GENERALIZED PREDICTIVE CONTROL IN FAST-RATE SINGLE-RATE AND INPUT MULTIPLEX TYPE MULTIRATE SYSTEM

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Abstract: This paper discusses the designs of Generalized Predictive Control (GPC) scheme. GPC is designed in two cases; the first is a multirate system, where the sampling interval of a plant output is an integer multiple of the holding interval of a control input, and the second is a fast-rate single-rate system, where both the holding and sampling intervals are equivalent to the holding interval of the multirate system. Furthermore, the relation between them is investigated. This study gives the conditions that the fast-rate single-rate and the multirate GPC become equivalent.

Keywords: generalized predictive control, single-rate system, multirate system, lifting, fast-rate system, single-rate system

1. INTRODUCTION

In digital control a system is called a single-rate system, where the holding interval of a control input is equal to the sampling interval of a plant output. On the other hand, a system is called a multirate system, where these intervals are not equal. The multirate system is also called as a dual-rate system. This paper discusses a single-input single-output multirate linear time-invariant system, where the sampling interval of a plant output is an integer multiple of the holding interval of a control input. In this case, a control input is updated at a fast-rate, but a plant output is sampled at a slow-rate.

Generally, a fast-rate single-rate system, where input and output intervals are equivalent and are sampled or updated at a fast-rate, is superior to a slow-rate single-rate system, where input and output intervals are equivalent and are sampled or updated at a slow-rate. However, these intervals cannot be always set arbitrarily due to constraints or specifications of actuators or sensors. Then, a fast-rate single-rate system cannot be obtained, but only a slow-rate single-rate or a dual-rate system can be realized. In that case it is expected that a multirate system is better than the slow-rate single-rate system. The design of the dual-rate system is obtained by a similar way to the design of the slow-rate single-rate system, that is, first, the dual-rate system can be transformed into a multi-input single-output slow-rate single-rate system using the lifting (Chen and Francis, 1995), then the dual-rate system is designed the same as the slow-rate single-rate system.

However, in the dual-rate system, since ripples may emerge between sampled outputs (Tangirala *et al.*, 1999), and the ripples should be suppressed. Generalized Predictive Control (GPC) (Clarke *et al.*, 1987*a*; Clarke *et al.*, 1987*b*) in a multirate system was proposed (Scattolini, 1992), and the ripples can be suppressed by using this multirate GPC because an integrator is included in the process of the design. Sheng *et al.* extended the design method of GPC considering intersample performance (Masuda *et al.*, 1997) into a multirate system (Sheng *et al.*, 2001*a*). Because this design method takes care of intersample output, the ripples are suppressed. In this paper, to suppress the ripples, GPC is designed considering the variation in control input between sampled instants. This paper also discusses the relation between the fast-rate single-rate and the dual-rate GPC and shows the conditions that the fast-rate single-rate and the dual-rate GPC are equivalent. This study results in an unified design method of the fast-rate single-rate and the dual-rate GPC.

This paper is organized as follows. In section 2, a plant model is given. A fast-rate single-rate and a dual-rate GPC law are derived in section 3 and 4, respectively. By comparing the GPC laws in the fast-rate single-rate and the dual-rate system, the relation between them is shown in section 5.

2. PLANT MODEL

Consider a plant with the following discrete-time model

$$\bar{\boldsymbol{x}}[k+1] = \bar{A}\bar{\boldsymbol{x}}[k] + \bar{\boldsymbol{b}}u[k] \tag{1}$$

$$y[k] = \bar{\boldsymbol{c}}^T\bar{\boldsymbol{x}}[k] \tag{2}$$

where, $u[k]$ and $y[k]$ the control input and the plant output, and n-th order vector $\bar{\boldsymbol{x}}[k]$ is the state variable. $\bar{A}$, $\bar{\boldsymbol{b}}$ and $\bar{\boldsymbol{c}}^T$ are an $n \times n$ matrix and n-th vectors, respectively.

