & - x^\top(t) \tilde{Q} x(t) + 2\big(1 + \mu^*\big) \sigma(t) \\ &+ \frac{1}{4}\big(1 + \mu^*\big) \sigma(t) \Big[\, |\psi^* |^2 + |\phi^* |^2 \Big] \qquad (19)\end{aligned}$$

where

$$\tilde{Q} \;:=\; Q - \alpha^{-1}(1 + r) I \;>\; 0 \qquad (20)$$

Moreover, letting

$$\begin{aligned}\tilde{x}(t) &:= \left[\, x^\top(t) \quad \tilde{\psi}(t) \quad \tilde{\phi}(t) \,\right]^\top \\ \tilde{\varepsilon} &:= \frac{1}{4}\big(1 + \mu^*\big) \Big(8 + |\psi^* |^2 + |\phi^* |^2\Big)\end{aligned}$$

one can obtain from (19) that for any $t \quad t_0$,

$$\frac{dV(\tilde{x}(t))}{dt} \quad - \lambda_{\min}(\tilde{Q}) \quad x(t) \quad ^2 + \tilde{\varepsilon}\sigma(t) \qquad (21)$$

On the other hand, in the light of (16), there always exist two positive constants $\delta_{\min}$ and $\delta_{\max}$ such that for any $t \quad t_0$,

$$\tilde{\gamma}_1(\quad \tilde{x}(t) \quad) \qquad V(\tilde{x}(t)) \qquad \tilde{\gamma}_2(\quad \tilde{x}(t) \quad) \qquad (22)$$

where

$$\begin{aligned}\tilde{\gamma}_1(\quad \tilde{x}(t) \quad) &:= \delta_{\min} \quad \tilde{x}(t) \quad ^2 \\ \tilde{\gamma}_2(\quad \tilde{x}(t) \quad) &:= \delta_{\max} \quad \tilde{x}(t) \quad ^2 \\ &+ \sum_{j=1}^{r} \alpha^{-1} h_j \sup_{\tau \in [t - h_j,\, t]} \quad x_j(\tau) \quad ^2\end{aligned}$$

Now, from (21) and (22), one want to show that the solutions $\tilde{x}(t)$ of (13) and (14) are uniformly bounded, and that the state $x(t)$ converges asymptotically to zero.

By the continuity of the systems described by (13) and (14), it is obvious that any solution $(x, \tilde{\psi}, \tilde{\phi})(t;\ t_0, x(t_0), \tilde{\psi}(t_0), \tilde{\phi}(t_0))$ of the system is continuous.

It follows from (21) and (22) that for any $t \quad t_0$,

$$
\begin{aligned}
0 \quad & \tilde{\gamma}_1(\ \tilde{x}(t)\ ) \quad V(\tilde{x}(t)) \\
&= V(\tilde{x}(t_0)) + \int_{t_0}^{t} \dot{V}(\tilde{x}(\tau))d\tau \\
& \tilde{\gamma}_2(\ \tilde{x}(t_0)\ ) - \int_{t_0}^{t} \tilde{\gamma}_3(\ x(\tau)\ )d\tau + \int_{t_0}^{t} \tilde{\varepsilon}\sigma(\tau)d\tau
\end{aligned}
\tag{23}
$$

where the scalar function $\tilde{\gamma}_3(\ x(t)\ )$ is defined as

$$
\tilde{\gamma}_3(\ x(t)\ ) := \lambda_{\min}(\tilde{Q})\ \ x(t)\ \ ^2 \tag{24}
$$

Therefore, from (23) one can obtain the following two results. First, taking the limit as t approaches infinity on both sides of inequality (23), one can obtain that

$$
\lim_{t\to\infty} \int_{t_0}^{t} \tilde{\gamma}_3(\ x(\tau)\ )d\tau \quad \tilde{\gamma}_2(\ \tilde{x}(t_0)\ ) + \tilde{\varepsilon}\bar{\sigma} \tag{25}
$$

On the other hand, from (23) one also have

$$
0 \quad \tilde{\gamma}_1(\ \tilde{x}(t)\ ) \quad \tilde{\gamma}_2(\ \tilde{x}(t_0)\ ) + \tilde{\varepsilon}\bar{\sigma} \tag{26}
$$

which implies that $\tilde{x}(t)$ is uniformly bounded. Since $\tilde{x}(t)$ has been shown to be continuous, it follows that $\tilde{x}(t)$ is uniformly continuous, which implies that $x(t)$ is uniformly continuous. Therefore, it follows from the definition that $\tilde{\gamma}_3(\ x(t)\ )$ is also uniformly continuous. Applying the Barbalat lemma (Slotine and Li, 1991) to inequality (25) yields that

$$
\lim_{t\to\infty} \tilde{\gamma}_3(\ x(t)\ ) = 0 \tag{27}
$$

Furthermore, since $\tilde{\gamma}_3(\cdot)$ is a positive definite scalar function, it is obvious from (27) that one can have (15). Thus, one can complete the proof of this theorem.

Remark 3.1. In the proof of *Theorem 3.1*, it is assumed for the constant α to satisfy (20). However, the adaptive output feedback controllers given in (11) with (12) are independent of this constant. Thus, it is not necessary for the designer to know or choose the constant α. In fact, the control gain adjusts automatically to counter the destabilizing effects of the delayed state perturbations, uncertainties, and disturbances.

Remark 3.2. The proposed robust output feedback control laws are memoryless, and the adaptive schemes given in (12) are independent of the time delays. Therefore, in the light of the proof given above, it can be known that the time–delay constants $h_j,\ j = 1, 2, \ldots, r$, are not required to be known for the system designer.