The extended model with an integrator is given as (Scattolini, 1992)

$$\boldsymbol{x}[k+1] = A\boldsymbol{x}[k] + \boldsymbol{b}\Delta u[k] \tag{3}$$

$$y[k] = \boldsymbol{c}^T\boldsymbol{x}[k] \tag{4}$$

where,

$$A = \begin{bmatrix} \bar{A} & \bar{\boldsymbol{b}} \\ \boldsymbol{0}_{1,n} & 1 \end{bmatrix} \tag{5}$$

$$\boldsymbol{b} = \begin{bmatrix} \bar{\boldsymbol{b}} \\ 1 \end{bmatrix} \tag{6}$$

$$\boldsymbol{c}^T = \begin{bmatrix} \bar{\boldsymbol{c}}^T & 0 \end{bmatrix} \tag{7}$$

$$\boldsymbol{x}[k] = \begin{bmatrix} \bar{\boldsymbol{x}}[k] \\ u[k-1] \end{bmatrix} \tag{8}$$

$$\Delta = 1 - z^{-1}. \tag{9}$$

z^{-1} is the one-step backward shift operator.

When the plant output is sampled every step and the control input is updated every step, the control system is called a fast-rate single-rate system, and when both of the intervals are l steps, the system is called a slow-rate single-rate system. On the other hand, the system is called a dual-rate system when the control input is updated every step, but the plant output is sampled at interval of l steps.

The following are assumed in this paper.

[A.1] The model of a plant is known.
[A.2] A reference input is given as a step type.
[A.3] The control input is updated every step.
[A.4] In the fast-rate single-rate system, the plant output is sampled every step.
[A.5] In the dual-rate system, the plant output is sampled every l steps.

The fast-rate single-rate system is simply described as the single-rate system hereafter.

3. SINGLE-RATE GPC

In this section a GPC law is derived in the single-rate system.

3.1 Performance function

The performance function of the single-rate GPC is given by the following

$$J_s = E\left[\sum_{j=N_{s,1}}^{N_{s,2}} \mu_{s,j}\{y[k+j] - w[k+j]\}^2 + \sum_{j=1}^{N_{s,u}} \lambda_{s,j}\Delta u[k+j-1]^2\right] \tag{10}$$

where, $N_{s,1}$, $N_{s,2}$ and $N_{s,u}$ are minimum prediction horizon, maximum prediction horizon and control horizon, respectively. $\mu_{s,j}$ is a weighting factor of the error between the reference input $w[k+j]$ and the plant output, and $\lambda_{s,j}$ is a weighting factor of the variation in the control input, respectively. A GPC law minimizing J_s is derived in the single-rate system.

3.2 Reference Input

The reference input in the single-rate GPC is given as follows (Clarke *et al.*, 1987*a*).

$$w[k] = y[k] \tag{11}$$

$$w[k+j] = (1-\alpha_s)r + \alpha_s w[k+j-1] \tag{12}$$

$$(0 \le \alpha_s < 1)$$

where, r is the set-point, and α_s is a design parameter. The future reference input is rewritten by (13) when the set-point is given as step type (Sato and Inoue, 2006).

$$w[k+j] = \alpha_s^j y[k] + (1-\alpha_s^j) r \tag{13}$$

3.3 Predictive Output

One step and two steps forward predictive output are given as

$$y[k+1] = \boldsymbol{c}^T A \boldsymbol{x}[k] + \boldsymbol{c}^T \boldsymbol{b} \Delta u[k] \tag{14}$$

$$y[k+2] = \boldsymbol{c}^T A^2 \boldsymbol{x}[k] + \boldsymbol{c}^T A \boldsymbol{b} \Delta u[k] + \boldsymbol{c}^T \boldsymbol{b} \Delta u[k+1]. \tag{15}$$

Repeating these calculations, j steps forward predictive output is calculated by

$$y[k+j] = \boldsymbol{c}^T A^j \boldsymbol{x}[k] + \boldsymbol{c}^T \sum_{i=0}^{j-1} A^i \boldsymbol{b} \Delta u[k+j-1-i]. \tag{16}$$

3.4 Derivation of Control Law

The future control input series minimizing (10) is given as follows.

$$\Delta \boldsymbol{u}_s[k] = (G_s^T M_s G_s + \Lambda_s)^{-1} \times G_s^T M_s (\boldsymbol{w}_s[k] - H_s \boldsymbol{x}[k]) \tag{17}$$

where,

$$\Delta \boldsymbol{u}_s[k] = \begin{bmatrix} \Delta u[k] \\ \Delta u[k+1] \\ \vdots \\ \Delta u[k+N_{s,u}-1] \end{bmatrix} \tag{18}$$

$$\boldsymbol{w}_s[k] = \begin{bmatrix} w[k+N_{s,1}] \\ w[k+N_{s,1}+1] \\ \vdots \\ w[k+N_{s,2}] \end{bmatrix} \tag{19}$$