4. ILLUSTRATIVE EXAMPLE

Consider the following numerical example.

$$
\begin{aligned}
\frac{dx(t)}{dt} &= \left(\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} + \Delta A(\upsilon, t) \right) x(t) \\
&+ \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \Delta B(\xi, t) \right) u(t) \\
&+ \sum_{j=1}^{3} \Delta E_j(\zeta, t) x(t - h_j) + q(\nu, t)
\end{aligned}
\tag{28a}
$$

$$
y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(t) \tag{28b}
$$

where

$$
\Delta A(\upsilon, t) = \begin{bmatrix} 0 & 0 \\ \upsilon(t) & 2\upsilon(t) \end{bmatrix}, \quad \Delta B(\xi, t) = \begin{bmatrix} 0 \\ \xi(t) \end{bmatrix}
$$

$$
\Delta E_1(\zeta, t) = \begin{bmatrix} 0 & 0 \\ 0.2\zeta(t) & 0.5\zeta(t) \end{bmatrix}
$$

$$
\Delta E_2(\zeta, t) = \begin{bmatrix} 0 & 0 \\ 0 & 0.3\zeta(t) \end{bmatrix}
$$

$$
\Delta E_3(\zeta, t) = \begin{bmatrix} 0 & 0 \\ 0.5\zeta(t) & 0 \end{bmatrix}, \quad q(\nu, t) = \begin{bmatrix} 0 \\ 0.5\nu(t) \end{bmatrix}
$$

The problem is to determine a control law in the form (11) with (12), that will stabilizes the time–delay system described by (28) in the presence of the delayed state perturbations, uncertainties, and external disturbances.

It can known from (28) that if the output control gain matrix is given by

$$
F = 2
$$

then its transfer function matrix

$$
T(s) = FC(sI - A)^{-1}B
$$

is strictly positive real ((Narendra and Taylor, 1973), (Narendra and Annaswamy, 1989), (Khalil, 1996)).

Thus, one can construct an adaptive robust output feedback controller. In this numerical example, for the adaptation laws and output feedback controller, one select the following parameters:

$$
\eta = 2, \qquad \gamma_1 = 0.5, \qquad \gamma_2 = 0.2
$$

$$
\sigma(t) = 20 \exp\{\ -0.5t\ \}
$$

Therefore, for uncertain time–delay system (28), from (11) with (12) one can obtain an adaptive robust output feedback controllers, by which the system state $x(t)$ can decrease uniformly asymptotically to zero.

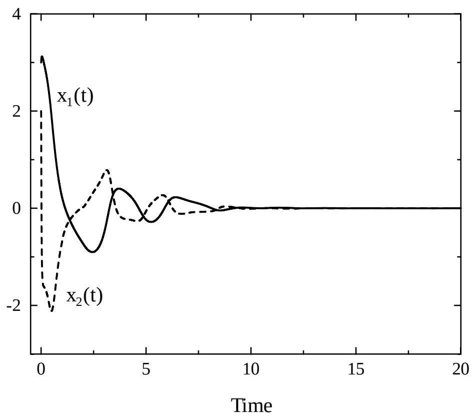

Fig. 1. Response of state variable $x(t)$.

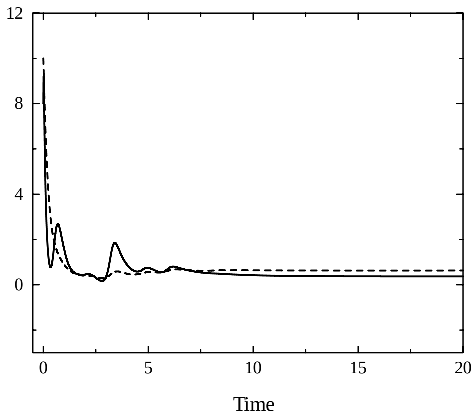

Fig. 2. History of the updating parameters $\hat{\psi}(t)$ (solid) and $\hat{\phi}(t)$ (dash).

For simulation, the uncertain parameters and initial conditions are given as follows.

$$\upsilon(t) = 0.2\sin(3t), \qquad \xi = 0.3\cos(5t)$$
$$\zeta = 0.5\sin(2t), \qquad \nu = 0.2\sin(3t)$$
$$h_1 = 1.0, \qquad h_2 = 2.0, \qquad h_3 = 3.0$$
$$x(t) = [3.0\cos(t)\ \ 2.0\cos(t)]^\top, \quad t \quad [-\bar{h},\ 0]$$
$$\hat{\psi}(0) = 8.0, \qquad \hat{\phi}(0) = 10.0$$

With the chosen parameter settings, the results of simulation are shown in *Fig.1* and *Fig.2* for this numerical example.

It can be observed from *Fig.1* that the adaptive robust output feedback controllers can indeed stabilize system (34), and the states $x(t)$ decrease asymptotically to zero. On the other hand, it can be known from *Fig.2* that similar to the conventional adaptation laws with σ–modification, the improved ones make the estimate values of the unknown parameters decreasing.

5. CONCLUDING REMARKS

The problem of robust stabilization has been considered for a class of systems with the delayed state perturbations, uncertainties, and external disturbances. In this paper, It has been assumed that the upper bounds of the delayed state perturbations, uncertainties, and external disturbances, are unknown, and that the states of the systems to be controlled are not measured. An improved adaptation law with σ–modification have been introduced to estimate these unknown bounds. Then, by making use of the updated values of these unknown bounds. a class of adaptive robust output feedback controllers has been constructed. It has also been shown from the Kalman–Yakubovitch lemma that the solutions of the resulting adaptive closed–loop time-delay dynamical system can be guaranteed to be uniformly bounded, and the states decreases uniformly asymptotically to zero.

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