$$H_s = \begin{bmatrix} \boldsymbol{c}^T A^{N_{s,1}} \\ \boldsymbol{c}^T A^{N_{s,1}+1} \\ \vdots \\ \boldsymbol{c}^T A^{N_{s,2}} \end{bmatrix} \tag{20}$$

$$\Lambda_s = \mathrm{diag}\{\lambda_{s,1}, \lambda_{s,2}, \cdots, \lambda_{s,N_{s,u}}\} \tag{21}$$

$$M_s = \mathrm{diag}\{\mu_{s,N_{s,1}}, \mu_{s,N_{s,1}+1}, \cdots, \mu_{s,N_{s,2}}\} \tag{22}$$

$$(i,j) \text{ element of } G_s = \begin{cases} \boldsymbol{c}^T A^{N_{s,1}+i-1-j} \boldsymbol{b} & (N_{s,1}+i-1-j \geq 0) \\ 0 & (N_{s,1}+i-1-j < 0) \end{cases}. \tag{23}$$

The standard single-rate GPC utilizes only the first element of the obtained control inputs $\Delta \boldsymbol{u}_s[k]$ because of the use of Receding Horizon.

On the other hand, in order to show the relation with the dual-rate GPC derived in the next section, the first l elements ($\Delta \boldsymbol{u}[k]$) of $\Delta \boldsymbol{u}_s[k]$ are utilized in this paper; therefore the following control law is obtained.

$$\Delta \boldsymbol{u}[k] = \left[\, I_l \ \boldsymbol{0}_{l,(N_{s,u}-l)} \,\right] (G_s^T M_s G_s + \Lambda_s)^{-1} \times G_s^T M_s (\boldsymbol{w}_s[k] - H_s \boldsymbol{x}[k]) \tag{24}$$

where,

$$\Delta \boldsymbol{u}[k] = \begin{bmatrix} \Delta u[k] \\ \Delta u[k+1] \\ \vdots \\ \Delta u[k+l-1] \end{bmatrix}. \tag{25}$$

4. DUAL-RATE GPC

In the dual-rate system, the plant output is sampled every l steps due to the assumption. Hence, the dual-rate system is transformed into the slow-rate single-rate system by using the lifting (Chen and Francis, 1995), and a GPC law is derived as l-inputs single-output single-rate system.

4.1 Lifted System

Using the lifting (Chen and Francis, 1995), single-input single-output fast-rate single-rate system (3) and (4) is transformed into the following l-inputs single-output slow-rate single-rate system.

$$\boldsymbol{x}[k+l] = A_l \boldsymbol{x}[k] + B_l \Delta \boldsymbol{u}[k] \tag{26}$$

$$y[k] = \boldsymbol{c}^T \boldsymbol{x}[k] \tag{27}$$

where,

$$A_l = A^l \tag{28}$$

$$B_l = \left[\, A^{l-1}\boldsymbol{b} \ A^{l-2}\boldsymbol{b} \ \cdots \ A\boldsymbol{b} \ \boldsymbol{b} \,\right]. \tag{29}$$

The dual-rate GPC is designed using this lifted single-rate system.

4.2 Performance function

The dual-rate GPC derives future control input series minimizing the following performance function.

$$J_m = E\left[\sum_{j=N_{m,1}}^{N_{m,2}} \mu_{m,j} \{y[k+jl] - w[k+jl]\}^2 + \sum_{j=1}^{N_{m,u}} ||\Delta \boldsymbol{u}[k+(j-1)l]||^2_{\bar{\Lambda}_{m,j}} \right] \tag{30}$$

$$\bar{\Lambda}_{m,j} = \mathrm{diag}\{\lambda_{m,(j-1)l+1}, \lambda_{m,(j-1)l+2}, \cdots, \lambda_{m,jl}\} \tag{31}$$

where, $N_{m,1}$, $N_{m,2}$ and $N_{m,u}$ are minimum prediction horizon, maximum prediction horizon and control horizon of the dual-rate GPC, respectively. $\mu_{m,j}$ and $\lambda_{m,j}$ are weighting factors of the error and the variation in the control input, respectively.

4.3 Reference Input

In designing the dual-rate GPC, a reference input is given by the following.

$$w[k] = y[k] \quad (32)$$

$$w[k+jl] = (1-\alpha_m)r + \alpha_m w[k+(j-1)l] \quad (33)$$

$$(0 \leq \alpha_m < 1)$$

The reference input in the dual-rate system is rewritten by (34) (Sato and Inoue, 2005).

$$w[k+jl] = \alpha_m^j y[k] + (1-\alpha_m^j)r \quad (34)$$

4.4 Predictive Output

In the dual-rate system, the plant output is sampled at interval of l steps, and l steps and $2l$ steps forward predictive output are given as

$$y[k+l] = \boldsymbol{c}^T A_l \boldsymbol{x}[k] + \boldsymbol{c}^T B_l \Delta \boldsymbol{u}[k] \quad (35)$$

$$y[k+2l] = \boldsymbol{c}^T A_l^2 \boldsymbol{x}[k] + \boldsymbol{c}^T A_l B_l \Delta \boldsymbol{u}[k] + \boldsymbol{c}^T B_l \Delta \boldsymbol{u}[k+l]. \quad (36)$$

Repeating these calculations, jl steps forward predictive output of the lifted system is calculated by

$$y[k+jl] = \boldsymbol{c}^T A_l^j \boldsymbol{x}[k] + \boldsymbol{c}^T \sum_{i=0}^{j-1} A_l^i B_l \Delta \boldsymbol{u}[k+(j-1-i)l]. \quad (37)$$

4.5 Derivation of Control Law

With the predictive output (37), the future control input series minimizing the performance function is obtained by the followings (Sheng *et al.*, 2001*b*).

$$\Delta \boldsymbol{u}_m[k] = (G_m^T M_m G_m + \Lambda_m)^{-1} \times G_m^T M_m(\boldsymbol{w}_m[k] - H_m \boldsymbol{x}[k]) \quad (38)$$

where,

$$\Delta \boldsymbol{u}_m[k] = \begin{bmatrix} \Delta \boldsymbol{u}[k] \\ \Delta \boldsymbol{u}[k+l] \\ \vdots \\ \Delta \boldsymbol{u}[k+(N_{m,u}-1)l] \end{bmatrix} \quad (39)$$

$$\boldsymbol{w}_m[k] = \begin{bmatrix} w[k+N_{m,1}l] \\ w[k+(N_{m,1}+1)l] \\ \vdots \\ w[k+N_{m,2}l] \end{bmatrix} \quad (40)$$

$$H_m = \begin{bmatrix} \boldsymbol{c}^T A_l^{N_{m,1}} \\ \boldsymbol{c}^T A_l^{N_{m,1}+1} \\ \vdots \\ \boldsymbol{c}^T A_l^{N_{m,2}} \end{bmatrix} \quad (41)$$

$$\Lambda_m = \text{block diag}\{\bar{\Lambda}_{m,1}, \bar{\Lambda}_{m,2}, \cdots, \bar{\Lambda}_{s,N_{m,u}}\} \quad (42)$$

$$M_m = \text{diag}\{\mu_{m,N_{m,1}}, \mu_{m,N_{m,1}+1}, \cdots, \mu_{m,N_{m,2}}\} \quad (43)$$

(i,j) block of block matrix $G_m =$

$$\begin{cases} \boldsymbol{c}^T A_l^{N_{m,1}-1+i-j} B_l & (N_{m,1}+i-1-j \geq 0) \\ \boldsymbol{0}_{1,l} & (N_{m,1}+i-1-j < 0) \end{cases}. \quad (44)$$

Because the lifted single-rate system is l-inputs system, the first l elements of $\Delta \boldsymbol{u}_m[k]$, that is $\Delta \boldsymbol{u}[k]$, are utilized. Thus, multiplying both sides of (38) by $[I_l \ \ \boldsymbol{0}_{l,(N_{m,u}-1)}]$ from the left, the next dual-rate GPC law is obtained.

$$\Delta \boldsymbol{u}[k] = \left[\, I_l \ \ \boldsymbol{0}_{l,(N_{m,u}-1)} \,\right] (G_m^T M_m G_m + \Lambda_m)^{-1} \times G_m^T M_m(\boldsymbol{w}_m[k] - H_m \boldsymbol{x}[k]) \quad (45)$$

5. CONDITIONS

In this section, we give the conditions that the performance functions in the single-rate and the dual-rate system become equivalent.

With (16), jl steps forward of the prediction output in the single-rate system is rewritten as

$$y[k+jl] = \boldsymbol{c}^T A^{jl} \boldsymbol{x}[k] + \boldsymbol{c}^T \sum_{i=0}^{jl-1} A^i \boldsymbol{b} \Delta u[k+jl-1-i]. \quad (46)$$

Further the second term in the right hand of (46) is rewritten as

$$\begin{aligned} &\boldsymbol{c}^T \sum_{i=0}^{jl-1} A^i \boldsymbol{b} \Delta u[k+jl-1-i] \\ &= \boldsymbol{c}^T \sum_{i=0}^{j-1} \sum_{h=il}^{(i+1)l-1} A^h \boldsymbol{b} \Delta u[k+jl-1-h] \\ &= \boldsymbol{c}^T \sum_{i=0}^{j-1} A_l^i B_l \Delta \boldsymbol{u}[k+(j-1-i)l]. \quad (47) \end{aligned}$$

Hence, it follows from (46) and (47) that jl steps forward predictive outputs of the fast-rate single-rate and the dual-rate GPC are equivalent.

It follows from (13) and (34) that the weighting factor of the reference input is determined by

$$\alpha_s^l = \alpha_m. \quad (48)$$

Then, the reference input in the single-rate and the dual-rate system are equivalent.

In the dual-rate system, because the plant output is sampled at interval of l steps, the weighting factor $\mu_{s,j}$ of the single-rate GPC is designed as (49) so as to evaluate the control error between the reference input and the future predictive output at the same step as the dual-rate system.

$$\mu_{s,j} = \begin{cases} \mu_{m,j/l} & (\mathrm{rem}(j,l) = 0) \\ 0 & (\text{others}) \end{cases} \quad (49)$$

where j is divisible by l with remainder $\mathrm{rem}(j, l)$.

The following conditions of prediction and control horizon are required so that the performance functions of the dual-rate and the single-rate GPC become equivalent.

$$N_{s,1} = lN_{m,1} \quad (50)$$
$$N_{s,2} = lN_{m,2} \quad (51)$$
$$N_{s,u} = lN_{m,u} \quad (52)$$

(53) gives the condition that the weighting factor of the control input in the single-rate and the dual-rate system are equivalent.

$$\lambda_{s,j} = \lambda_{m,j} \quad (j = 1, \cdots, N_{s,u}) \quad (53)$$

where $N_{s,u}$ is designed so that (52) is satisfied.

If the design parameters of GPC satisfy the conditions (48) $\sim$ (53), the single-rate and the dual-rate GPC law become equivalent. As mentioned above, the dual-rate GPC law can be taken as the fast-rate single-rate GPC law satisfying the conditions. Further, l control inputs of the dual-rate GPC designed as the slow-rate single-rate system using the lifting can be equivalent to the first l elements of the future control input series of the fast-rate single-rate GPC.

6. NUMERICAL EXAMPLE

Consider a plant described by the following linear time-invariant continuous-time model.

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} -0.14 & -0.0040 \\ 1.00 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \quad (54)$$
$$y(t) = \begin{bmatrix} 0 & 0.0040 \end{bmatrix} \boldsymbol{x}(t) \quad (55)$$

We will show the responses with the single-rate and the dual-rate GPC. The simulation length is $50[s]$, and the set-point r is 1. The control input is updated at interval of $1[s]$, but the plant output is sampled at interval of $2[s]$. The dual-rate GPC is designed first, and then the single-rate GPC whish is equivalent to the dual-rate GPC is designed.

The design parameters are set as: $N_{m,1} = 1$, $N_{m,2} = 5$, $N_{m,u} = 2$, $\alpha_m = 0.75^2$, $\lambda_{m,j} = 0.005$ $(j = 1, \cdots, N_{m,u})$, $\mu_{m,j} = 1$ $(j = N_{m,1}, \cdots, N_{m,2})$. With these parameters the dual-rate GPC law is given by the following.

$$\Delta\boldsymbol{u}[k] = \begin{bmatrix} 0.71 & 1.6 & 1.9 & 1.6 & 0.98 \\ -0.20 & 0.20 & 0.64 & 1.1 & 1.5 \end{bmatrix} \times \left(\begin{bmatrix} 0.56y[k] + 0.44r \\ 0.32y[k] + 0.68r \\ 0.18y[k] + 0.82r \\ 0.10y[k] + 0.90r \\ 0.056y[k] + 0.94r \end{bmatrix} - \begin{bmatrix} 0.0070 & 0.0040 & 0.0073 \\ 0.012 & 0.0039 & 0.027 \\ 0.016 & 0.0038 & 0.055 \\ 0.019 & 0.0036 & 0.089 \\ 0.020 & 0.0035 & 0.13 \end{bmatrix} \boldsymbol{x}[k] \right) \quad (56)$$

Next, the single-rate GPC law is designed using (48) $\sim$ (53). The design parameters of the single-rate GPC is set as: $N_{s,1} = 2$, $N_{s,2} = 10$, $N_{s,u} = 4$, $\alpha_s = 0.75$, $\lambda_{s,j} = 0.005$ $(j = 1, \cdots, N_{s,u})$,

$$\mu_{s,j} = \begin{cases} 1 & (\mathrm{rem}(j,2) = 0) \\ 0 & (\text{others}) \end{cases} \quad (57)$$

$(j = N_{s,1}, \cdots, N_{s,2})$. Then, the dual-rate GPC law is given by the following.

$$\Delta\boldsymbol{u}[k] = \begin{bmatrix} 0.71 & 0 & 1.63 & 0 & 1.9 & 0 & 1.6 & 0 & 0.98 \\ -0.20 & 0 & 0.20 & 0 & 0.64 & 0 & 1.1 & 0 & 1.5 \end{bmatrix} \times \left(\begin{bmatrix} 0.56y[k] + 0.44r \\ 0 \\ 0.32y[k] + 0.68r \\ 0 \\ 0.18y[k] + 0.82r \\ 0 \\ 0.10y[k] + 0.90r \\ 0 \\ 0.056y[k] + 0.94r \end{bmatrix} - \begin{bmatrix} 0.0070 & 0.0040 & 0.0073 \\ 0 & 0 & 0 \\ 0.012 & 0.0039 & 0.027 \\ 0 & 0 & 0 \\ 0.016 & 0.0038 & 0.055 \\ 0 & 0 & 0 \\ 0.019 & 0.0036 & 0.089 \\ 0 & 0 & 0 \\ 0.020 & 0.0035 & 0.13 \end{bmatrix} \boldsymbol{x}[k] \right) \quad (58)$$

Using the derived control laws the plant is controlled. Output and input results are illustrated in Fig. 1 and Fig. 2, respectively. The sampled outputs in the single-rate and the dual-rate system are plotted by dot and circle in Fig. 1, respectively. Because the dual-rate and the single-rate GPC law calculate the same control inputs shown in Fig. 2, the sampled outputs at interval of $2[s]$ are the same. It follows from the simulation results that the single-rate GPC law is designed equivalent to the dual-rate GPC law using (48) $\sim$ (53).

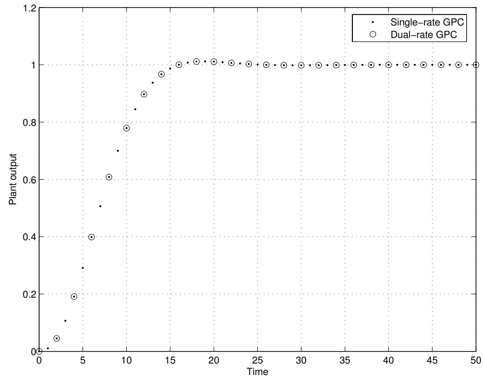

Fig. 1. Output results with the single-rate and the dual-rate GPC

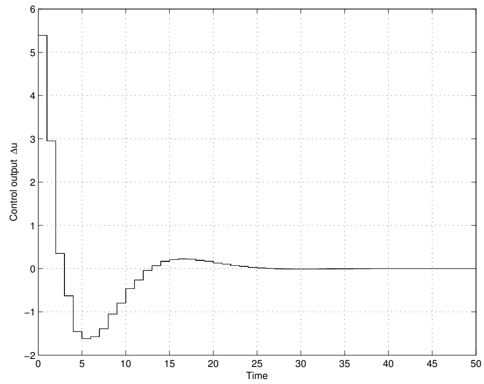

Fig. 2. Input results with the single-rate and the dual-rate GPC

7. CONCLUSION

We designed GPC in both a fast-rate single-rate and a dual-rate system and discussed the relation between them, and the conditions for identifying the fast-rate single-rate GPC as the dual-rate GPC were shown. It follows from our research result that the fast-rate single-rate GPC can be equivalent to the dual-rate GPC by selecting the design parameters. Consequently, the dual-rate GPC can be designed by the same way as the standard single-rate system. Further, the relation between the fast-rate single-rate and the dual-rate GPC is made clear. Finally, in order to illustrate the effectiveness of the proposed method, the numerical examples have been shown.

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8. ACKNOWLEDGMENT

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